Discussion of Coupling

David Robin

Outline

- **Motivation**
- **Coupling resonance**
- **Resonant Excitation**

Coupling

Skew quadrupole field errors generate betatron coupling between horizontal and vertical equations of motion.

4x4 transfer matrix for a quadrupole rotated by a small angle ϕ

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -q & 1 & -2q\phi & 0 \\ 0 & 0 & 1 & 0 \\ -2q\phi & 0 & q & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}$$

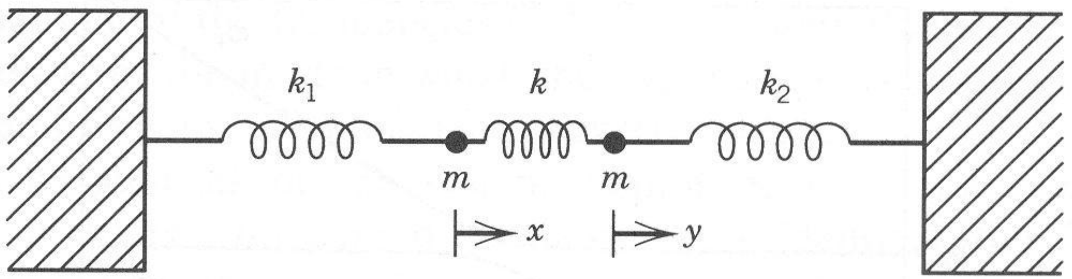
Coupled Motion

Coupled equations

$$x'' - Kx = -K_s y \qquad y'' + Ky = -K_s x$$

$$K = \frac{1}{B\rho}\frac{\partial B_y}{\partial x} \qquad K_s = \frac{1}{B\rho}\frac{\partial B_x}{\partial x}$$

Analogy with springs



$$m\ddot{x} + (k_1 + k)x - ky = 0,$$

$$m\ddot{y} + (k_2 + k)y - kx = 0,$$

Exciting the linear coupling resonance

- Resonance theory (Guignard, CERN 76-06 1976)
  - Difference coupling resonance (that skew quad spatial harmonic that samples horizontal oscillations to resonantly drive vertical oscillations.)

$$\kappa = \frac{1}{4\pi}\int ds \; K_s \sqrt{\beta_x \beta_y}\, e^{i\phi_D}$$

$$\frac{\phi_D}{2\pi} = \mu_x(s) - \mu_y(s) - \frac{s}{C}\Delta_r \qquad \Delta_r = (\nu_x - \nu_y - N)$$

  - Vertical emittance near difference resonance:

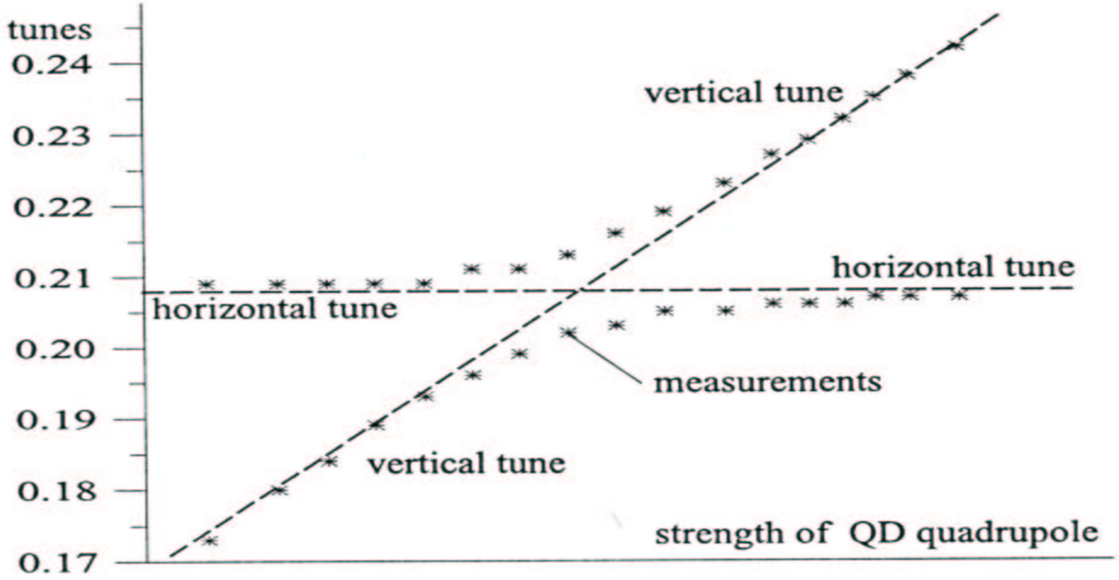
$$\frac{\varepsilon_y}{\varepsilon_x} = \frac{|\kappa|^2}{|\kappa|^2 + \Delta_r^2/2}$$

κ is resonance strength, Δ_r is distance from resonance.

Measures of driving term

❑ Tune split at difference resonance:

$$(\nu_x - \nu_y)_{min} = 2|\kappa|$$



Courtesy

H. Wiedemann

Resonance correction of the sum and difference resonance

To correct coupling, tweak orthogonal harmonic knobs for both difference resonance phases. Minimize tune split.

Sum resonance also generates linear coupling.

$$\kappa_{sum} = \frac{1}{4\pi}\int ds \; K_s \sqrt{\beta_x \beta_y}\, e^{i\phi_s}$$

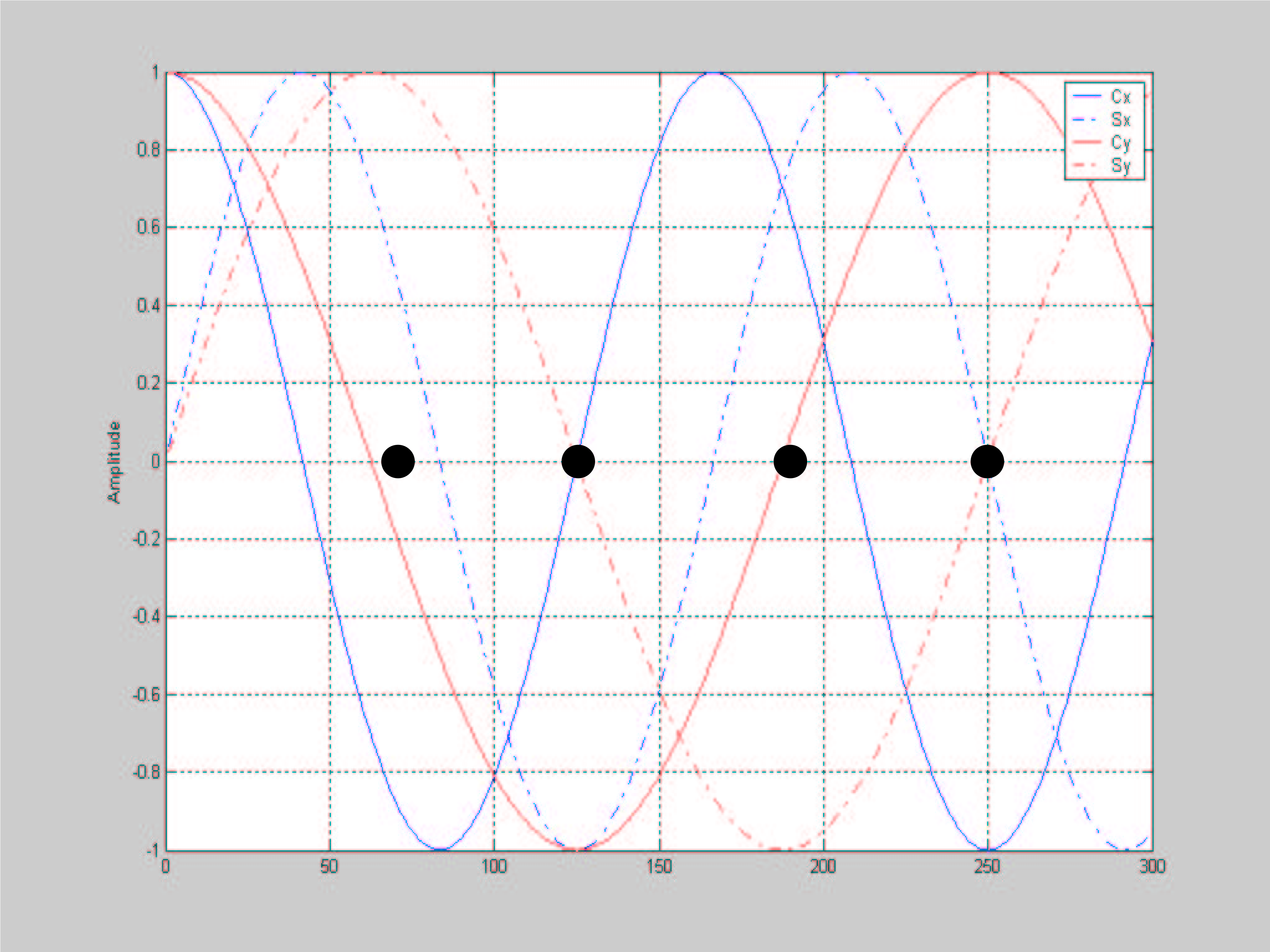
$$\frac{\phi_s}{2\pi} = \mu_x(s) + \mu_y(s) - \frac{s}{C}\Delta_r \qquad \Delta_r = (\nu_x + \nu_y - N)$$

