Model Independent Analysis

Using physical correlation between multiple measurements to reduce noise of individual measurement

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Based on work by J. Irwin, C.-X. Wang

Contributions by Y.T. Yan, Y. Cai, A. Wolski

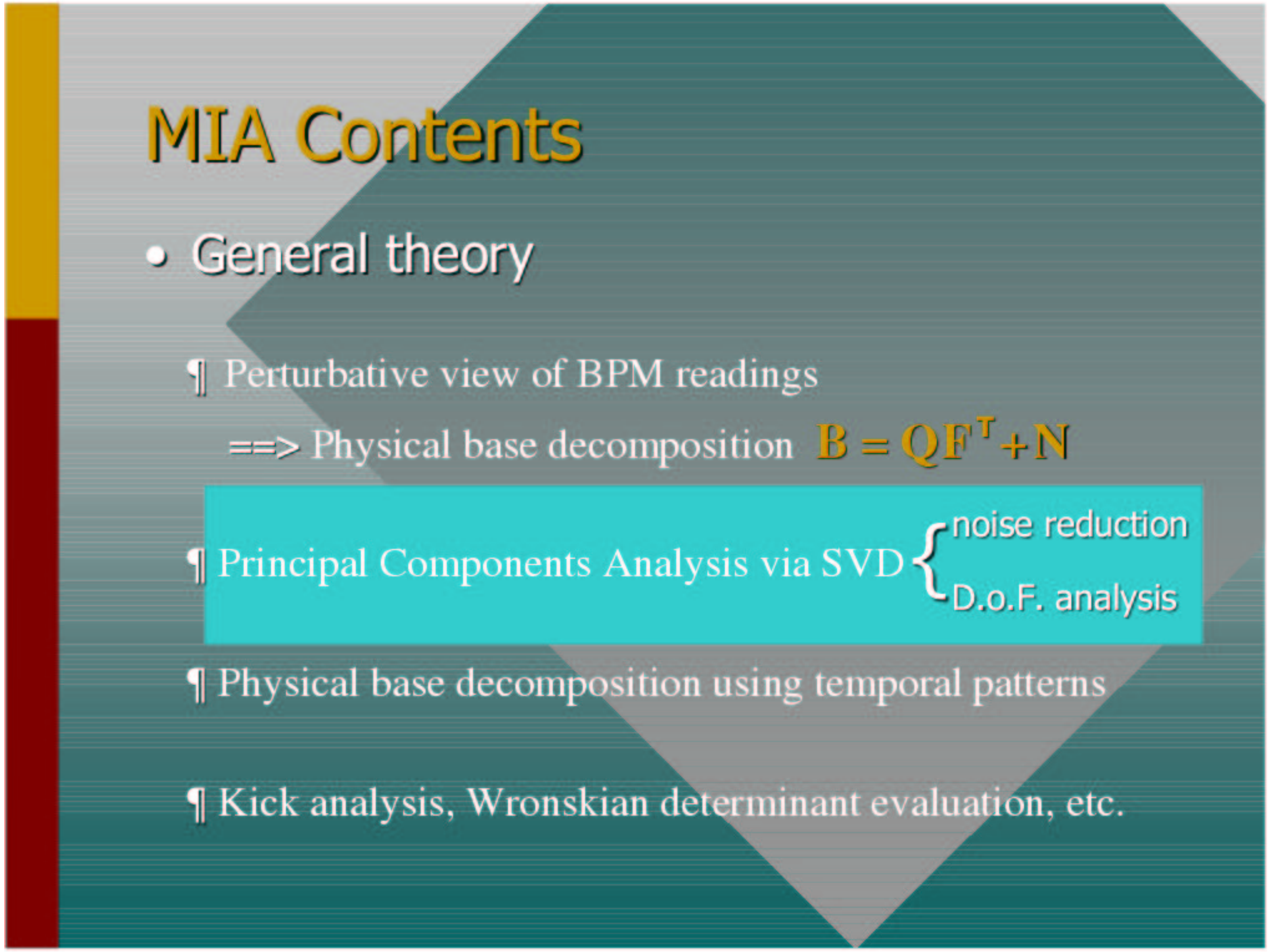
- Motivation
- List of the concepts of MIA
- SVD
- Experimental Results
- Summary

Motivation

- Idea is to use the correlation between BPMs to reduce the statistical noise of individual BPM measurements.
- Originally developed for LINACs (single pass beamlines), making use of statistical averaging over many shots – Very good method for this case.
- Also applicable to storage rings (averaging over many turns). Most applications in that case are not model independent anymore (comparison with model necessary). Compared to response matrix analysis it requires better knowledge of BPM calibration/tilt. Phase advance measurements are achieving the noise reduction by analyzing with a fixed frequency (either known from the excitation or measured using many BPMs).
- Ideally should make it possible to precisely measure nonlinearities in transfer maps (not demonstrated, yet).
- Will first talk about LINAC case and later show some examples from storage rings (ATF, PEP-II, ALS).

