Model Independent Analysis

- Summary of work done by C. X. Wang, J. Irwin, Y.T. Yan, X. Huang and many others.
- Uses variations in successive BPM readings to debug accelerators.
- The correlations between BPM readings along a linac are used to improve BPM noise rejection.
- First developed for linacs. More recently extended to storage rings.
- For storage rings, much the same analysis as phase advance measurements, but with lower noise betatron phase measurement.

Statistical benefits of using a large number of BPMs

C.X. Wang

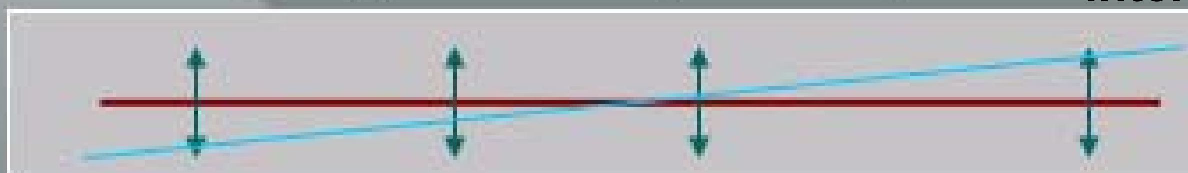
Noise reduction factor — Number of BPMs

- All BPMs are at the same location

$\sqrt{\frac{1}{M}}$ trivial

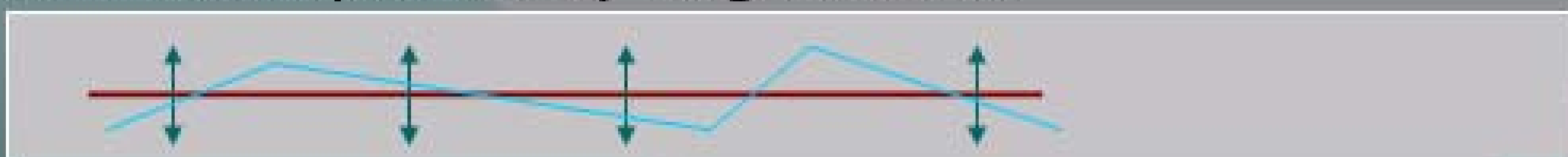
Slope + intercept

- BPMs are separated by drift spaces



$\sqrt{2/M}$ easy

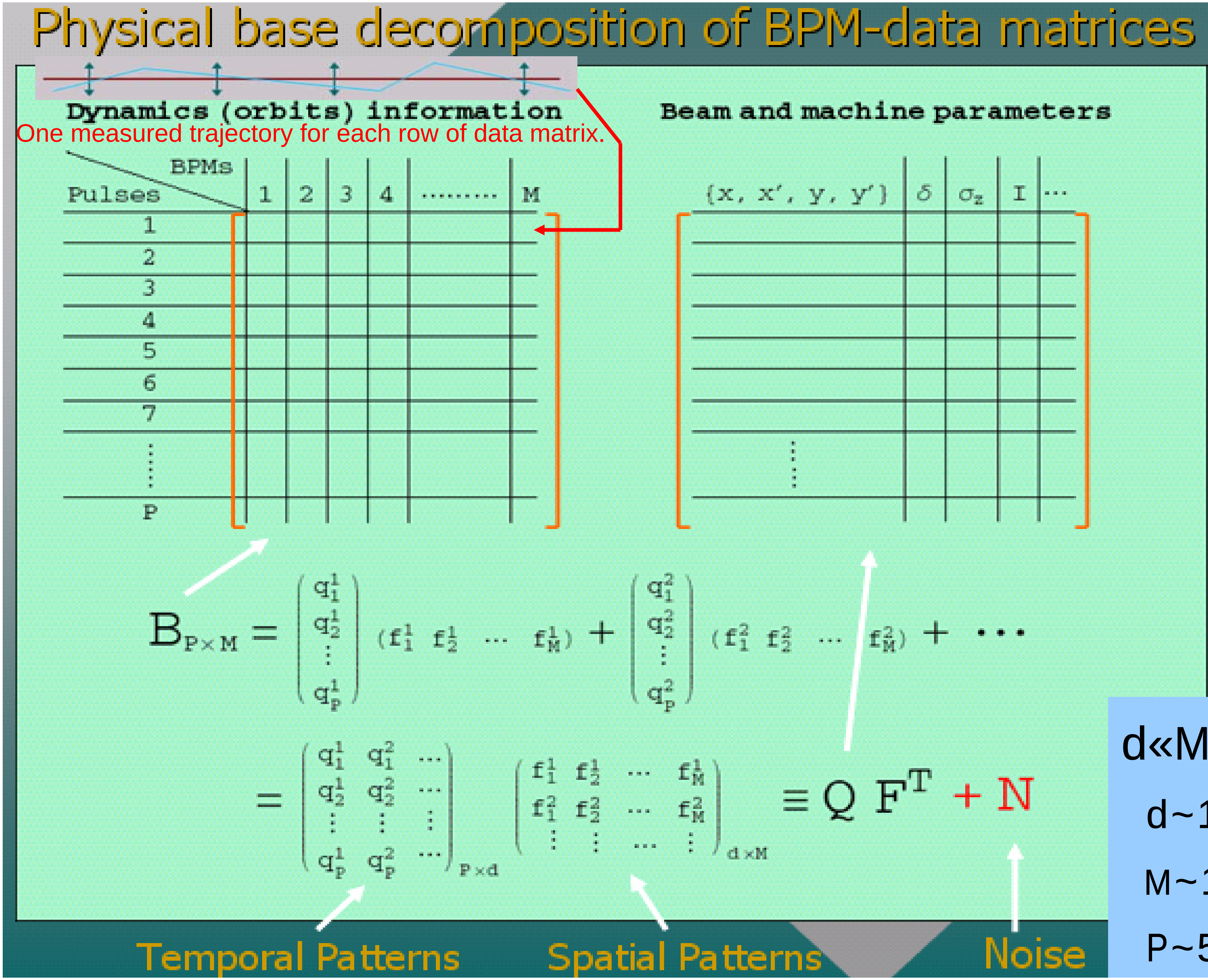
- BPMs are separated by magnets, etc.