Coupling correction – minimize measured vertical beam size as a function of skew quad strengths:

$$\sigma_{y,meas}(K_{s,1}, K_{s,2}, \ldots)$$

Good to use orthogonal harmonic knobs:

$$\sigma_{y,meas}(\kappa_{diff,\cos_N}, \kappa_{diff,\sin_N}, \kappa_{sum,\cos_N}, \kappa_{sum,\sin_N}, \kappa_{\eta_y,\cos_N}, \kappa_{\eta_y,\sin}\ldots)$$



Cx
Sx
Cy
Sy
Amplitude
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
50
100
150
200
250
300

Vertical dispersion

$$\eta_y'' + K\eta_y = \frac{1}{\rho_y} - K_s\eta_x$$

- Nonzero η_y in dipoles generates vertical emittance.
- Skew quads or vertical steerers generate or correct η_y.
- η_y knobs orthogonal to coupling knobs.

$$\kappa_{\eta_y} = \int ds\, K_s\eta_x\sqrt{\beta_y}\, e^{i\phi_{\eta_y}}$$

$$\frac{\phi_{\eta_y}}{2\pi} = \mu_y(s) - \frac{s}{C}(\nu_y - 5)$$

Beam envelope formalism – the way our codes calculate emittance

(K. Brown et al., TRANSPORT
K. Ohmi et al., PRE 49, No 1, 1994)

- Transport matrix for individual trajectories

$$x_i(s) = R_{ij}\, x_j(s_0)$$

$$\vec{x}^T = (x, x', y, y')$$

- Beam envelope matrix, Σ

$$\Psi(\vec{x}) = \frac{1}{(2\pi)^3\sqrt{\det(\Sigma)}}\exp\left(-\frac{1}{2}\Sigma_{ij}^{-1} x_i x_j\right)$$

$$\Sigma_{ij} = \langle x_i x_j \rangle$$

$$\Sigma(s) = R(s, s_0)\,\Sigma(s_0)\,R^T(s, s_0)$$

Normal mode decomposition

The 4x4 single turn matrix **T** maps phase space

$$x_i(1) = T_{ij} x_j(0)$$

$$\boldsymbol{x} = (x, x', y, y')$$

V transforms to normal mode coordinates

$$\mathbf{T} = \mathbf{V}\mathbf{U}\mathbf{V}^{-1}$$

$$\mathbf{U} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \gamma \mathbf{I} & \mathbf{C} \\ \mathbf{C}^{+} & \gamma \mathbf{I} \end{bmatrix}$$

A, **B** and **C** are 2x2 matrices. **A** and **B** propagate the normal modes.

V=**I**, **C**= **0** means the normal modes are aligned with the *x* and *y* axes.

C is a measure of local coupling.

Edwards and Teng, IEEE Trans. Nucl. Sci. 20-3, 1973

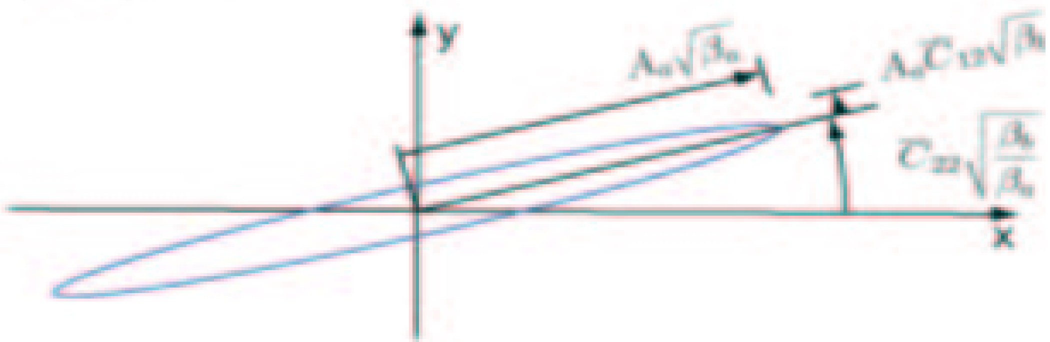
Billing, Cornell Report No. CBM 85-2, 1985

Sagan and Rubin, PRST-AB, Vol 2, 1999

The C matrix

The physical interpretation of the C matrix is that for excitation of the horizontal-like normal mode the C_{22} component is a measure of the vertical motion that is in phase with the horizontal motion while the C_{12} component is a measure of the out of phase part of the vertical motion. For the excitation of the vertical-lie normal mode, C_{11} gives the in phase component and C_{12} gives the out of phase component of the horizontal motion with respect to the vertical motion.

$$\boldsymbol{C} = \begin{pmatrix} \boldsymbol{C}_{11} & \boldsymbol{C}_{12} \\ \boldsymbol{C}_{21} & \boldsymbol{C}_{22} \end{pmatrix}$$



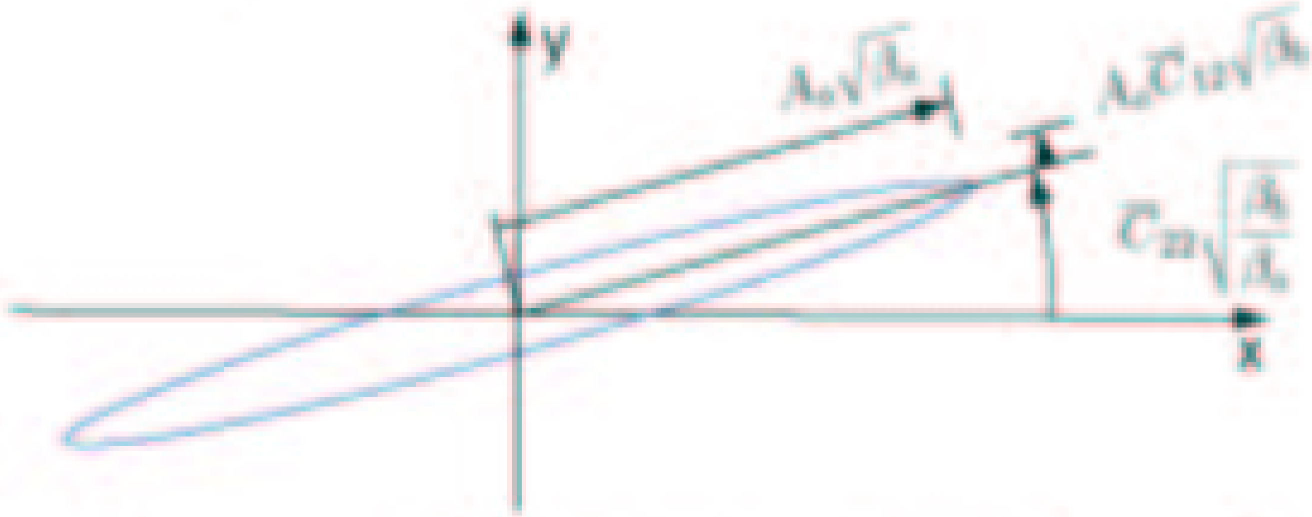
Here $\bar{C}_{12}$ gives the out–of–phase component, $\bar{C}_{22}$ gives the in–phase component.