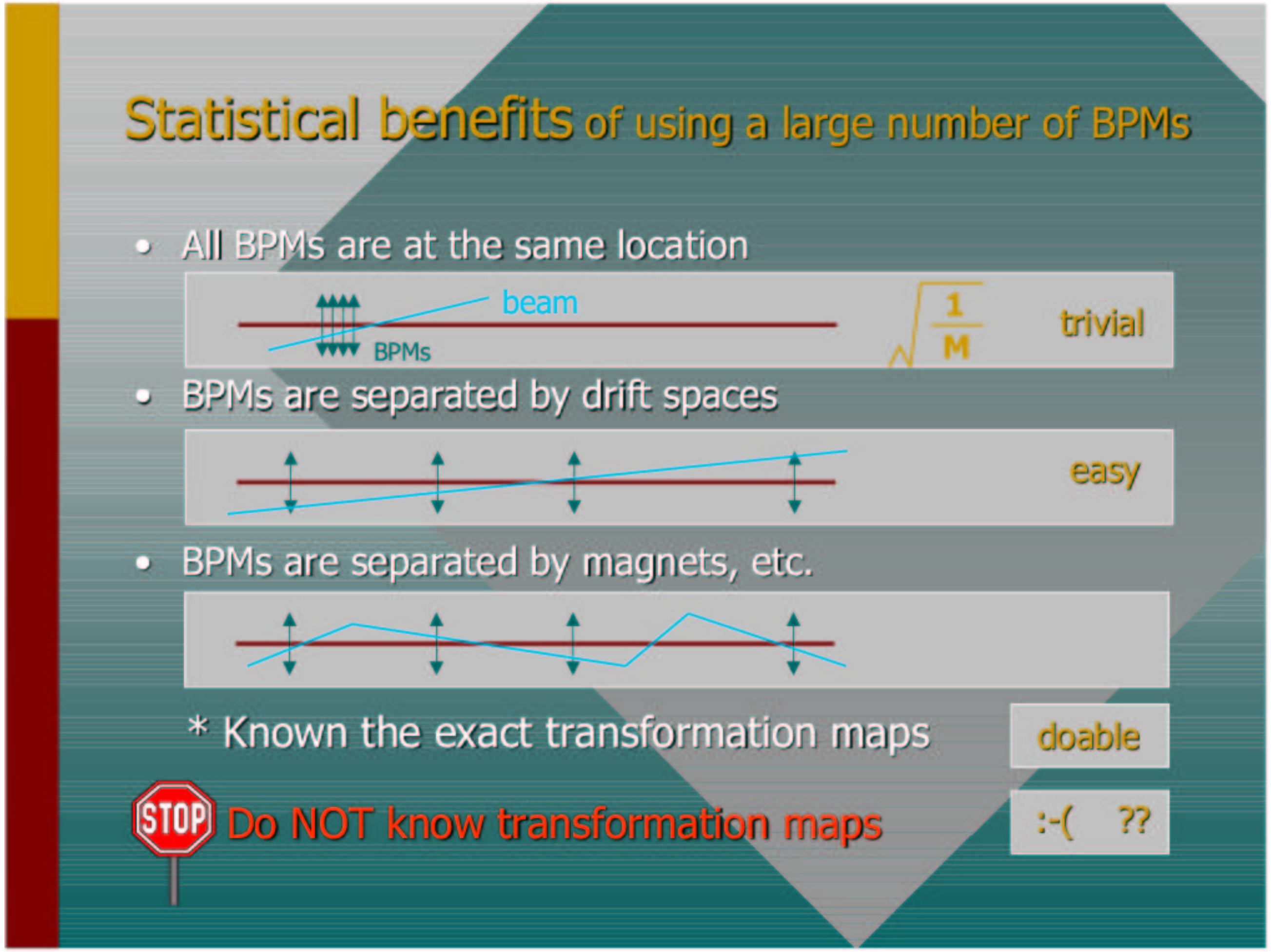
Components of MIA

❖ Courtesy of C.-X. Wang



Making use of BPM correlation

❖ Courtesy of C.-X. Wang



Singular Value Decomposition

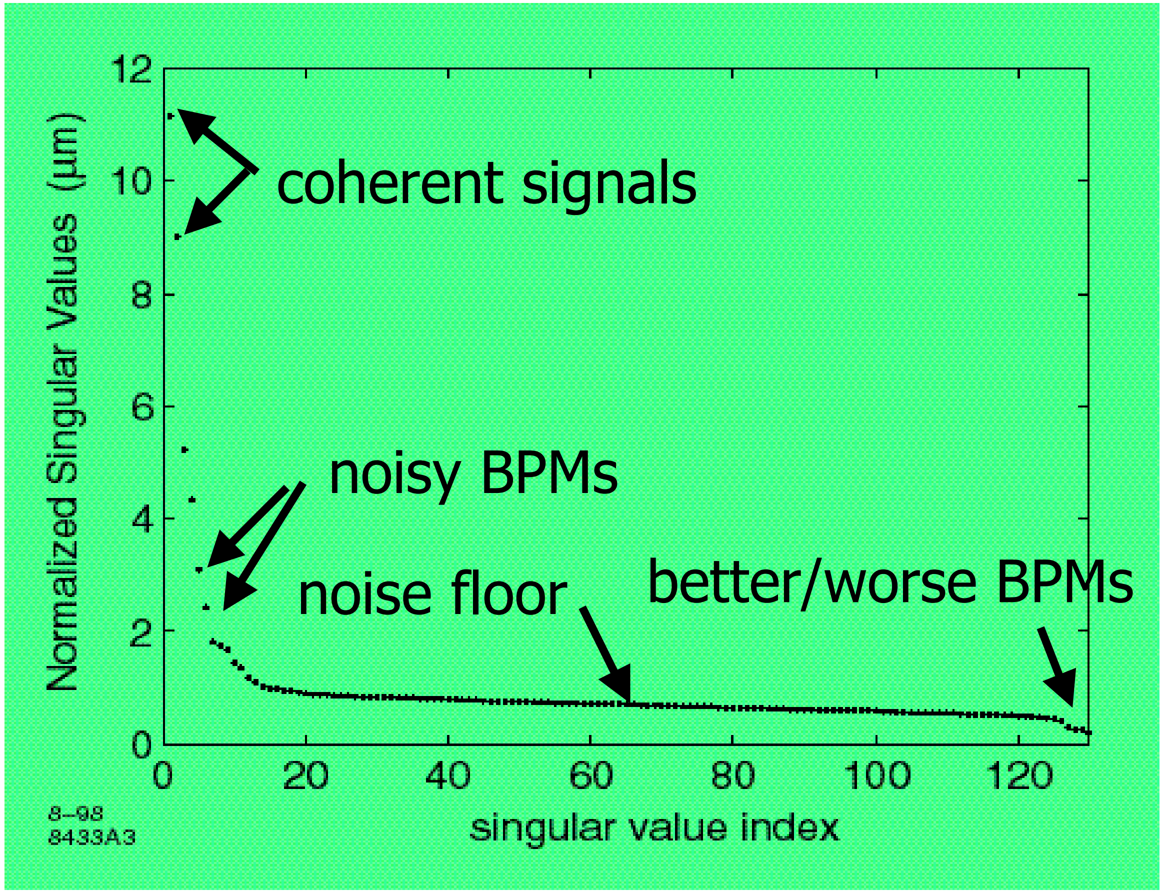
- Any Matrix M can be decomposed (SVD)

$$M = U \cdot S \cdot V^T = \sum_i \vec{u}_i \sigma_i \vec{v}_i^T,$$

- Where U and V are orthogonal matrices (I.e. $U \cdot U^T = 1,\; V \cdot V^T = 1$) and S is diagonal and contains the (σ_i) singular values of M.
- Examples:
  - M is the orbit response matrix
    - U contains an orthonormal set of BPM vectors
    - V contains an orthonormal set of corrector magnet vectors
  - M is a set of many (single turn/single pass) orbit measurements
    - U contains an orthonormal set of spatial vectors
    - V contains an orthonormal set of temporal vectors

Singular Value Spectrum

- Real physical modes (sine/cosine, nonlinearities)
- Faulty BPMs (individual)
- Noise Floor (of all BPMs)
- Some BPM with different noise

