Determination of the Linear Lattice through Analysis of turn-by-turn orbit data

David Robin

Outline

- **Motivation**
- **Survey of Different Techniques**
- **Resonant Excitation**

Motivation

Desire to understand and control the linear lattice

- **Beamsize and divergence**
- **Nonlinear dynamics is determined by the linear lattice functions and the sextupoles**

Linear lattice overview-
Normal mode decomposition

The 4x4 single turn matrix ***T*** maps phase space

$$x_i(1) = T_{ij}\, x_j(0)$$

$$\mathbf{x} = (x, x', y, y')$$

Without coupling

$$\mathbf{U} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{bmatrix}$$

$$\mathbf{A} = \begin{pmatrix} \cos\theta_a + \alpha_a \sin\theta_a & \beta_a \sin\theta_a \\ -\gamma_a \sin\theta_a & \cos\theta_a - \alpha_a \sin\theta_a \end{pmatrix}$$

Measurement Techniques

- **Vary quadrupole strengths and look at tune-changes – (Monday's talk)**
- **Fit orbit response matrix data – (J. Safranek)**
- **Ping the beam and analyze turn-by-turn data**
- **Resonantly excite the beam and look at turn-by-turn data**

Variable quadrupole strengths

Vary quadrupole strengths and look at tune-changes

β is computed via

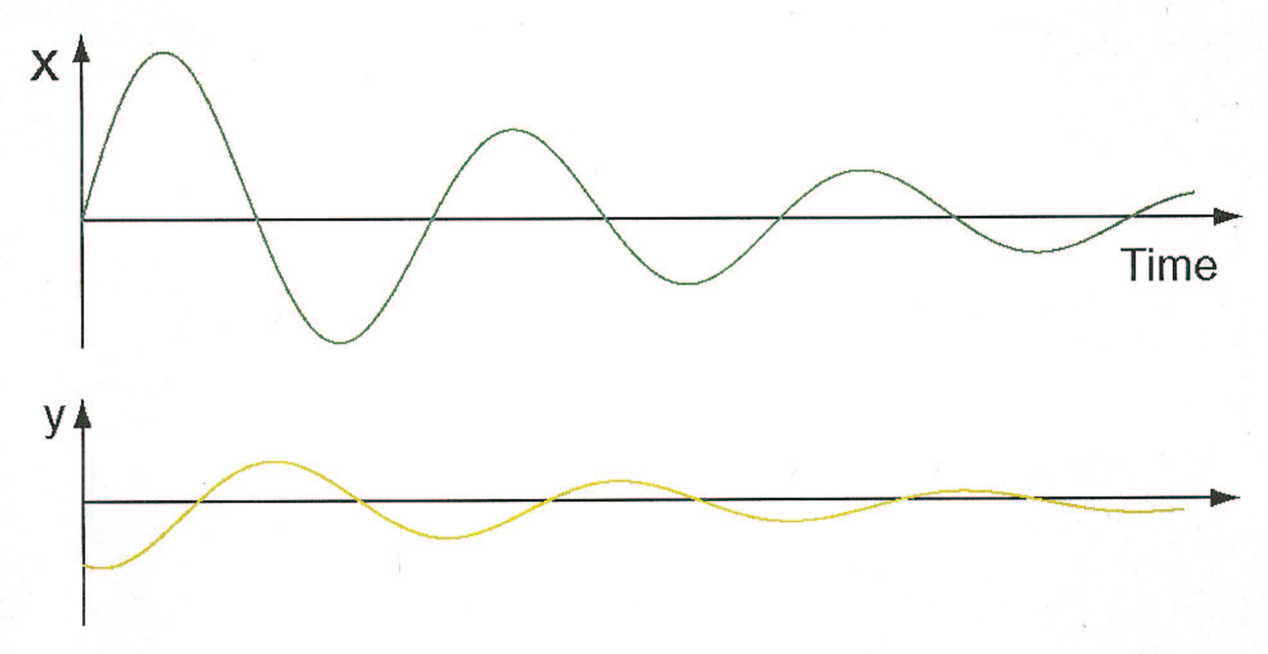
$$\delta\nu_{x,y} = \frac{\beta_{h,v}}{4\pi}\Delta kl$$