- To characterize the linear lattice need:

$$\beta_{u}, \quad \beta_{v}, \quad \alpha_{u}, \quad \alpha_{v}, \quad \phi_{u}, \quad \phi_{v}, \quad \bar{\mathbf{C}}.$$

Determining the coupling terms

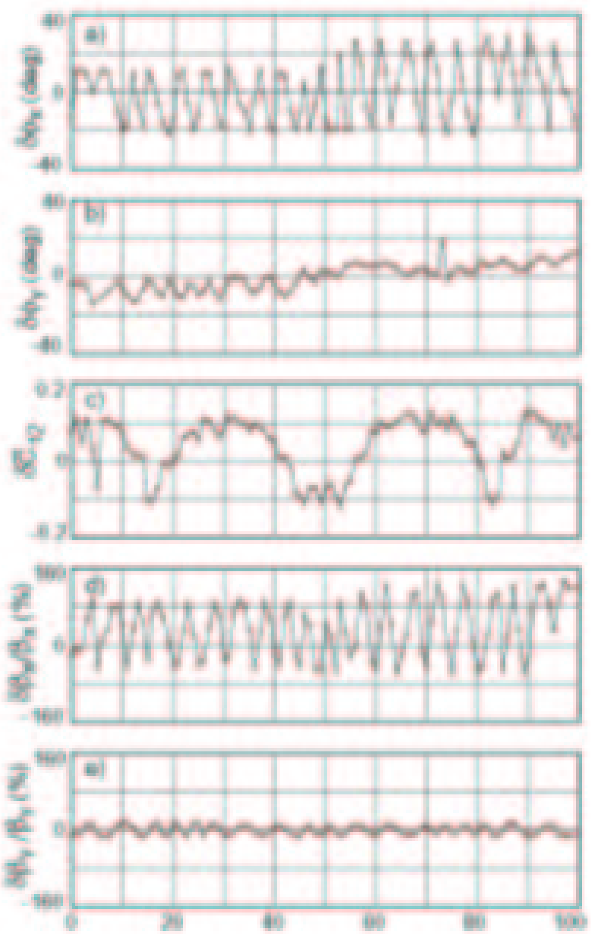
$$x = A_a \sqrt{\beta_a} \cos(n\omega_a + \phi_a),$$
$$y = -A_a \sqrt{\beta_b}(\overline{C}_{22} \cos(n\omega_a + \phi_a) + \overline{C}_{12} \sin(n\omega_a + \phi_a)).$$



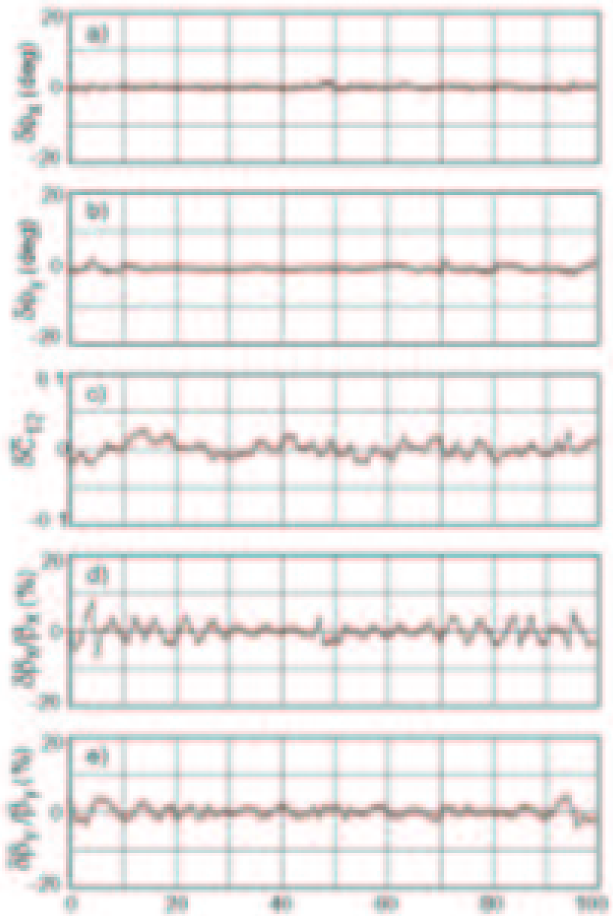
6. In practice assume $\beta = \beta$(design) and solve for ϕ and C_{ij}.

Measuring and correcting the coupling

- Before:



- After:



Coupling correction at CESR-

turn-by-turn BPM measurement of driven normal mode

P. Bagley and D. Rubin, PAC'87 and PAC'89.

D. Sagan, PAC'99

D. Sagan et al. PRST-AB, Vol. 3, 2000.

For viewgraphs, see D.Sagan viewgraph link from ABS'01 program.

Summary

Using resonance excitation and analyzing turn-by-turn data
• Lattice function measurements can be done quickly and accurately
- Single BPM sample time 800 msec (Cornell system)
- 100 BPM sample time 40 seconds (Cornell system)

Further reading

P. Castro et al. “Proceedings of the 1993 PAC Conference p2103 (1993)

D. Sagan et al
PRST V.2 074001 (1999)
PRST V.3 092801 (2000),
PRST V.3 102801 (2000)

Coupling correction using closed orbits

Closed orbit response between steering magnets and BPMs:

$$\begin{bmatrix} \vec{\mathbf{x}} \\ \vec{\mathbf{y}} \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{xx} & \mathbf{M}_{xy} \\ \mathbf{M}_{yx} & \mathbf{M}_{yy} \end{bmatrix} \begin{bmatrix} \vec{\boldsymbol{\theta}}_x \\ \vec{\boldsymbol{\theta}}_y \end{bmatrix}$$

Matices $\mathbf{M}_{xy}$ and $\mathbf{M}_{yx}$ give a measure of coupling, and should be zero in an ideal decoupled machine.

Safranek & Krinsky, PAC'93 and AIP Proc. 315, 1993.

Safranek, NIMA 388, p 27, 1997.

Steier & Robin, EPAC'00.

Nghiem & Tordeux, Coupling correction for the ESRF, SOLEIL internal report, 1999.

Nagaoka, EPAC'00.

Nagaoka & Farvacque, PAC'01.

D

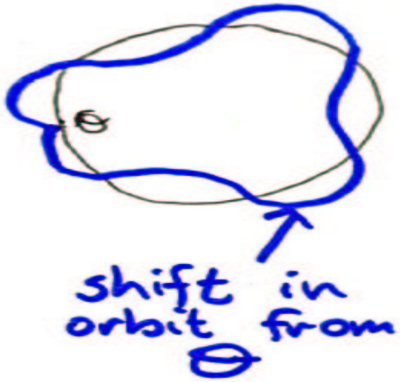
J. Safranek
S. Krinsky

METHOD OF REDUCING VERTICAL BEAM SIZE

M_{yx}

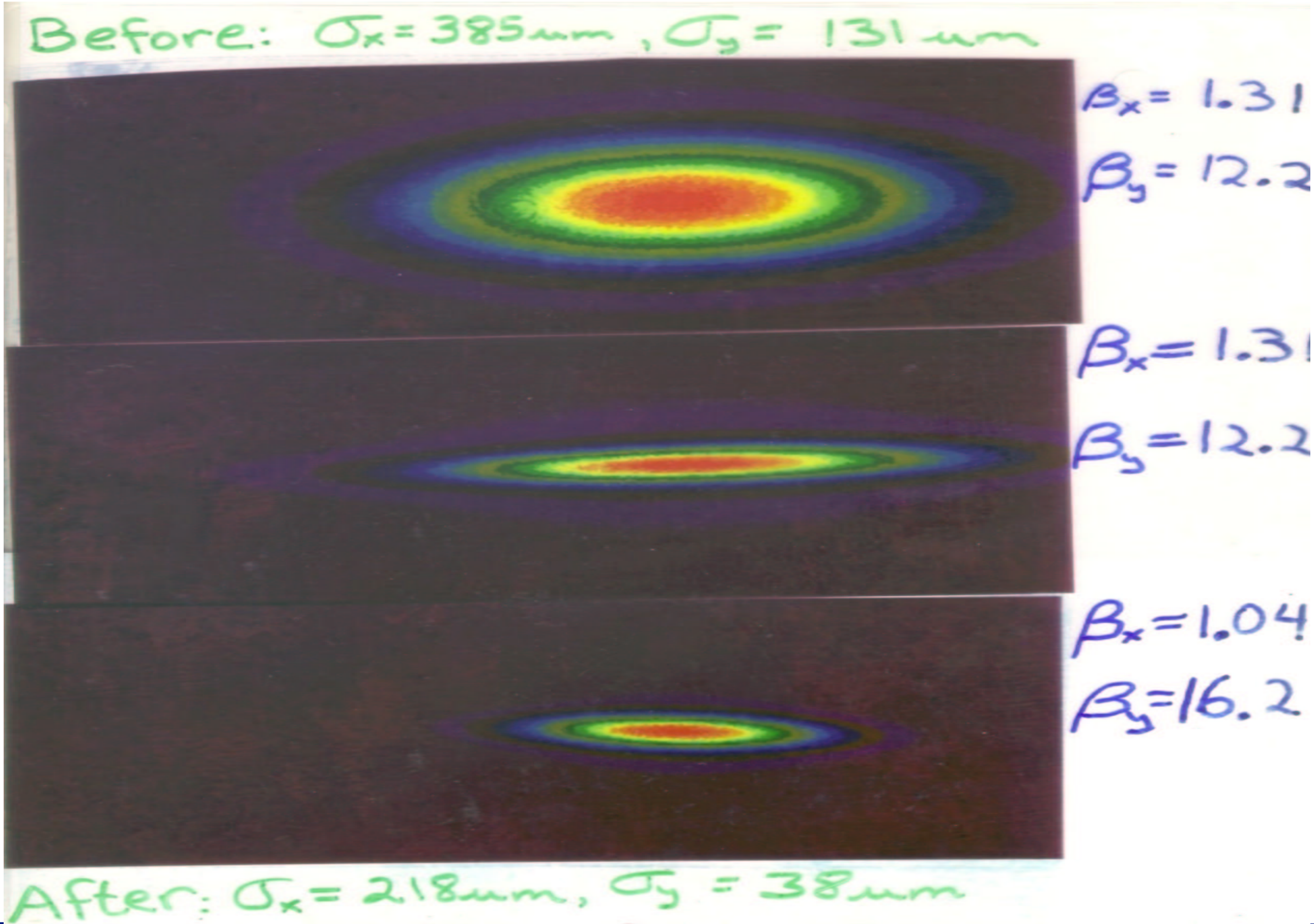
$$-\begin{pmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \\ \cdot \\ \cdot \\ \cdot \\ \boldsymbol{y}_{39} \\ \kappa\boldsymbol{\eta}_y \end{pmatrix} = A \;\; \Delta K_{SQ}$$