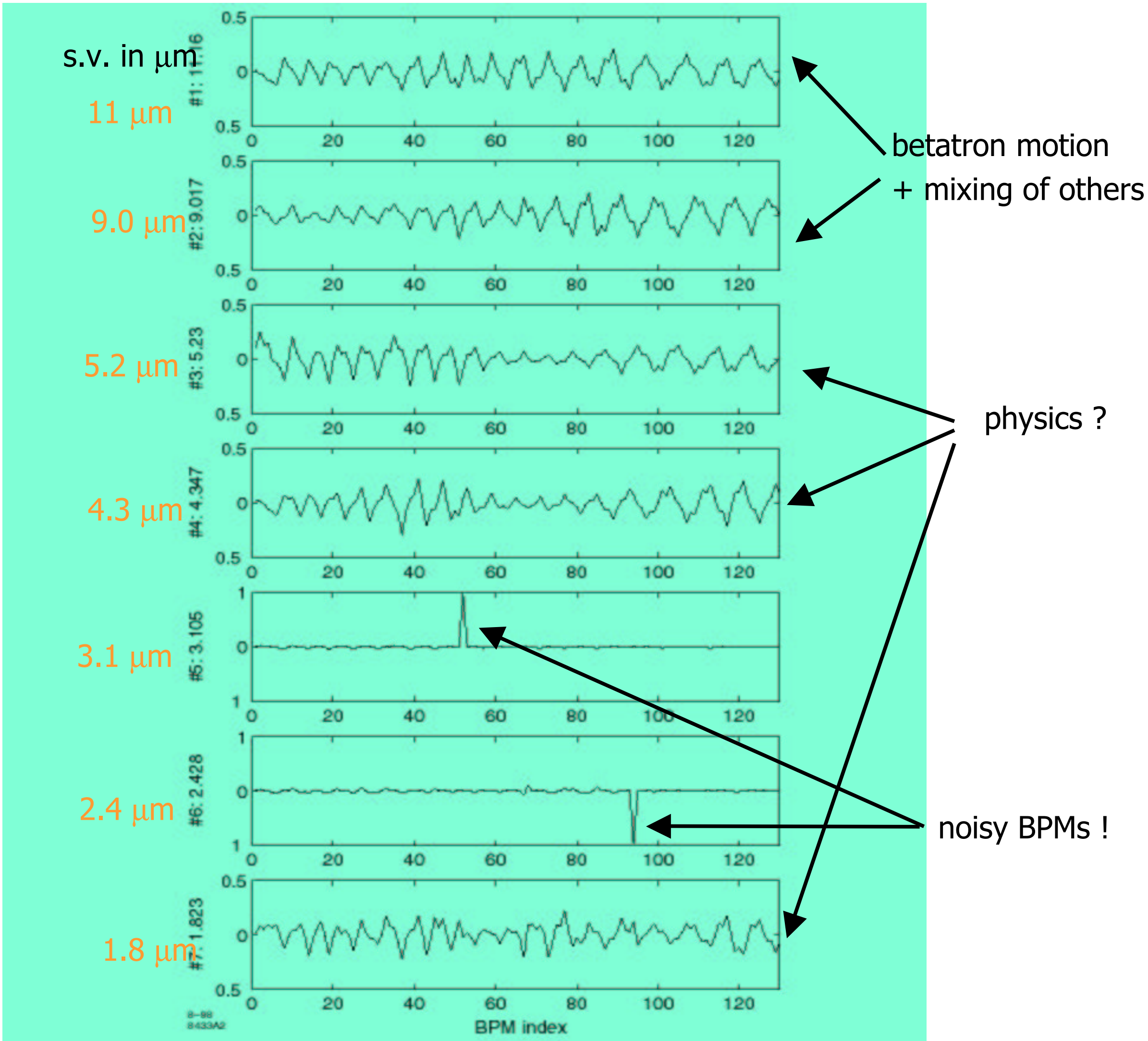


(SLC example, C.-X. Wang)

Spatial Singular Vectors

- Spatial vectors
  - Combine first two to get betafunctions
  - Identify faulty BPMs
  - Identify additional Physics

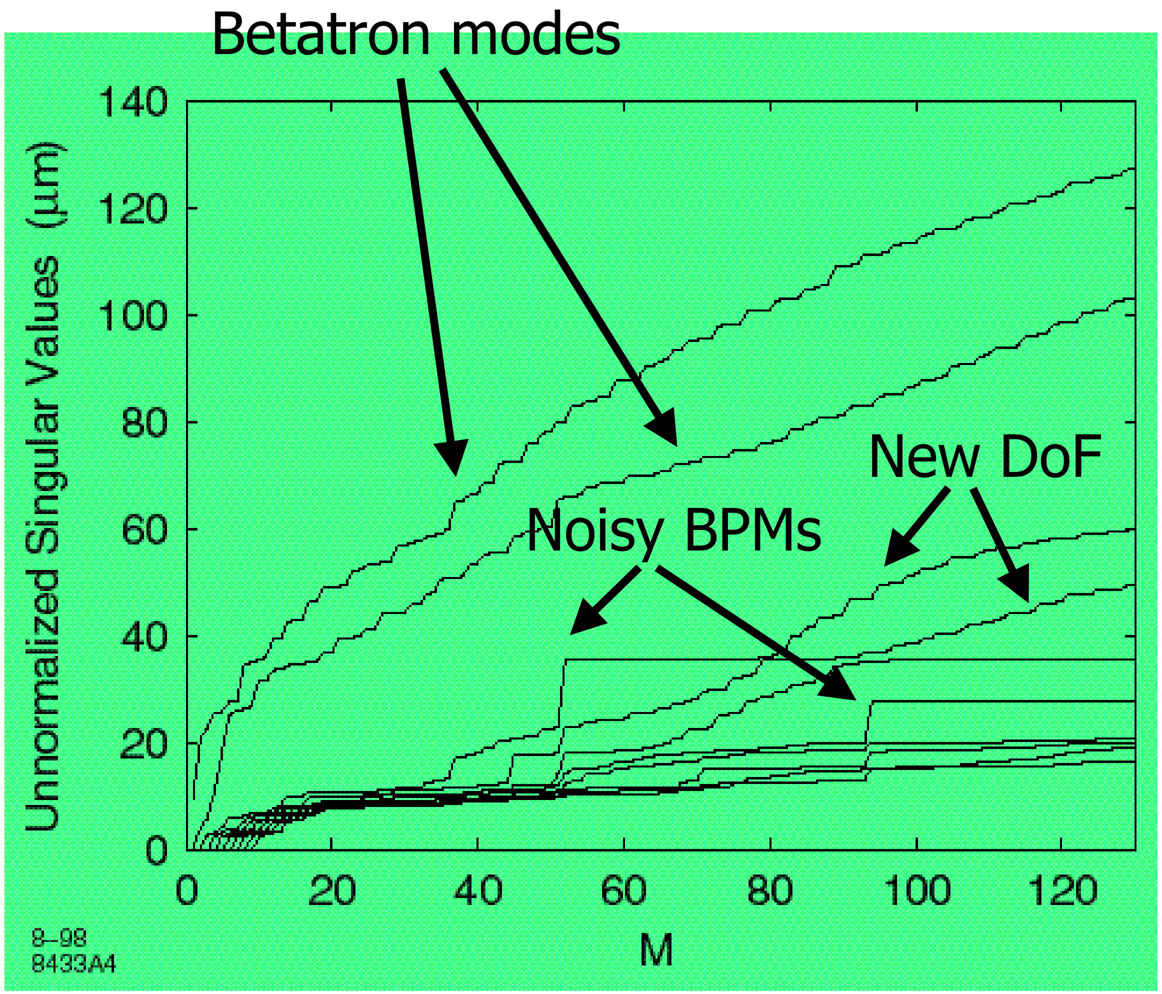
  (SLC example, C.-X. Wang)



Degree of Freedom Analysis

- ❖ Watching the evolution of singular values along a beamline allows to locate bad BPMs and especially to find the onset of new physics (wakefields, …)

  (SLC example, C.-X. Wang)



Temporal Analysis

- If you know the temporal pattern of external quantities, you can filter the system response to those quantities
- Example: know measured bunchlength, find wakefield kicks
- Energy, displacement, …
- Can be used by dithering quantities

Using measured temporal patterns, an example

Consider an ensemble of BPM readings given by

$$\{f_p = \Delta x_p f_x + \Delta y_p f_y \mid p = 1, \cdots, P\}$$

Suppose we know the temporal patterns f, Δx, and Δy.