Disadvantages

- **Hysterisis – accuracy**
- **Slow**
- **Limited information**

Ping and analyze turn-by-turn data

Ping the beam and record turn-by-turn orbit data



Advantages

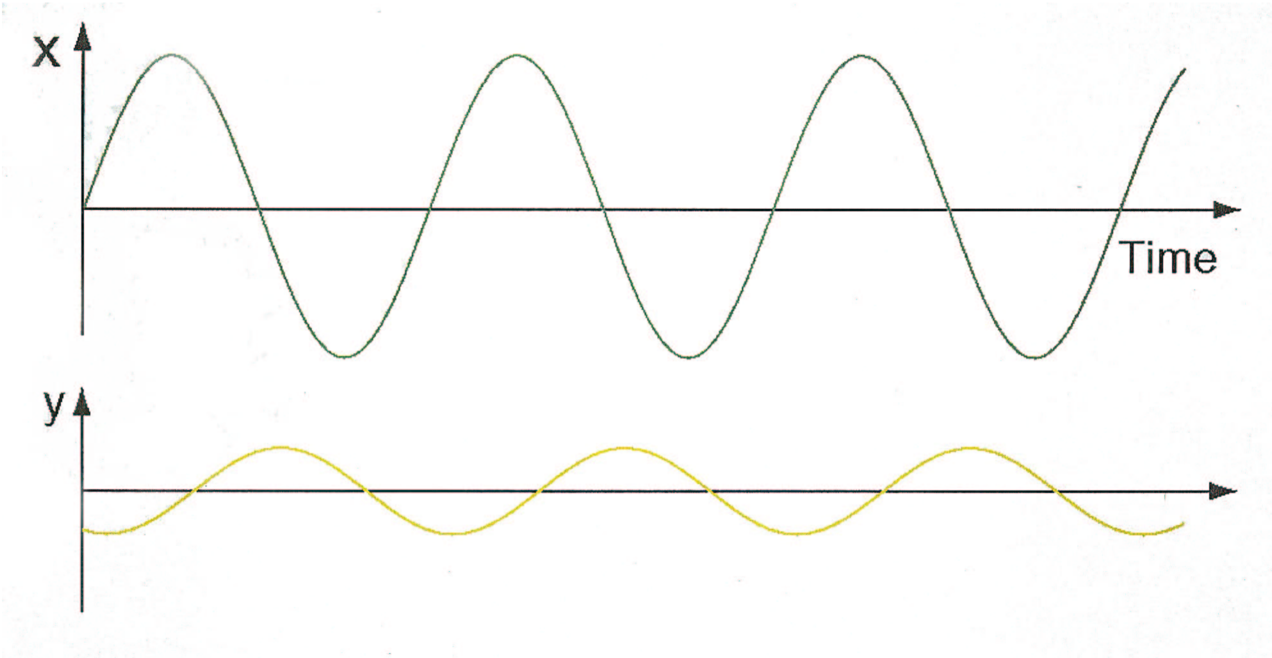
- Fast

Disadvantages

- Decoherence

Resonant excitation

Shake the beam at a betatron sideband and observe the beam motion at the BPMs



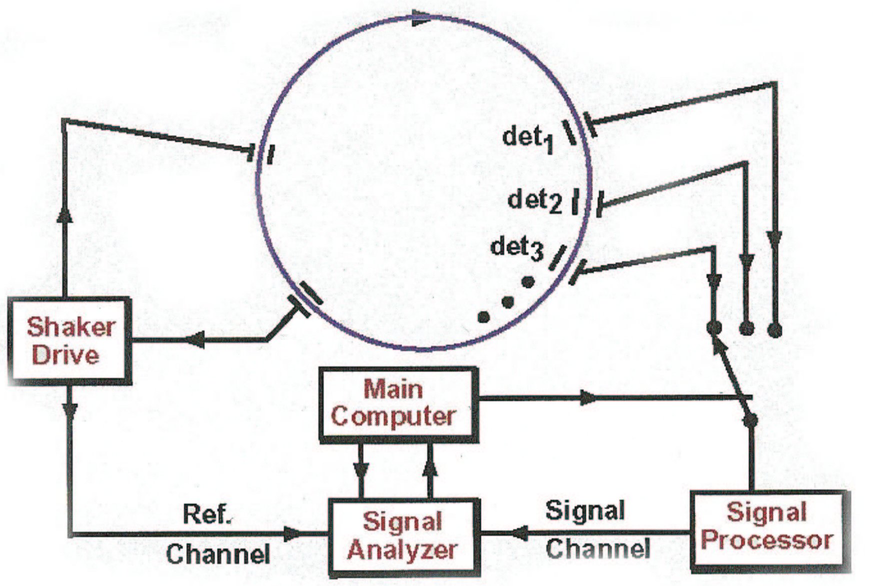
Advantages

- **Fast**
- **Not limited by damping and decoherence**

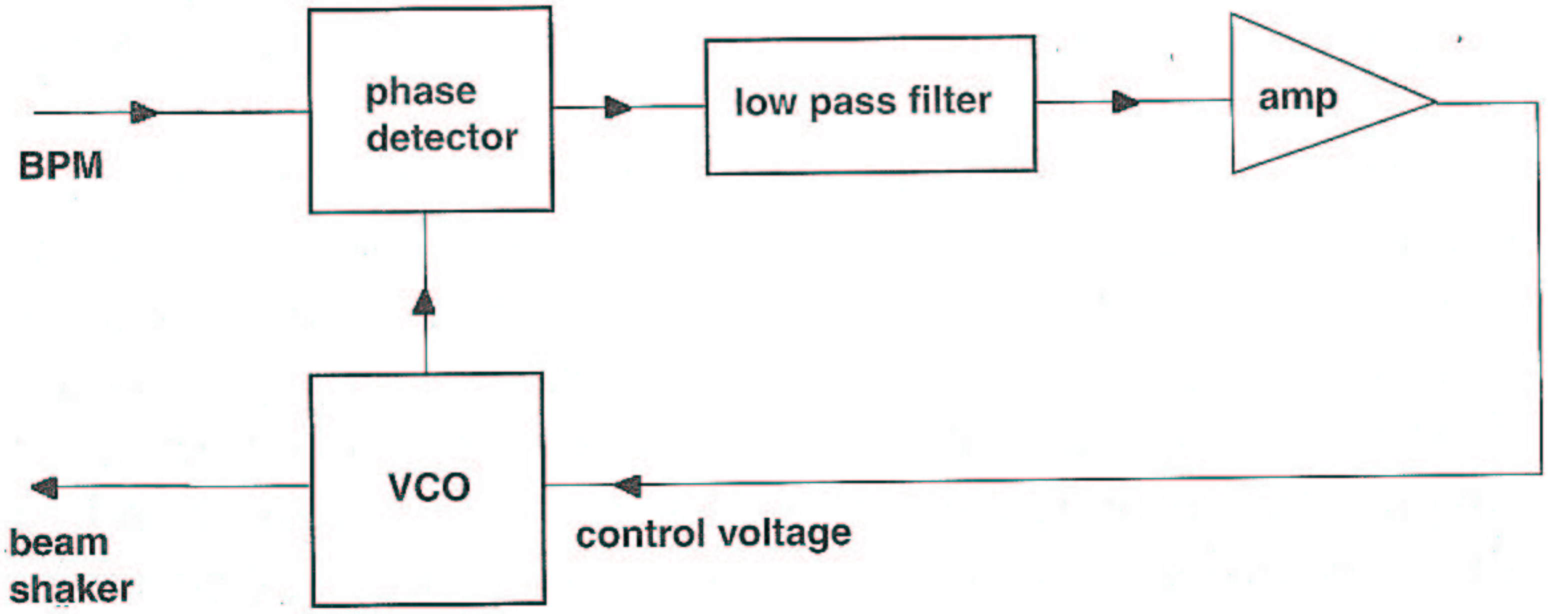
Resonant excitation

Cornell system:

- **shaker is phased locked to beam**
- **shake beam horizontally and vertically**
- **analyze the signals from the BPMs sequentially**



Phase locked loop



Phase detector compares the frequency of beam signal of beam and local oscillator, computes the frequency difference and adjusts the oscillator