η



- $\boldsymbol{y}_i$ is the vertical orbit shift with the i^{th} horizontal corrector
- $\boldsymbol{\eta}_y$ is the vertical dispersion
- κ is an adjustable weight for $\boldsymbol{\eta}_y$ correction
- A is the measured change in $(\boldsymbol{y}_i, \boldsymbol{\eta}_y)$ for each skew quadrupole
- ΔK_{SQ} is the desired change in skew quadrupole strength

X-Ray ring beam size reduction



LOCO

(linear optics from closed orbits)

Again use closed orbit response matrix:

$$\begin{bmatrix} \vec{\mathbf{x}} \\ \vec{\mathbf{y}} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \vec{\boldsymbol{\theta}}_x \\ \vec{\boldsymbol{\theta}}_y \end{bmatrix}$$

The parameters of a computer storage ring model are varied to minimize the χ^2 deviation between the model and measured response matrices ($\mathbf{M}_{mod}$ and $\mathbf{M}_{meas}$).

$$\chi^2 = \sum_{i,j} \frac{(M_{meas,ij} - M_{mod,ij})^2}{\sigma_i^2}$$

LOCO fit parameters

PARAMETERS VARIED TO FIT THE ORBIT RESPONSE MATRIX:

- 56 quadrupole gradients ☆
- 56 quadrupole rolls ☆
- 96 BPM gain ☆
- 48 BPM rolls ☆
- 48 BPM C-parameter
- 90 steering magnet calibration ☆
- 90 steering magnet rolls ☆
- 51 steering magnet longitudinal center
- 90 steering magnet fractional energy shift ☆

626 parameters

8640 data points

☆ (blue) = Standard parameter set, uncoupled
☆ (green) = Standard additions for coupling

LOCO BPM parameters

PARAMETERS USED FOR FITTING BPM DATA

Four parameters were varied for each BPM

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \frac{1}{\sqrt{1-C^2}} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & C \\ C & 1 \end{pmatrix} \begin{pmatrix} g_x x \\ g_y y \end{pmatrix}.$$

g_x is the horizontal gain

g_y is the vertical gain

θ is the BPM roll

C is a parameter associated with two diagonal buttons closer together than the other two

LOCO error bars

ERROR BARS ON THE FIT PARAMETERS DUE TO RANDOM ERROR IN THE MEASURED ORBIT.

The variations given in this table are the rms error bars on the fit parameters due to random orbit measurement errors. We measured the response matrix ten times, and fit a model to each response matrix. Then, for each of the parameters we took the average over the ten data sets and calculated the rms variation from the average.

Parameter	rms variation
quadrupole gradients	.04 %
quadrupole rolls	.4 mrad
BPM gain	.05 %
BPM rolls	.5 mrad
BPM C-parameter	.0004
steering magnet calibration	.05 %
steering magnet rolls	.8 mrad
steering magnet longitudinal center	2 mm
steering magnet fractional energy shift	3.4E-7
β functions	.08%

Further work

For further work using closed orbits and turn-by-turn BPM data for coupling correction, see Nagaoka and Farvacque web site link from the program of this workshop.

Acknowledgements

Thanks to D. Sagan, R. Nagaoka and L. Farvacque for providing viewgraphs, and to H.D. Nuhn for helping me with viewgraphs.

Linear lattice overview-
Normal mode decomposition

The 4x4 single turn matrix ***T*** maps phase space

$$x_i(1) = T_{ij}\, x_j(0)$$

$$\mathbf{x} = (x, x', y, y')$$

Without coupling

$$\mathbf{U} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{bmatrix}$$

$$\mathbf{A} = \begin{pmatrix} \cos\theta_a + \alpha_a \sin\theta_a & \beta_a \sin\theta_a \\ -\gamma_a \sin\theta_a & \cos\theta_a - \alpha_a \sin\theta_a \end{pmatrix}$$

Measurement Techniques

- Vary quadrupole strengths and look at tune-changes – (Monday's talk)
- Fit orbit response matrix data – (J. Safranek)
- Ping the beam and analyze turn-by-turn data
- Resonantly excite the beam and look at turn-by-turn data

Variable quadrupole strengths

Vary quadrupole strengths and look at tune-changes

β is computed via

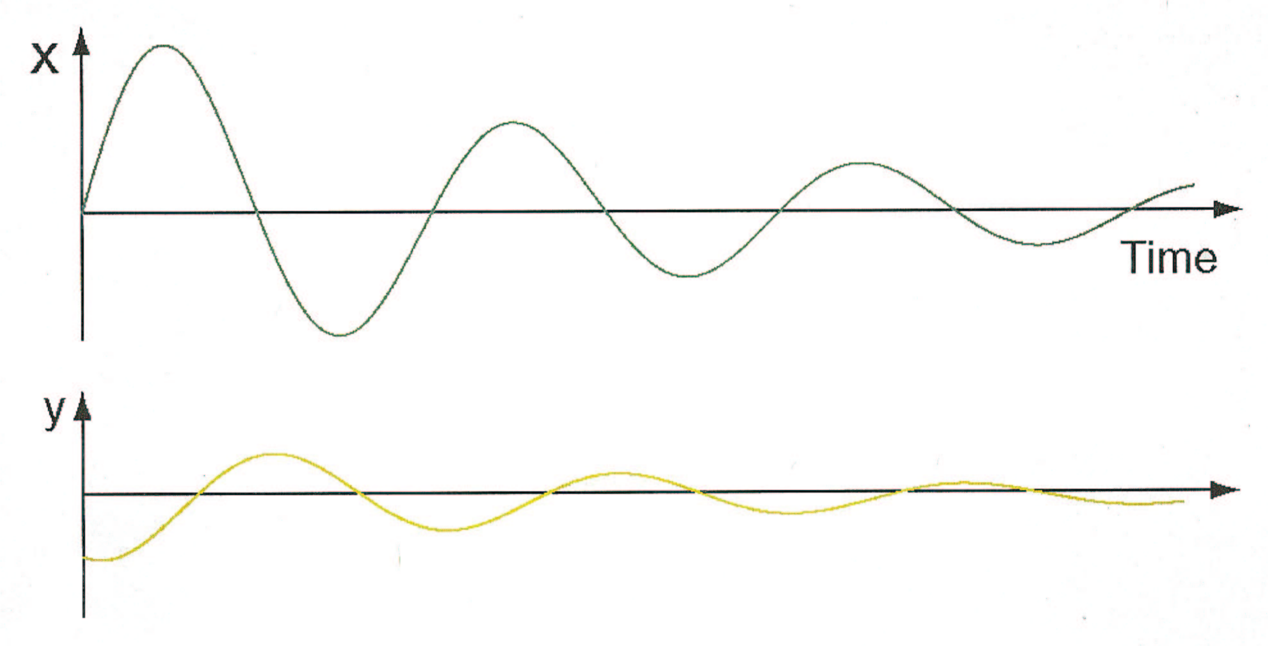
$$\delta\nu_{x,y} = \frac{\beta_{h,v}}{4\pi}\Delta kl$$