To extract f_x and f_y, compute $\langle \Delta x f \rangle_p$ and $\langle \Delta y f \rangle_p$

$$\begin{bmatrix} \langle \Delta x f \rangle \\ \langle \Delta y f \rangle \end{bmatrix} = \begin{bmatrix} \langle \Delta x^2 \rangle & \langle \Delta x \Delta y \rangle \\ \langle \Delta y \Delta x \rangle & \langle \Delta y^2 \rangle \end{bmatrix} \begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

Inversing the matrix gives the solution:

$$\begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{bmatrix} \langle \Delta x^2 \rangle & \langle \Delta x \Delta y \rangle \\ \langle \Delta y \Delta x \rangle & \langle \Delta y^2 \rangle \end{bmatrix}^{-1} \begin{bmatrix} \langle \Delta x f \rangle \\ \langle \Delta y f \rangle \end{bmatrix}$$

- Courtesy of C.-X. Wang

Singular Value Spectrum

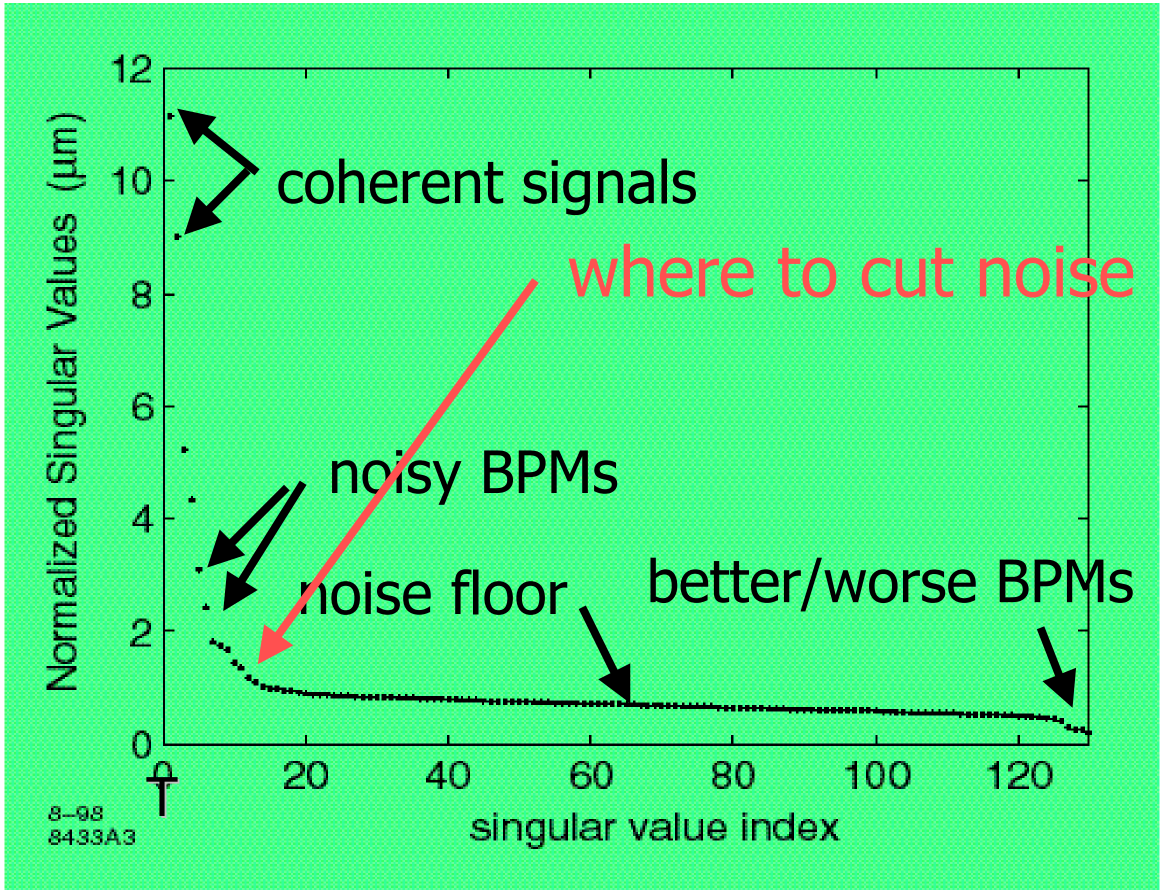
Reducing the random noise using Singular-Value Decomposition (SVD):