Determination of the Tunes

- Input signal is digitized
- Take N consecutive turns (say 1024)
- Compute frequency using fast Fourier transform and interpolation

Determination of the Tunes

Input: Turn-by-turn measured orbit data.

Analysis: Fourier transform of the turn-by-turn orbit data to compute the frequency, ν

$$x(n) = \sum_{j=1}^{N} \psi(\nu_i)\,\mathbf{exp}(2\pi i n \nu_i)$$

$$\psi(\nu_i) = \frac{1}{N}\sum_{n=1}^{N} x(n)\,\mathbf{exp}(-2\pi i n \nu_i)$$

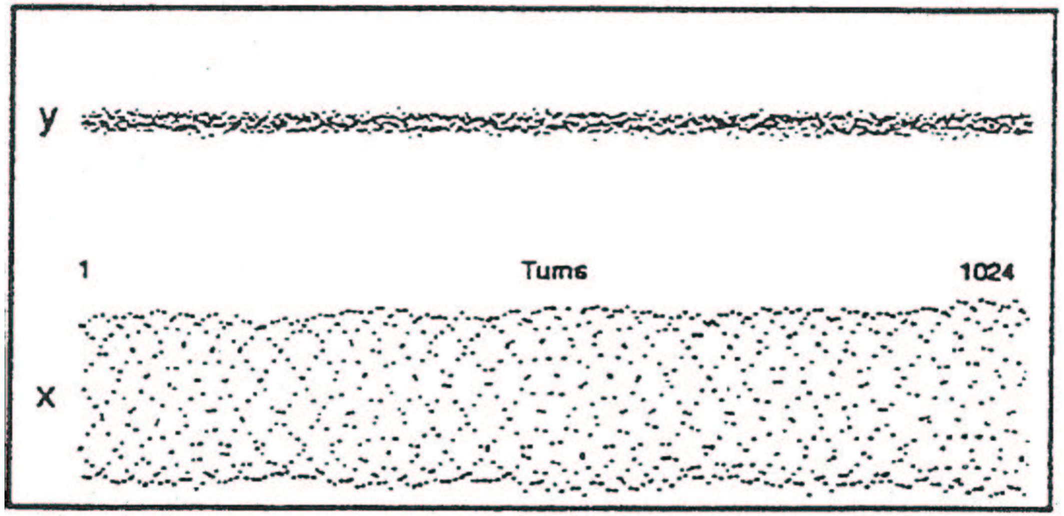


Figure 1: Single BPM recording the excited horizontal beam motion (scale: 8 mm peak to peak, time=88.9 μsec/turn)

Fast Fourier transform

The frequency corresponding to the largest value of ψ is taken as the approximate tune → $|\delta\nu| < 1/2N$

Improving the resolution

The resolution can be improved by an interpolated FFT.