* Known the exact transformation maps — doable

STOP Do NOT know transformation maps — :-(??

OR ... Measure many trajectories and see reproducible patterns (with some noise on each). Pull out reproducible part leaving noise behind = MIA.



d«M«P

d~10

M~100

P~5000

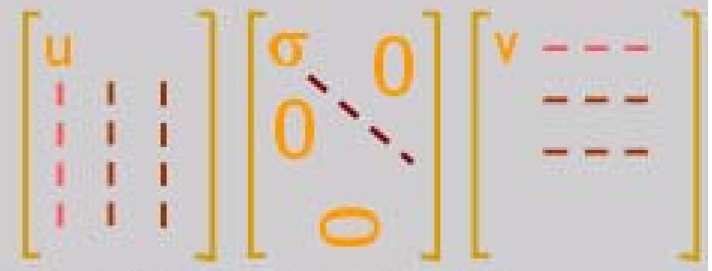
C.X. Wang

Apply SVD to the data matrix.

Singular Value Decomposition (SVD)

Mathematically, SVD of matrix B yields

$$B = USV^T = \sum_{i=1}^{d} \sigma_i u_i v_i^T$$



where $U_{P\times P} = [u_1, \cdots, u_P]$ and $V_{M\times M} = [v_1, \cdots, v_M]$ are orthogonal matrices, $S_{P\times M}$ is a diagonal matrix with nonnegative σ_i along the diagonal

- σ_i is the i-th largest singular value of B
- $d = \text{rank}(B)$ is the number of nonzero singular values.
- the vector u_i (v_i) is the i-th left (right) singular vector. u's and v's form orthogonal basis of the various spaces of B.

C.X. Wang

SVD and eigenvalue problems of real symmetric matrices

$$(B^T B)V = VS^2 \qquad \text{and} \qquad (BB^T)U = US^2$$

Therefore,

- the column vectors of V are eigenvectors of the real symmetric matrix $B^T B$
- the column vectors of U are eigenvectors of the real symmetric matrix BB^T
- eigenvalues are given by σ_i^2's.

Also, the covariance matrix of BPM readings can be decomposed as

$$C_B = VS^2V^T$$

Since C_B is a stationary quantity, V and S should be repeatable. (but not U)

C.X. Wang

- B^TB is the covariance matrix.
- If no real motion, just BPM noise, then singular values are simply BPM noise levels.
- With real motion added, get additional larger singular values associated with real motion.