Disadvantages

- **Hysterisis – accuracy**
- **Slow**
- **Limited information**

Ping and analyze turn-by-turn data

Ping the beam and record turn-by-turn orbit data



Advantages

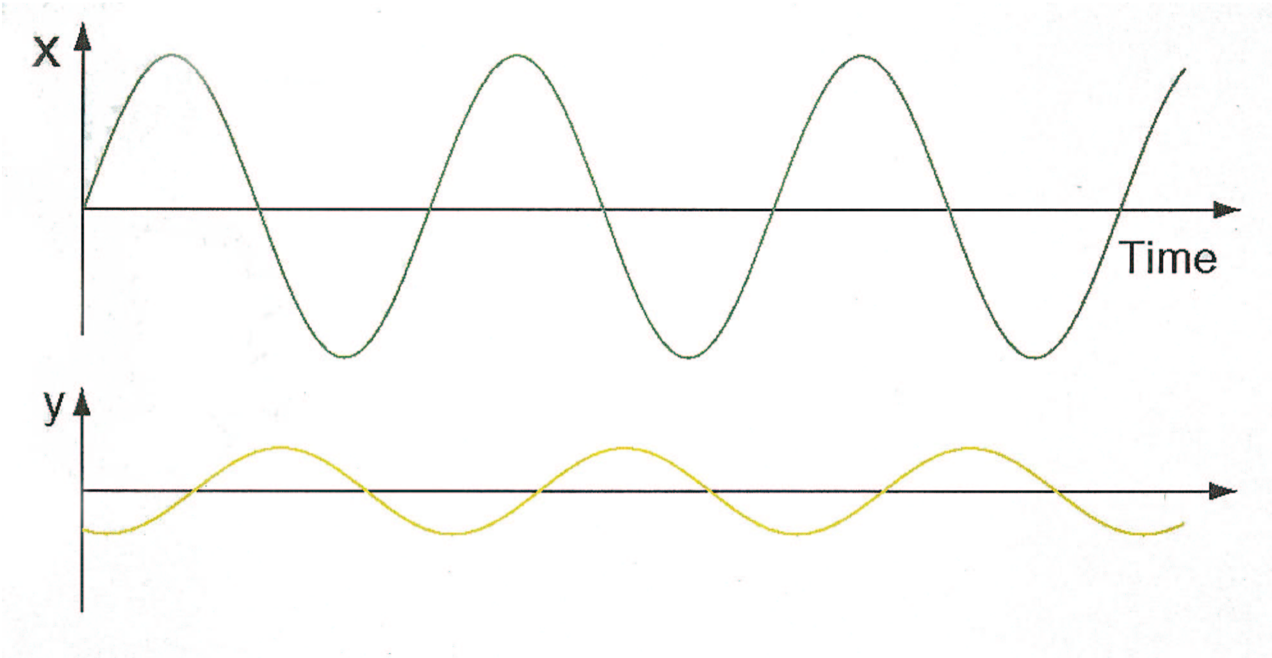
Fast

Disadvantages

Decoherence

Resonant excitation

Shake the beam at a betatron sideband and observe the beam motion at the BPMs



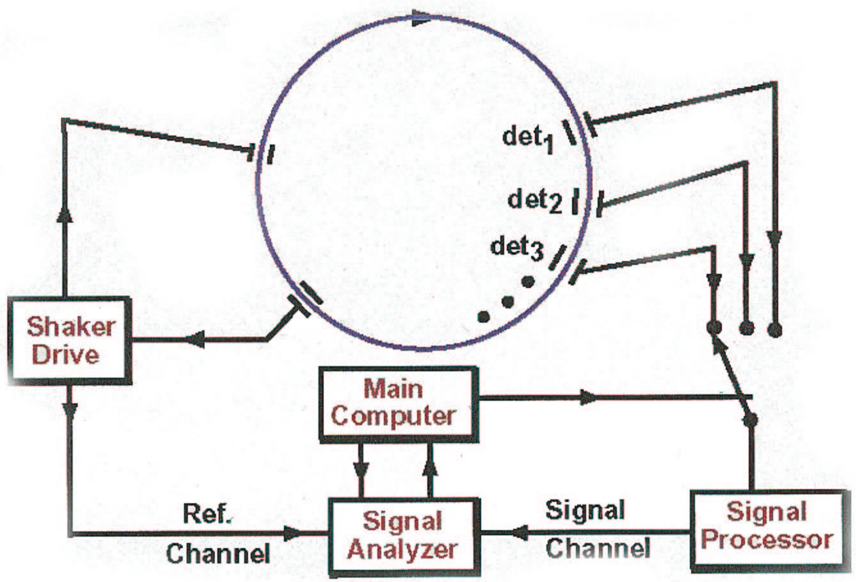
Advantages

- Fast
- Not limited by damping and decoherence

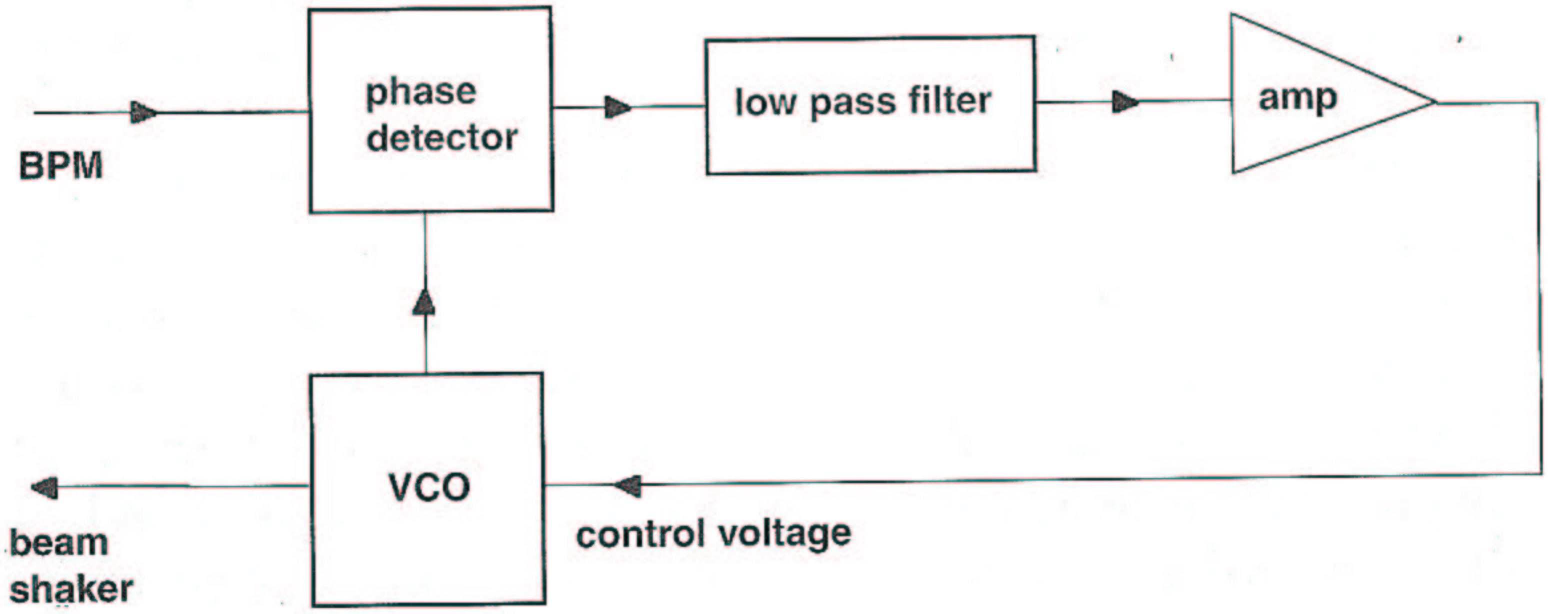
Resonant excitation

Cornell system:

- shaker is phased locked to beam
- shake beam horizontally and vertically
- analyze the signals from the BPMs sequentially



Phase locked loop



Phase detector compares the frequency of beam signal of beam and local oscillator, computes the frequency difference and adjusts the oscillator

Determination of the Tunes

- Input signal is digitized
- Take N consecutive turns (say 1024)
- Compute frequency using fast Fourier transform and interpolation

Determination of the Tunes

Input: Turn-by-turn measured orbit data.

Analysis: Fourier transform of the turn-by-turn orbit data to compute the frequency, ν

$$x(n) = \sum_{j=1}^{N} \psi(\nu_i)\exp(2\pi i n \nu_i)$$

$$\psi(\nu_i) = \frac{1}{N}\sum_{n=1}^{N} x(n)\exp(-2\pi i n \nu_i)$$

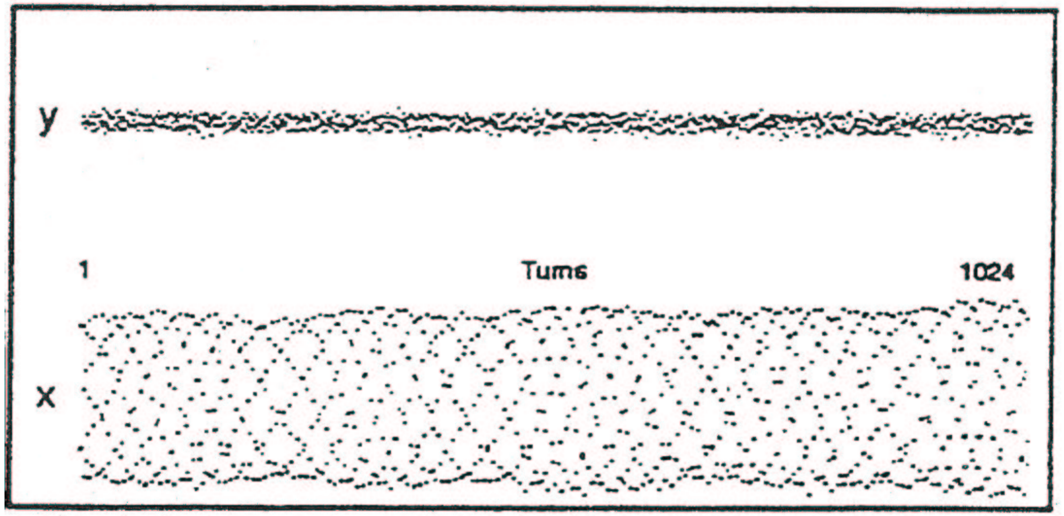


Figure 1: Single BPM recording the excited horizontal beam motion (scale: 8 mm peak to peak, time=88.9 μsec/turn)

Fast Fourier transform

The frequency corresponding to the largest value of ψ is taken as the approximate tune → $|\delta\nu|<1/2N$

Improving the resolution

The resolution can be improved by an interpolated FFT.