If one assumes that the shape of the Fourier spectrum is known and corresponds to that of a pure sinusoidal oscillation with tune, ν_{int}

$$\nu_{int} = \frac{1}{N}\left[k - 1 + \frac{A(k)}{A(k-1) + A(k)}\right], k - 1 \le N\nu \le k$$

with a sin window

$$y_k = x_k \sin\left(\frac{\pi k}{N}\right), k = 0, 1, 2, \ldots, N - 1$$

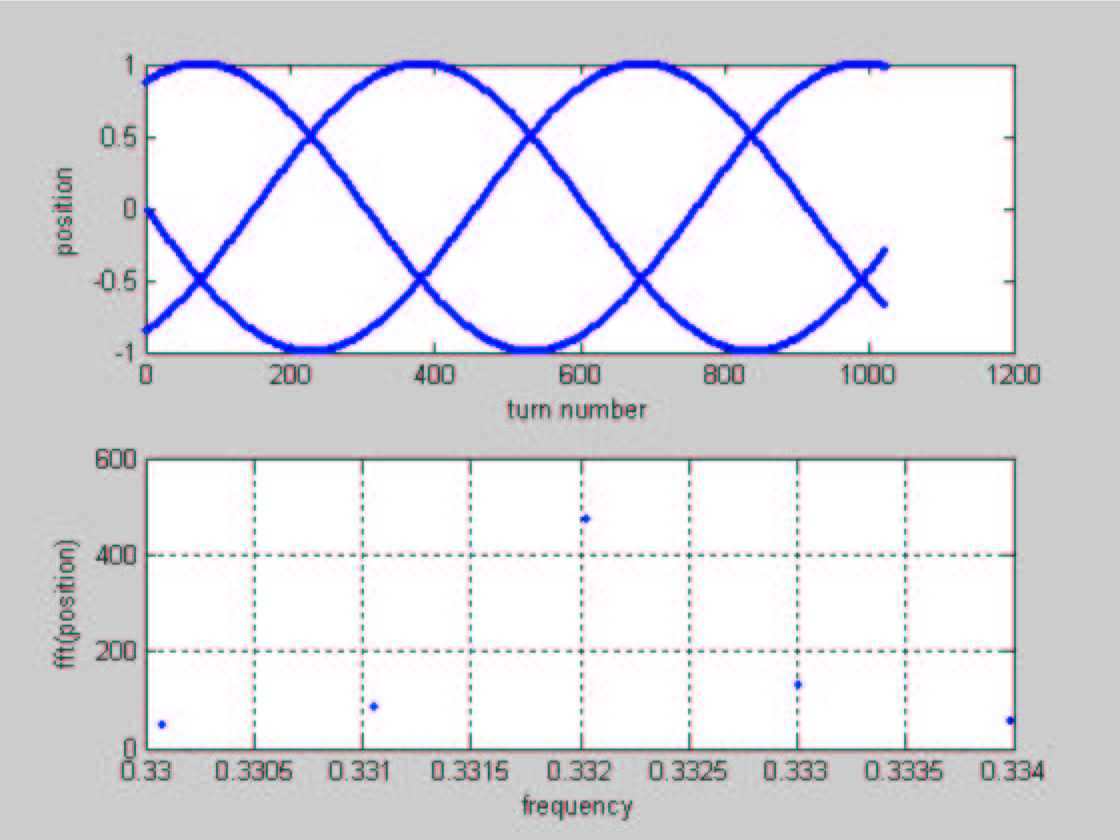
$$\nu_{int} = \frac{1}{N}\left[k - 1 + \frac{2A(k)}{A(k-1) + A(k)} - \frac{1}{2}\right]$$

(Asseo CERN PS Note 87-1 (1987))

Improving the resolution

Example: tune = 0.33224

$x(i) = \sin(2\pi(0.33224)i)$



Straight fft

$\nu = 0.332$

With interpolation

$\nu = 0.332239998$

Determination of the phases

One method (Castro et. al. PAC 1993)

Define two functions C and S using the turn-by-turn data x and analyzed frequency ν.