Identify the noise floor and zero the corresponding singular values

Re-multiply the matrices to generate noise-cut

Random noise is reduced by a factor of

$$\sqrt{\frac{d}{M}}$$

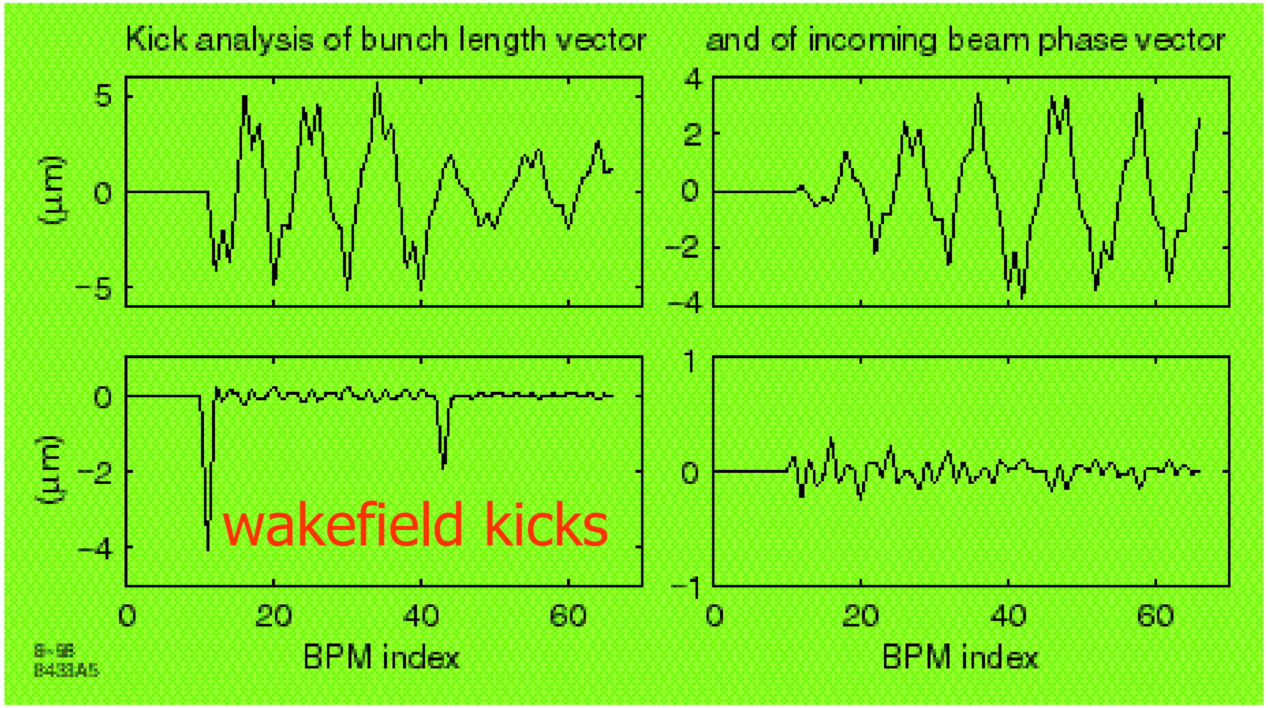


(SLC example, C.-X. Wang)

Simulated application: SLC wakefield kicks

- Kick analysis of spatial data allows location of wakefields (in simulation) due to bunchlength jitter
- Requires knowledge of unperturbed betatron trajectory (can be measured with MIA, e.g. dithering corrector at start of beamline)

  (SLC example, C.-X. Wang)



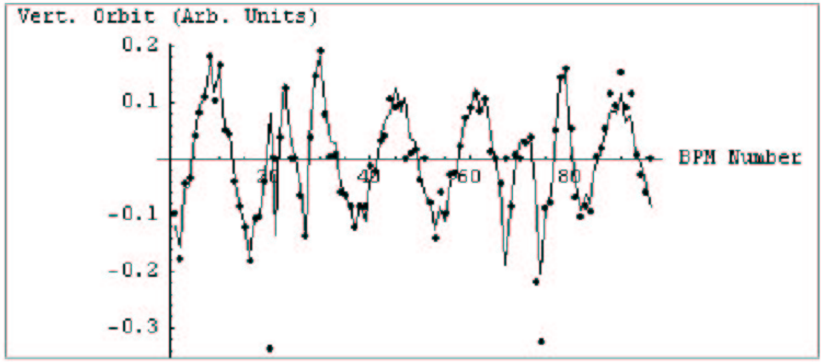
10% bunch length jitter   0.5   phase jitter

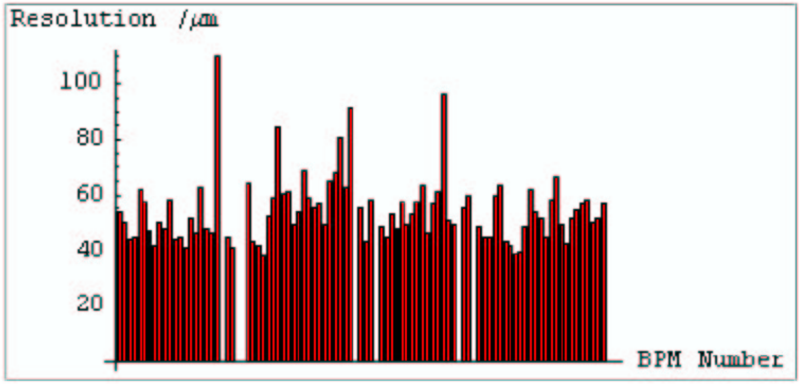
SLC simulation, 300 μm structure misalignments

Storage Rings: ATF example

- First tests, started dispersion/coupling correction based on MIA results

(Wolski, Ross, Woodley, Nelson)





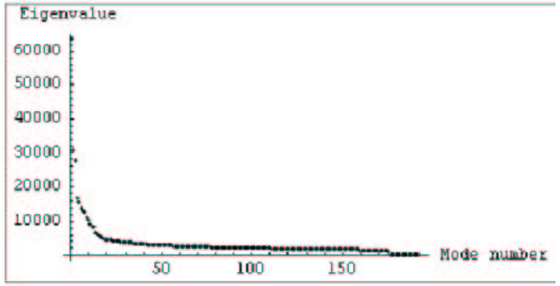


Figure 1: Eigenvalues from SVD of 7000 orbit vectors.