If one assumes that the shape of the Fourier spectrum is known and corresponds to that of a pure sinusoidal oscillation with tune, ν_{int}

$$\nu_{\text{int}} = \frac{1}{N}\left[k - 1 + \frac{A(k)}{A(k-1) + A(k)}\right], k - 1 \le N\nu \le k$$

with a sin window

$$y_k = x_k \sin\left(\frac{\pi k}{N}\right), k = 0, 1, 2, \ldots, N - 1$$

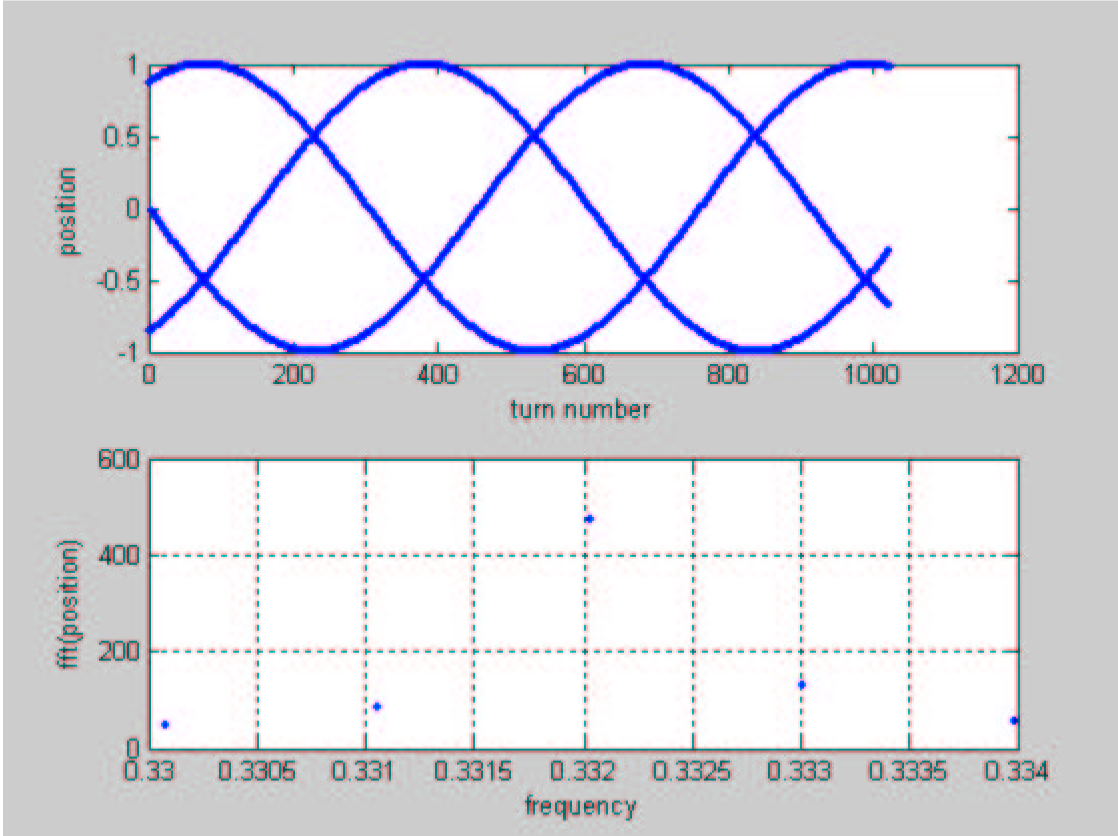
$$\nu_{\text{int}} = \frac{1}{N}\left[k - 1 + \frac{2A(k)}{A(k-1) + A(k)} - \frac{1}{2}\right]$$

(Asseo CERN PS Note 87-1 (1987))

Improving the resolution

Example: tune = 0.33224

x(i) = sin(2π(0.33224)i)



Straight fft

ν = 0.332

With interpolation

ν = 0.332239998

Determination of the phases

One method (Castro et. al. PAC 1993)

Define two functions C and S using the turn-by-turn data x and analyzed frequency ν.

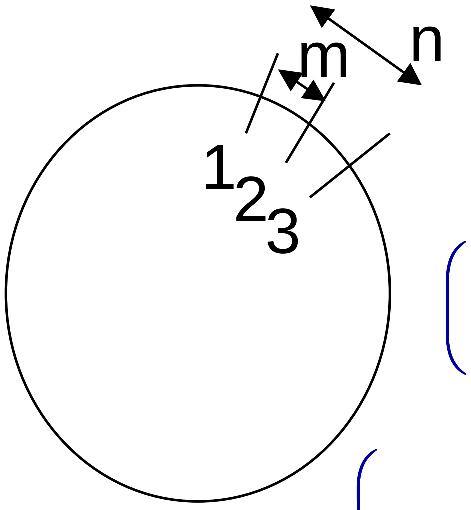
$$C = \sum_{i=1}^{N} x_i \cos(2\pi i\nu) \quad \text{and} \quad S = \sum_{i=1}^{N} x_i \sin(2\pi i\nu)$$

Then the amplitude, *A*, and phase μ are

$$A = \frac{2\sqrt{C^2 + S^2}}{N} \quad \text{and} \quad \mu = -\cot\left(\frac{S}{C}\right)$$

Amplitude is not as reliable as the phase

Determination of the β-functions – Method 1



(Castro et. al. PAC 1993)

$$\begin{pmatrix} x \\ x' \end{pmatrix}_2 = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_1 , \quad \begin{pmatrix} x \\ x' \end{pmatrix}_3 = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_1$$

$$R_{fi} = \begin{pmatrix} \sqrt{\frac{\beta_f}{\beta_i}}\left(\cos\varphi_{fi} + \alpha_i \sin\varphi_{fi}\right) & \sqrt{\beta_f \beta_i}\sin\varphi_{fi} \\ -\frac{1+\alpha_i\alpha_f}{\sqrt{\beta_f\beta_i}}\sin\varphi_{fi} + \frac{\alpha_i - \alpha_f}{\sqrt{\beta_f\beta_i}}\cos\varphi_{fi} & \sqrt{\frac{\beta_i}{\beta_f}}\left(\cos\varphi_{fi} - \alpha_f \sin\varphi_{fi}\right) \end{pmatrix}$$

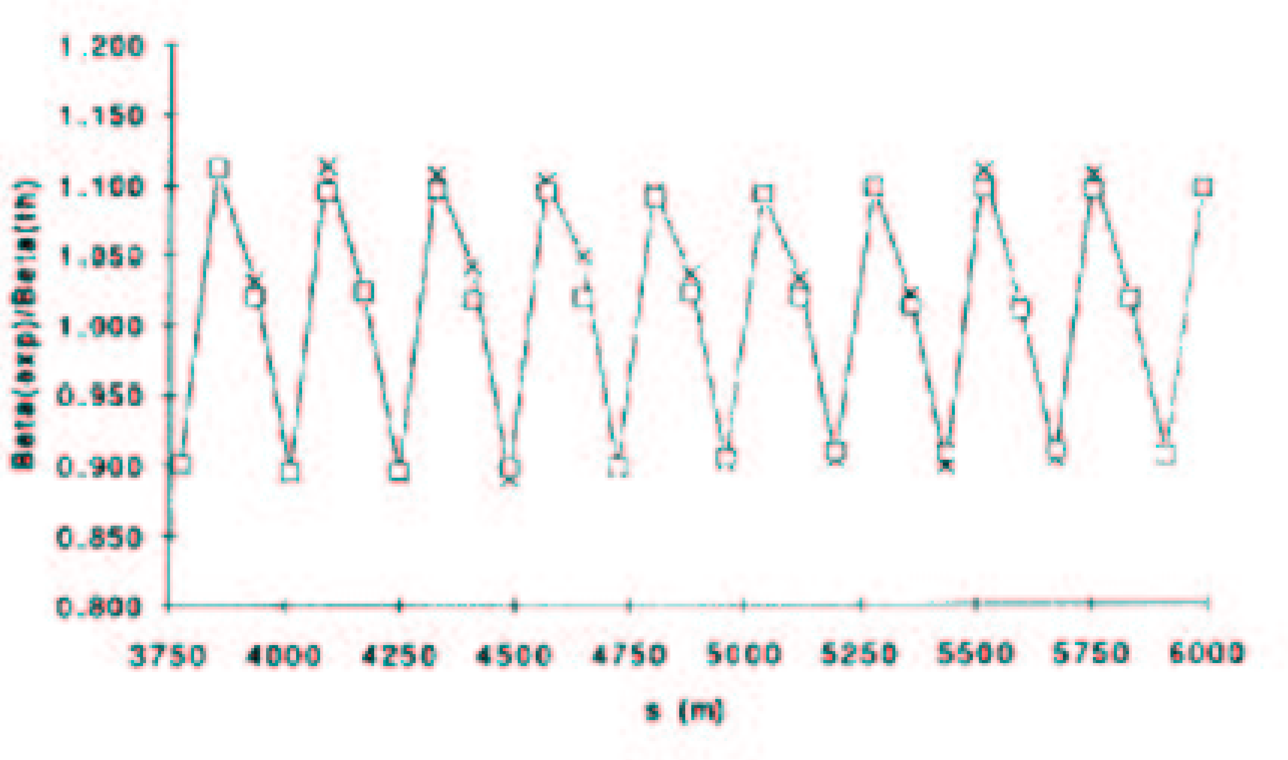
Using the ideal values for the machine and the measured phases