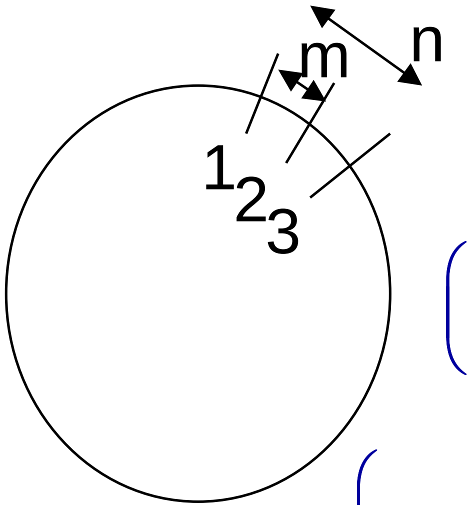
$$C = \sum_{i=1}^{N} x_i \cos(2\pi i\nu) \quad \text{and} \quad S = \sum_{i=1}^{N} x_i \sin(2\pi i\nu)$$

Then the amplitude, *A*, and phase μ are

$$A = \frac{2\sqrt{C^2 + S^2}}{N} \quad \text{and} \quad \mu = -\cot\left(\frac{S}{C}\right)$$

Amplitude is not as reliable as the phase

Determination of the β-functions – Method 1



(Castro et. al. PAC 1993)

$$\begin{pmatrix} x \\ x' \end{pmatrix}_2 = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_1 , \qquad \begin{pmatrix} x \\ x' \end{pmatrix}_3 = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_1$$

$$R_{fi} = \begin{pmatrix} \sqrt{\frac{\beta_f}{\beta_i}} \left(\cos\varphi_{fi} + \alpha_i \sin\varphi_{fi} \right) & \sqrt{\beta_f \beta_i} \sin\varphi_{fi} \\ -\frac{1+\alpha_i \alpha_f}{\sqrt{\beta_f \beta_i}} \sin\varphi_{fi} + \frac{\alpha_i - \alpha_f}{\sqrt{\beta_f \beta_i}} \cos\varphi_{fi} & \sqrt{\frac{\beta_i}{\beta_f}} \left(\cos\varphi_{fi} - \alpha_f \sin\varphi_{fi} \right) \end{pmatrix}$$

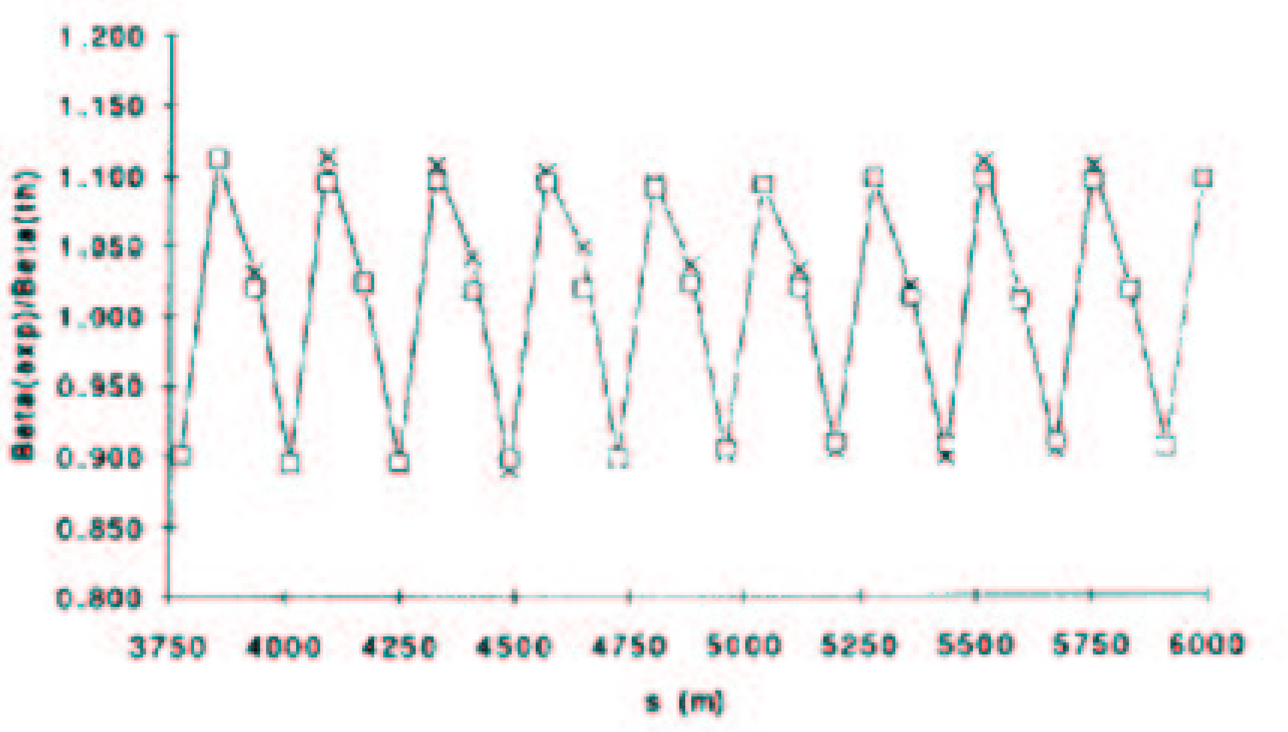
Using the ideal values for the machine and the measured phases

$$\beta_1^* = \beta_1 \frac{\left(\cot\psi_{12}^* - \cot\psi_{13}^*\right)}{\left(\cot\psi_{12} - \cot\psi_{13}\right)} \text{ and } \alpha_1^* = \alpha_1 \frac{\left(\cot\psi_{12}^* - \cot\psi_{13}^*\right) + \cot\psi_{12}^* \cot\psi_{13} - \cot\psi_{12} \cot\psi_{13}^*}{\left(\cot\psi_{12} - \cot\psi_{13}\right)}$$