3.1 Betatron Modes

The modes corresponding to the first two eigenvalues are shown in Figures 2 and 3.

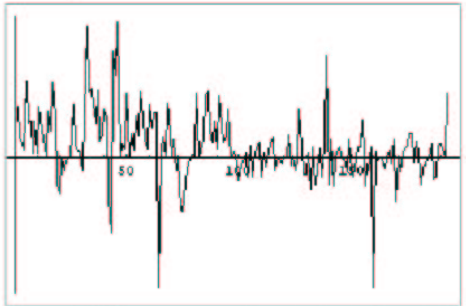


Figure 2: Mode corresponding to first eigenvalue.

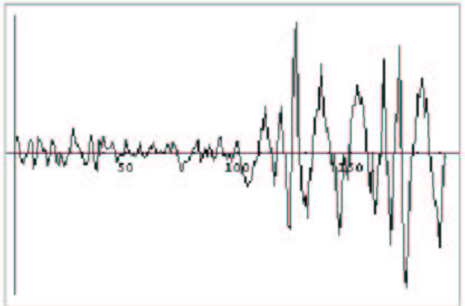


Figure 3: Mode corresponding to second eigenvalue.

Storage Rings: PEP-II example

- Has been used to correct beta beating
- Identify (SVD) four independent modes (coupling) for horizontal excitation and four for vertical excitation.
- Calculate phase advance and local Green's functions on those 'cleaned' modes.
- Fit lattice model to data.

(Yan, Cai, Irwin)