$$\beta_1^* = \beta_1 \frac{\left(\cot\psi_{12}^* - \cot\psi_{13}^*\right)}{\left(\cot\psi_{12} - \cot\psi_{13}\right)} \text{ and } \alpha_1^* = \alpha_1 \frac{\left(\cot\psi_{12}^* - \cot\psi_{13}^*\right) + \cot\psi_{12}^*\cot\psi_{13} - \cot\psi_{12}\cot\psi_{13}^*}{\left(\cot\psi_{12} - \cot\psi_{13}\right)}$$

Quantities with * are measured, those without are ideal

Beta beating at LEP

(Castro et. al. PAC 1993)



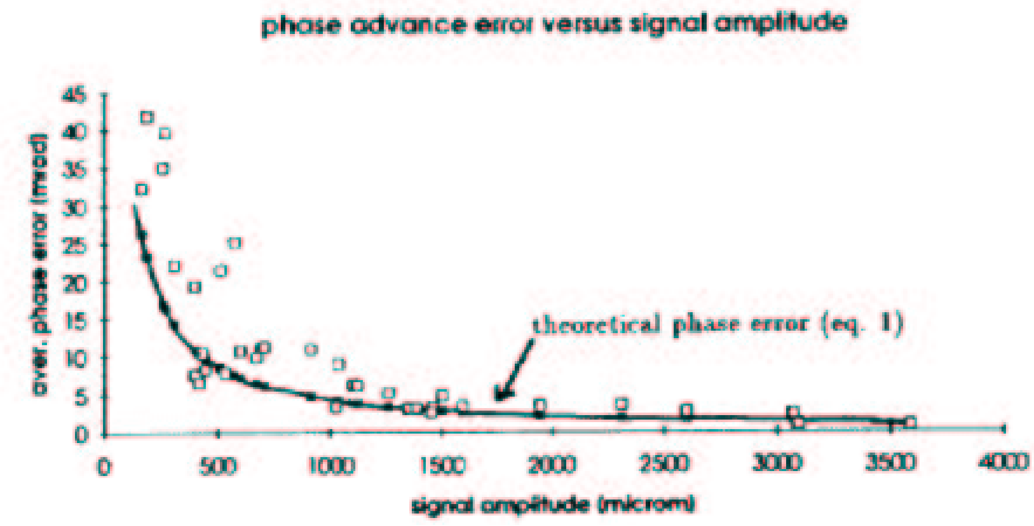
Error in the determination

Uncertainty in the phase

First there is noise of the BPMs, σ_x

The uncertainty in the phase , σ_μ, is then

$$\sigma_\mu = \frac{1}{A}\sqrt{\frac{2}{N}}\sigma_x$$



Determination of the β-functions – Method 2

Sagan et. al. PRST 2000

Beta is determined from the phase data

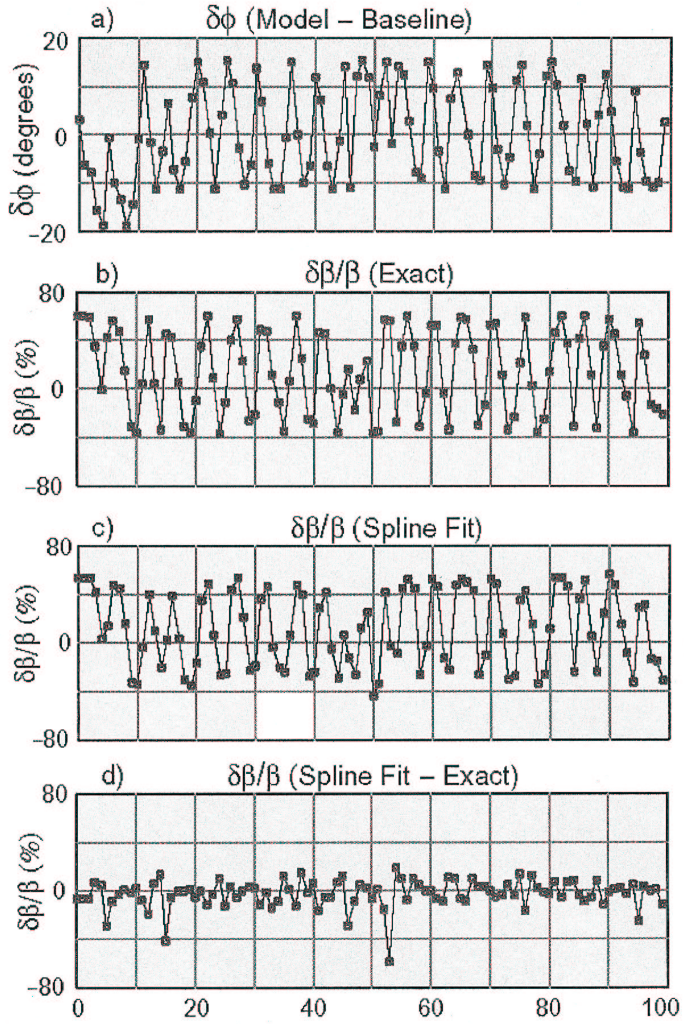
$$\frac{1}{\beta} = \frac{d\phi}{ds}$$

The relative error in the beta function is determined

$$\frac{\delta\beta}{\beta_{design}} = \frac{d(\delta\phi)}{d\phi_{design}}$$

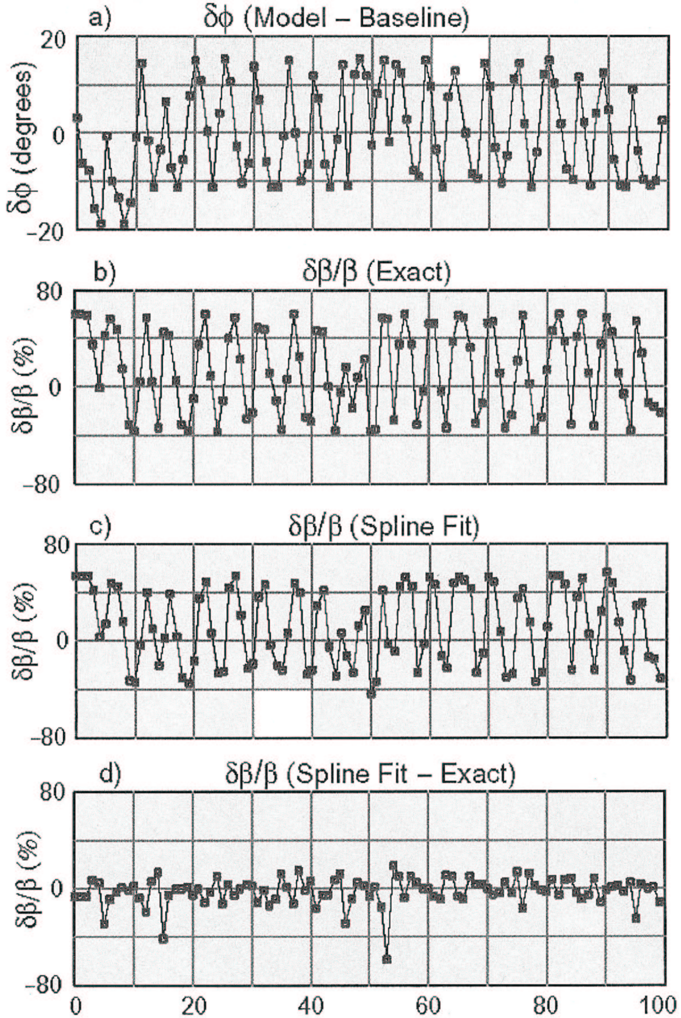
Determination of the β-functions – Method 2