Quantities with * are measured, those without are ideal

Beta beating at LEP

(Castro et. al. PAC 1993)



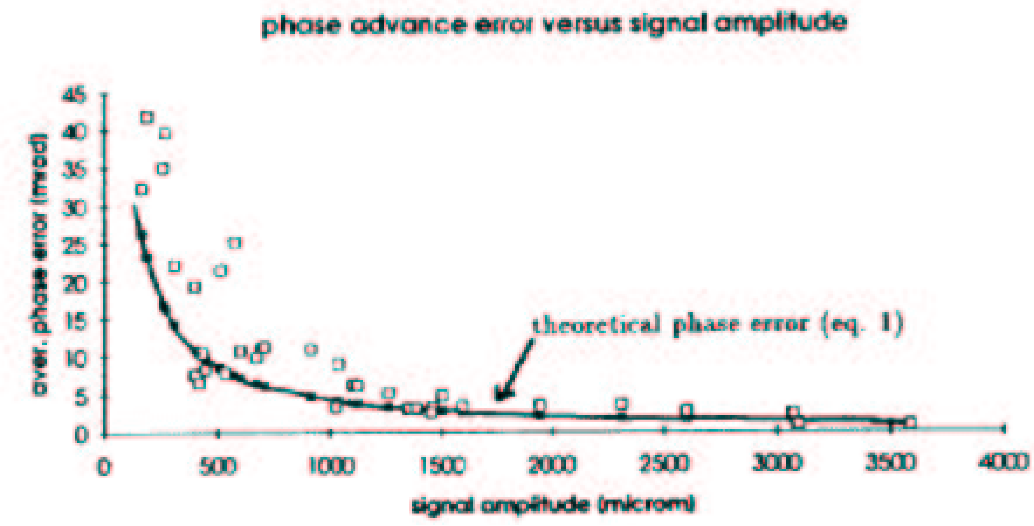
Error in the determination

Uncertainty in the phase

First there is noise of the BPMs, σ_x

The uncertainty in the phase , σ_μ, is then

$$\sigma_\mu = \frac{1}{A}\sqrt{\frac{2}{N}}\sigma_x$$



Determination of the β-functions – Method 2

Sagan et. al. PRST 2000

Beta is determined from the phase data

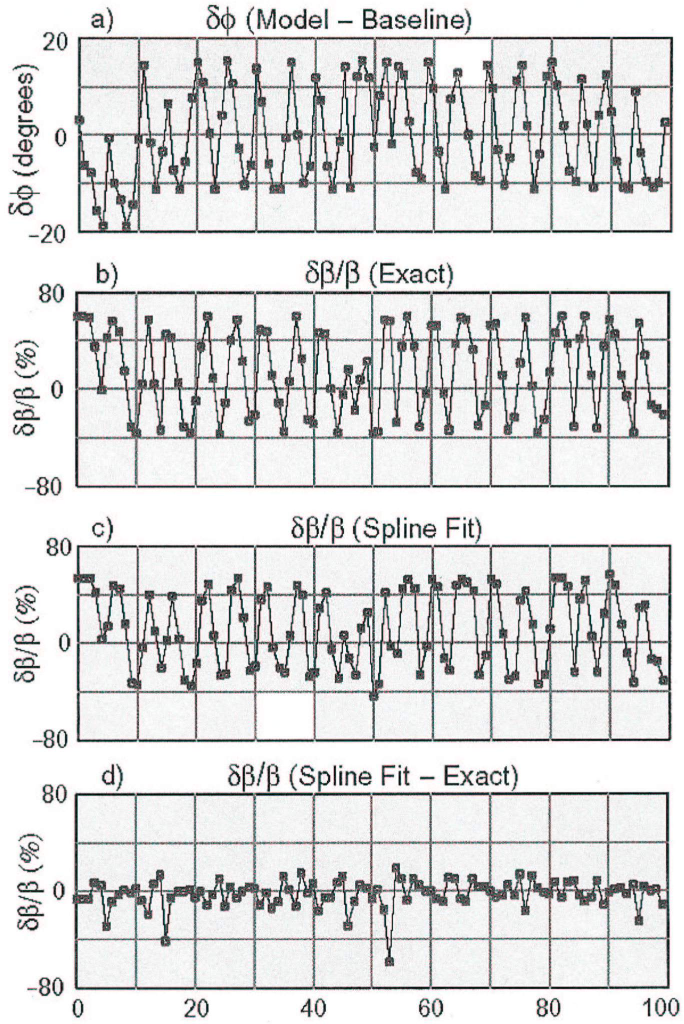
$$\frac{1}{\beta} = \frac{d\phi}{ds}$$

The relative error in the beta function is determined

$$\frac{\delta\beta}{\beta_{design}} = \frac{d(\delta\phi)}{d\phi_{design}}$$

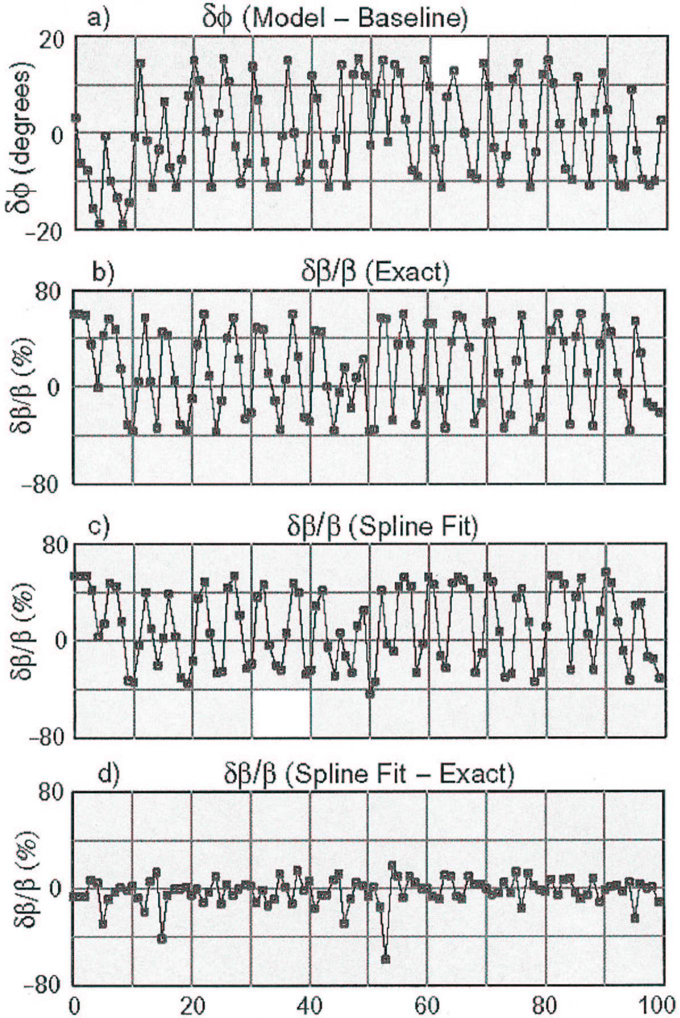
Determination of the β-functions – Method 2