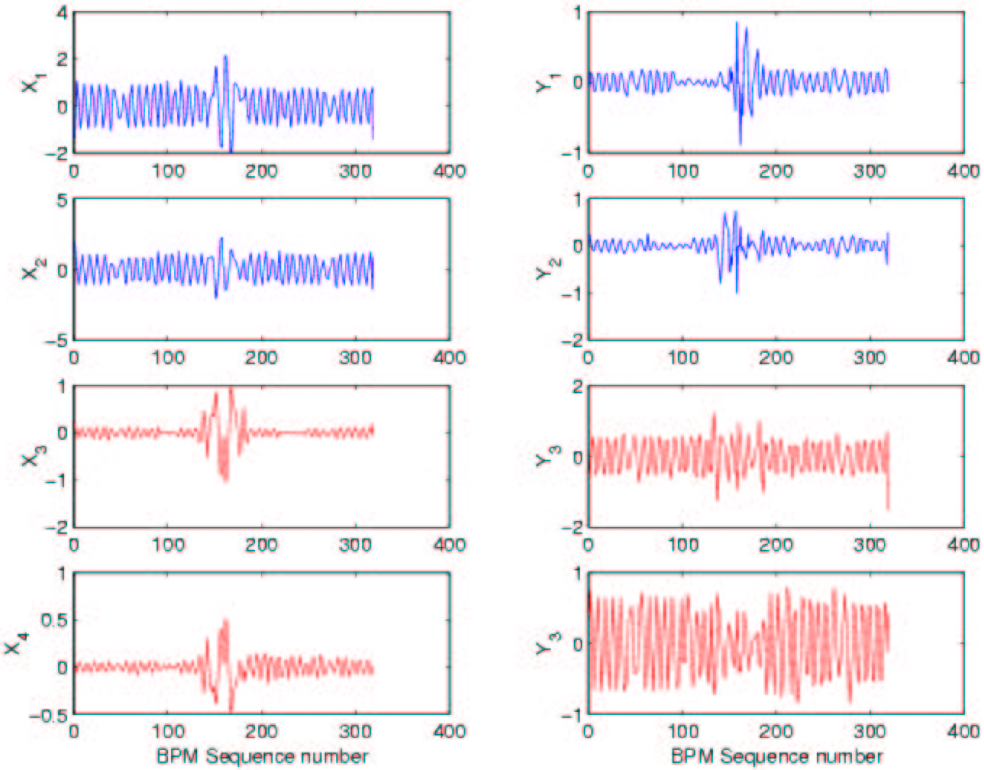
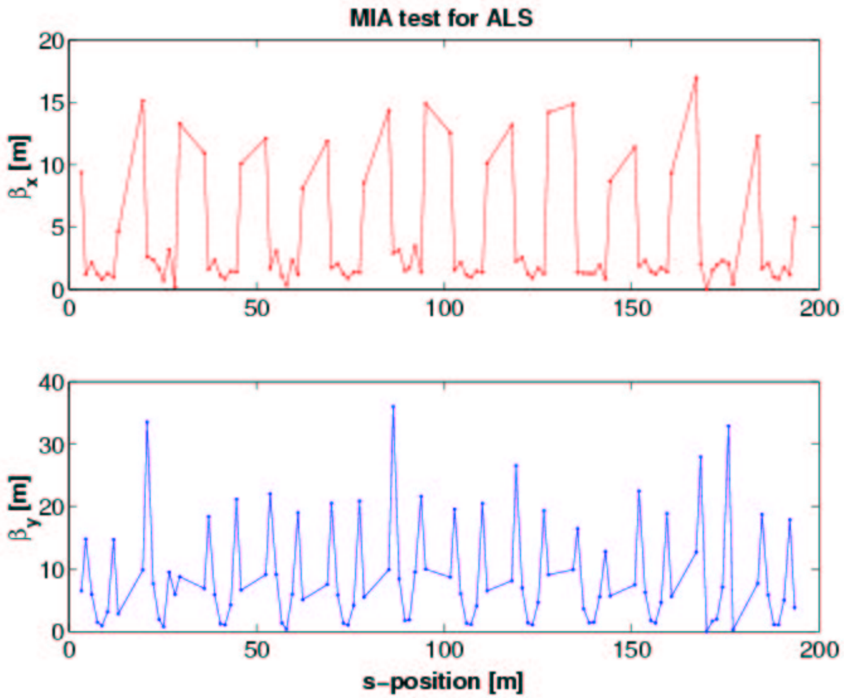
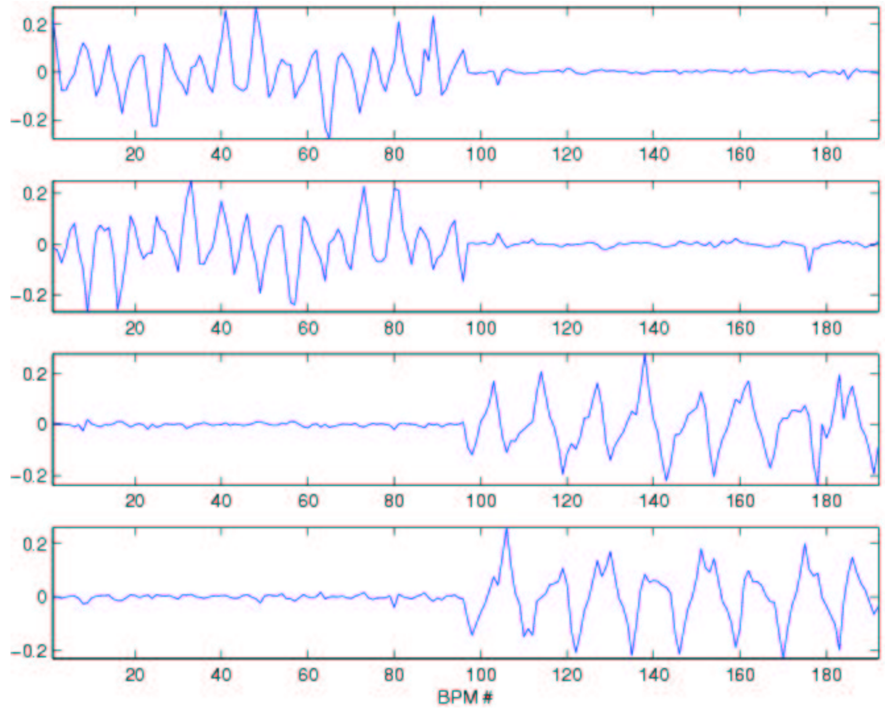


Figure 1: Four independent orbits extracted from PEP-II LER BPM buffer data. The first two orbits (x1, y1) and (x2, y2) are extracted from beam orbit excitation at the horizontal tune while the other two orbits (x3, y3) and (x4, y4) are from excitation at the vertical tune.

Storage Rings: ALS example





- ❖ MIA not used routinely so far
- ❖ Beta function measurement fast, but in turn-by-turn mode BPM gain errors large – would have to calibrate BPM gain independently
- ❖ Coupling terms visible in spatial vectors, could be used for coupling correction
- ❖ With BPM types at ALS response matrix analysis seems to be more accurate.

Summary

- MIA makes use of correlation between readings of multiple BPMs (or for multiple measurements) to reduce noise
- Useable for LINACs/beamlines and storage rings (multiple single turn measurements)
- Applications at LINACs appear very useful, but has been used successfully at storage rings as well (ATF, PEP-II).
- Those applications typically are not model independent anymore but require comparison with machine model (to compute corrections).
- In the storage ring cases a disadvantage over response matrix or phase advanced based methods is that one has to rely more on the correct absolute scaling and tilt of BPMs. Demonstrated level of optics correction so far not as good as the other methods (though MIA has only been around for a few years).

Further Reading

- J. Irwin, C.X. Wang, Y.T. Yan, et al., 'Model-Independent Beam Dynamics Analysis', Phys. Rev. Lett. 82, 1684 (1999).
- J. Irwin, and Y.T. Yan, 'Beamline model verification using model-independent analysis', SLAC-PUB-8515 (2000).
- Y. Cai, et al., ' Application of Model-Independent Analysis to PEP-II Rings', Proceedings of the 2001 Particle Accelerator Conference, Chicago, 3555 (2001).
- A. Wolski, et al., 'Initial Results from Model Independent Analysis of the KEK ATF', Proceedings of EPAC2002, Paris, France, 1205 (2002)