Sagan et. al. PRST 2000

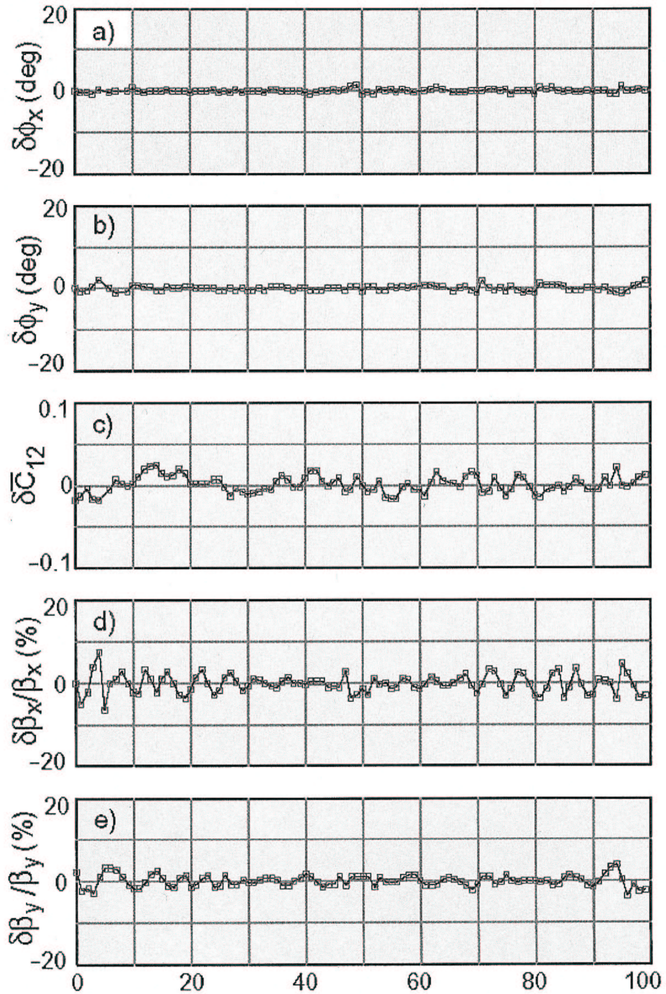


Correction of the beta beating – Method 2

Before



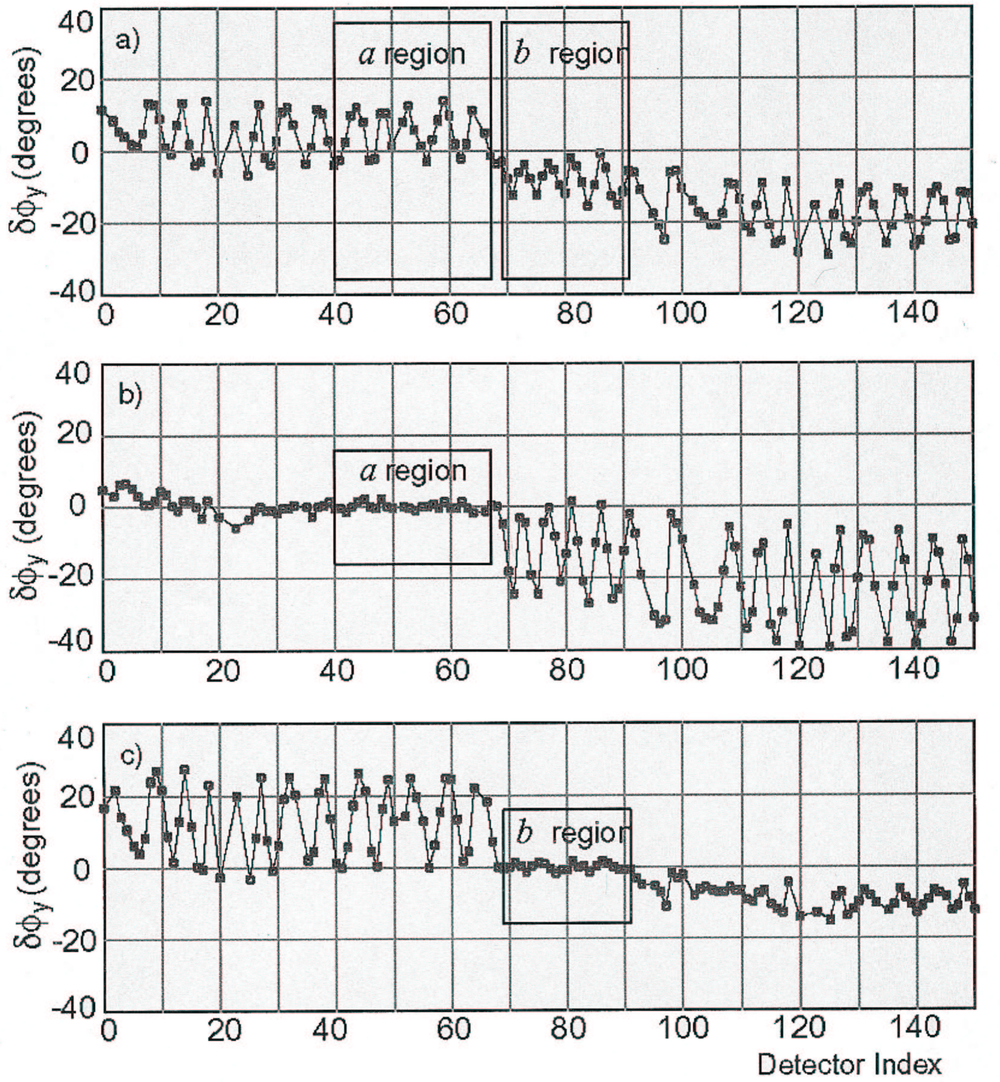
After



Note the change in scale

Location of Quadrupole Errors

Assume that one is suspicious about a certain area. Take two areas around the region and fit to free waves. See where the amplitude begins to change.



Summary

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- Lattice function measurements can be done quickly and accurately
  - Single BPM sample time 800 msec (Cornell system)
  - 100 BPM sample time 40 seconds (Cornell system)

Further reading

P. Castro et al. “Proceedings of the 1993 PAC Conference p2103 (1993)

D. Sagan et al
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V transforms to normal mode coordinates

$$\mathbf{T} = \mathbf{V}\mathbf{U}\mathbf{V}^{-1}$$

$$\mathbf{U} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \gamma\mathbf{I} & \mathbf{C} \\ \mathbf{C}^{+} & \gamma\mathbf{I} \end{bmatrix}$$

A, **B** and **C** are 2x2 matrices. **A** and **B** propagate the normal modes.

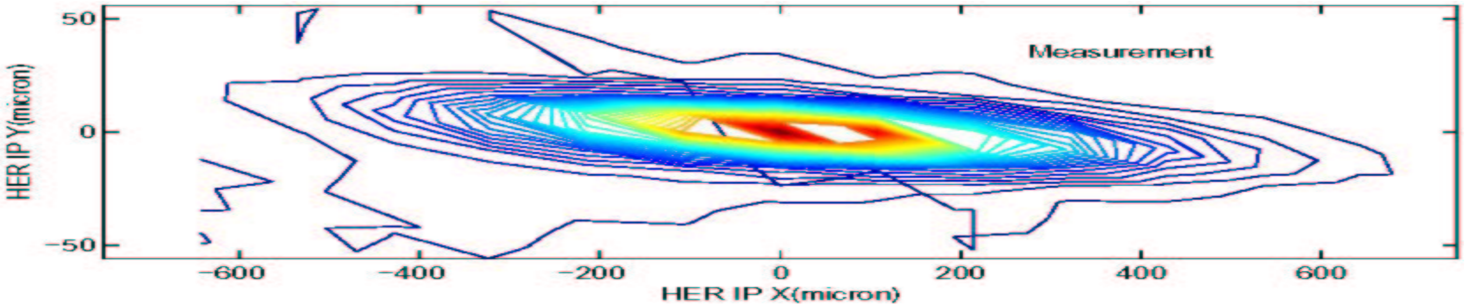
V=**I**, **C**= **0** means the normal modes are aligned with the *x* and *y* axes.

C is a measure of local coupling.

Edwards and Teng, IEEE Trans. Nucl. Sci. 20-3, 1973
Billing, Cornell Report No. CBM 85-2, 1985
Sagan and Rubin, PRST-AB, Vol 2, 1999

Measures of coupling

- Touschek lifetime
- Luminosity scan (Y. Cai, EPAC'00, p 400)



- Quadrupole moment detectors (A. Jansson et al., CERN-PS, PAC'99)