Sagan et. al. PRST 2000

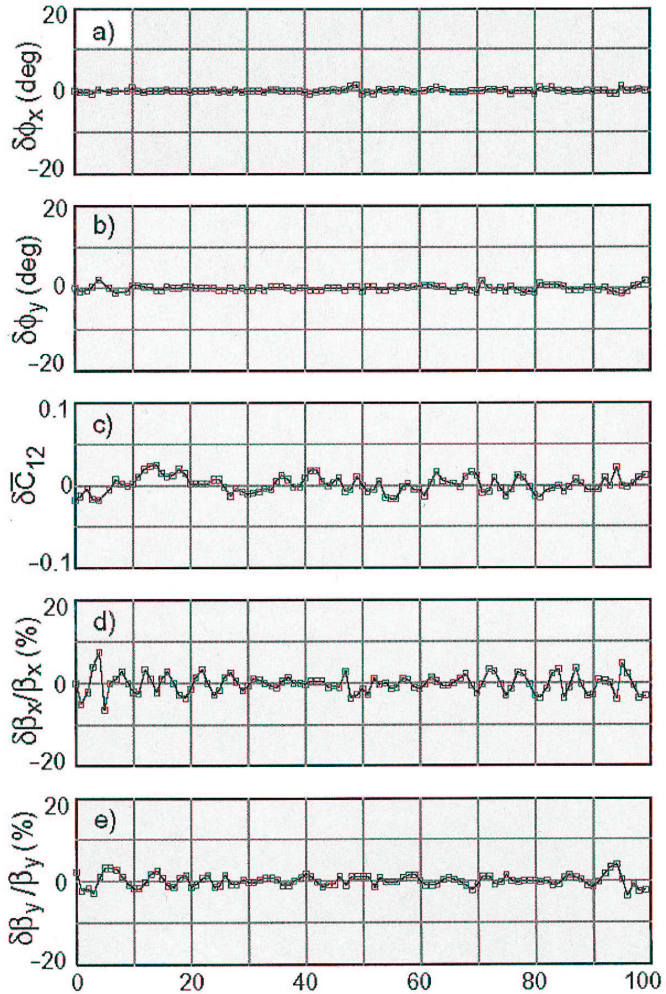


Correction of the beta beating – Method 2

Before



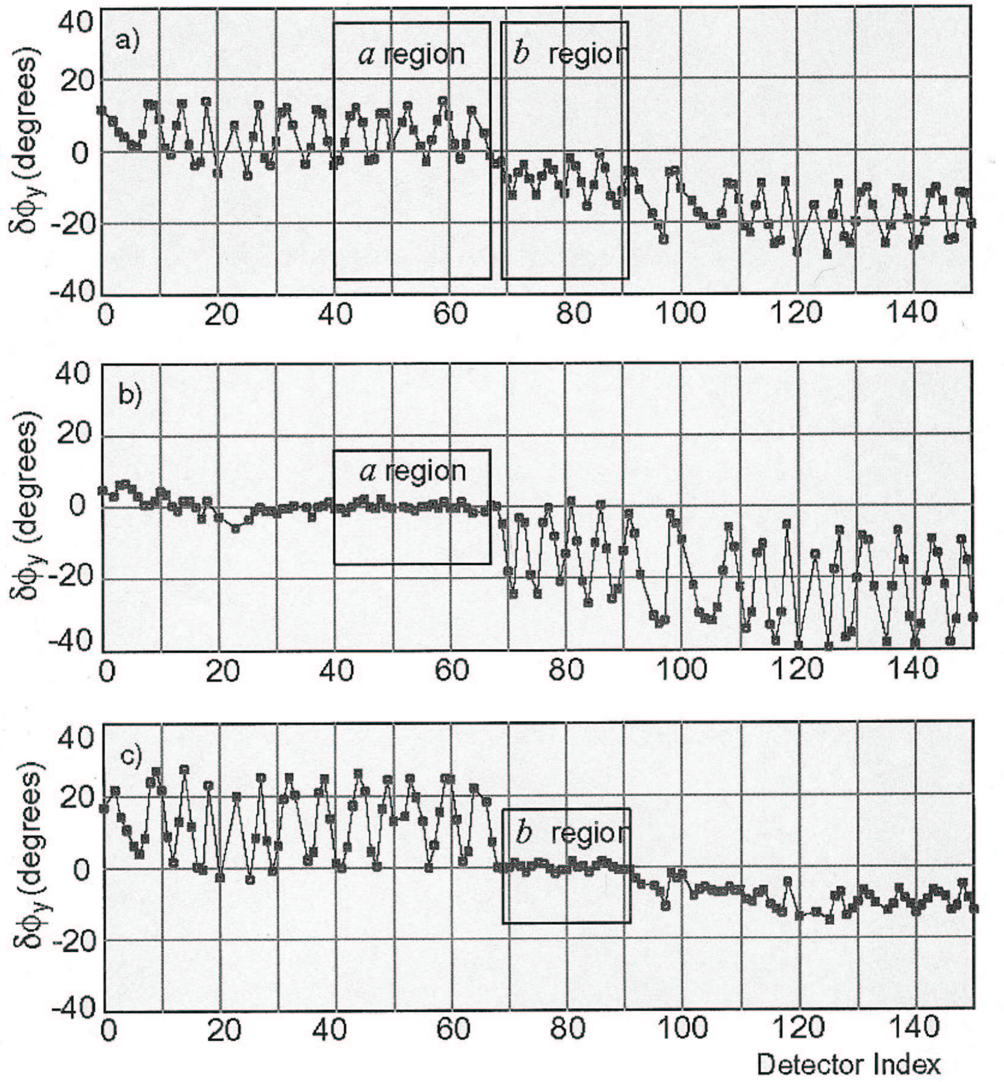
After



Note the change in scale

Location of Quadrupole Errors

Assume that one is suspicious about a certain area. Take two areas around the region and fit to free waves. See where the amplitude begins to change.



Summary

Using resonance excitation and analyzing turn-by-turn data

• Lattice function measurements can be done quickly and accurately

- Single BPM sample time 800 msec (Cornell system)
- 100 BPM sample time 40 seconds (Cornell system)

Further reading

P. Castro et al. "Proceedings of the 1993 PAC Conference p2103 (1993)

D. Sagan et al
PRST V.2 074001 (1999)
PRST V.3 092801 (2000),
PRST V.3 102801 (2000)

Linear lattice overview-
Normal mode decomposition

The 4x4 single turn matrix ***T*** maps phase space

$$x_i(1) = T_{ij}\, x_j(0)$$

$$\mathbf{x} = (x, x', y, y')$$

V transforms to normal mode coordinates

$$\mathbf{T} = \mathbf{V}\mathbf{U}\mathbf{V}^{-1}$$

$$\mathbf{U} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{bmatrix} \qquad \mathbf{V} = \begin{bmatrix} \gamma\mathbf{I} & \mathbf{C} \\ \mathbf{C}^{+} & \gamma\mathbf{I} \end{bmatrix}$$

A, **B** and **C** are 2x2 matrices. **A** and **B** propagate the normal modes.

V=**I**, **C**= **0** means the normal modes are aligned with the *x* and *y* axes.

C is a measure of local coupling.

Edwards and Teng, IEEE Trans. Nucl. Sci. 20-3, 1973
Billing, Cornell Report No. CBM 85-2, 1985
Sagan and Rubin, PRST-AB, Vol 2, 1999