Meaning of singular values in MIA

$$B^T B = \sum_{i=1}^{d} \sigma_i^2 v_i v_i^T = \begin{bmatrix} \mathrm{var}(\mathrm{BPM}_1) & \mathrm{cov}(\mathrm{BPM}_{12}) & \cdots \\ \mathrm{cov}(\mathrm{BPM}_{21}) & \mathrm{var}(\mathrm{BPM}_2) & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}$$

Comparing the diagonal terms we have

$$\mathrm{var}(\mathrm{BPM}_k) = \sum_{i=1}^{d} \sigma_i^2\, v_i(k)^2, \qquad k = 1, \cdots, M$$

Since the v vectors are normalized to 1, we have

$$\sigma_i^2 = M\, \overline{\mathrm{var}(\text{BPM readings due to } i\text{th mode})} \qquad \text{and} \qquad \sum_{i=1}^{d} \sigma_i^2 = \sum_{k=1}^{M} \mathrm{var}(\mathrm{BPM}_k)$$

where the overhead bar means average over all M BPMs.

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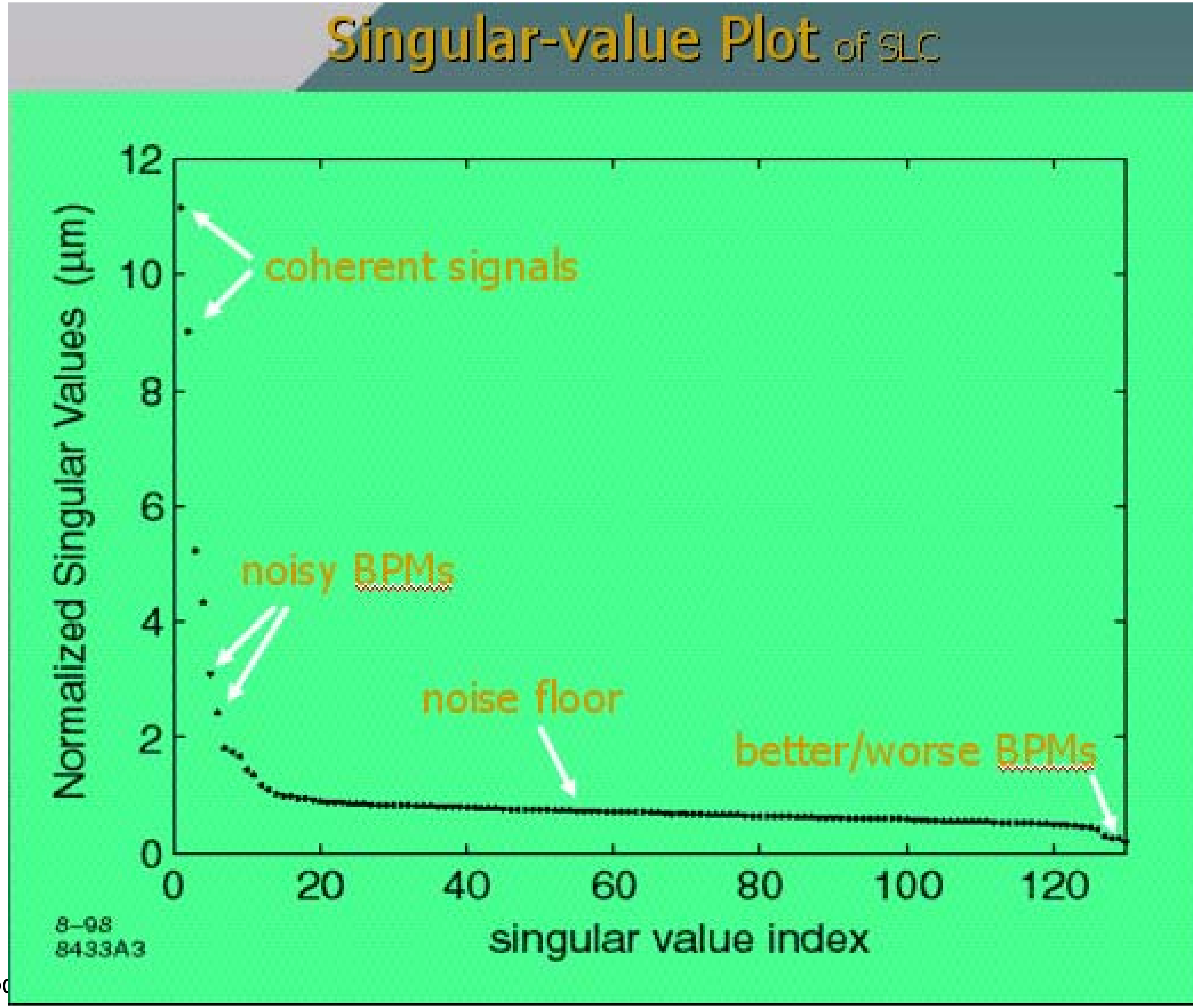
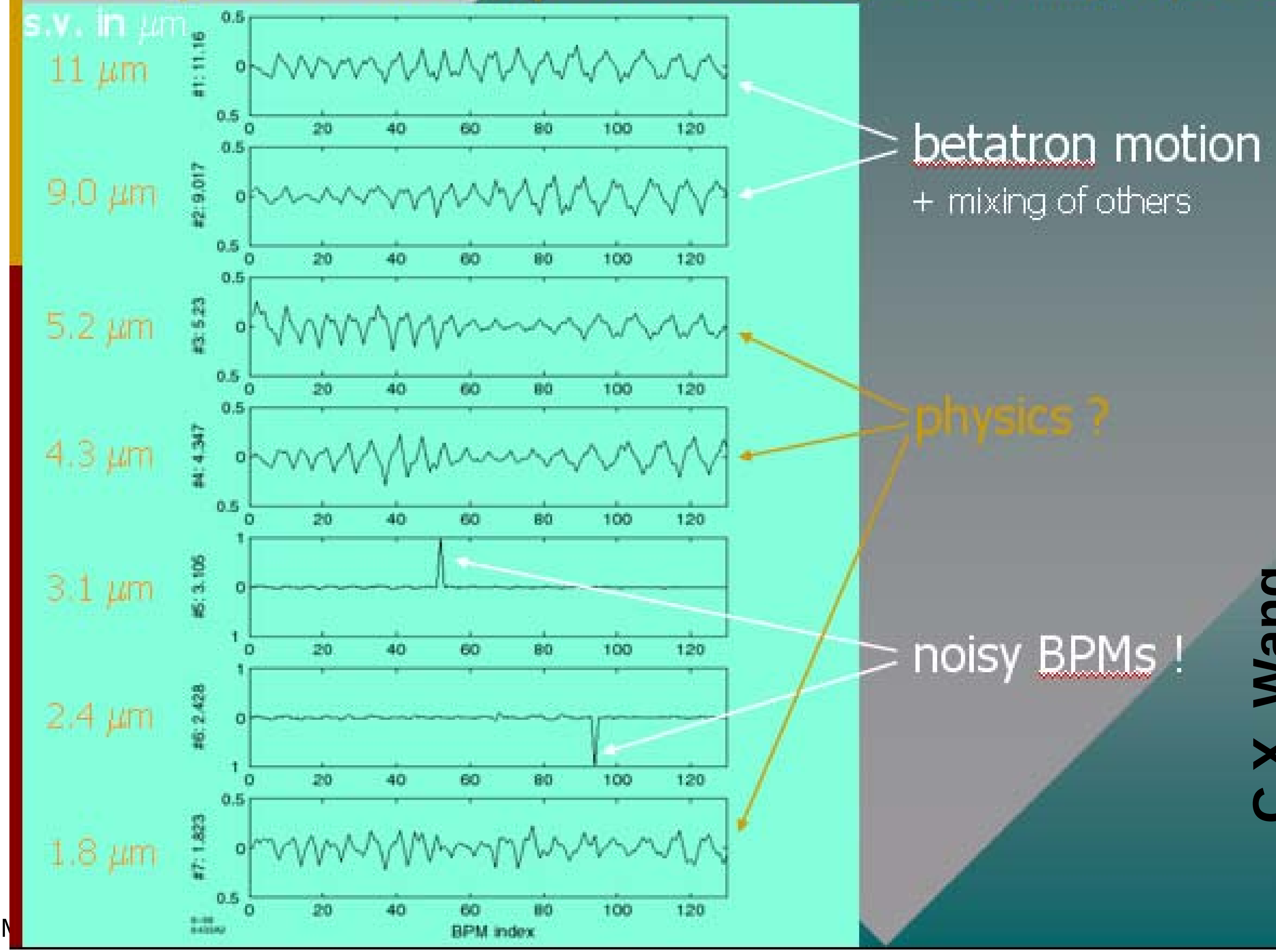

Singular-value Plot of SLC
Normalized Singular Values (µm)
singular value index
coherent signals
noisy BPMs
noise floor
better/worse BPMs
0
2
4
6
8
10
12
0
20
40
60
80
100
120
8–98
8433A3

C.X. Wang

First 7 spatial vectors/patterns of SLC first 3rd linac, x-plane



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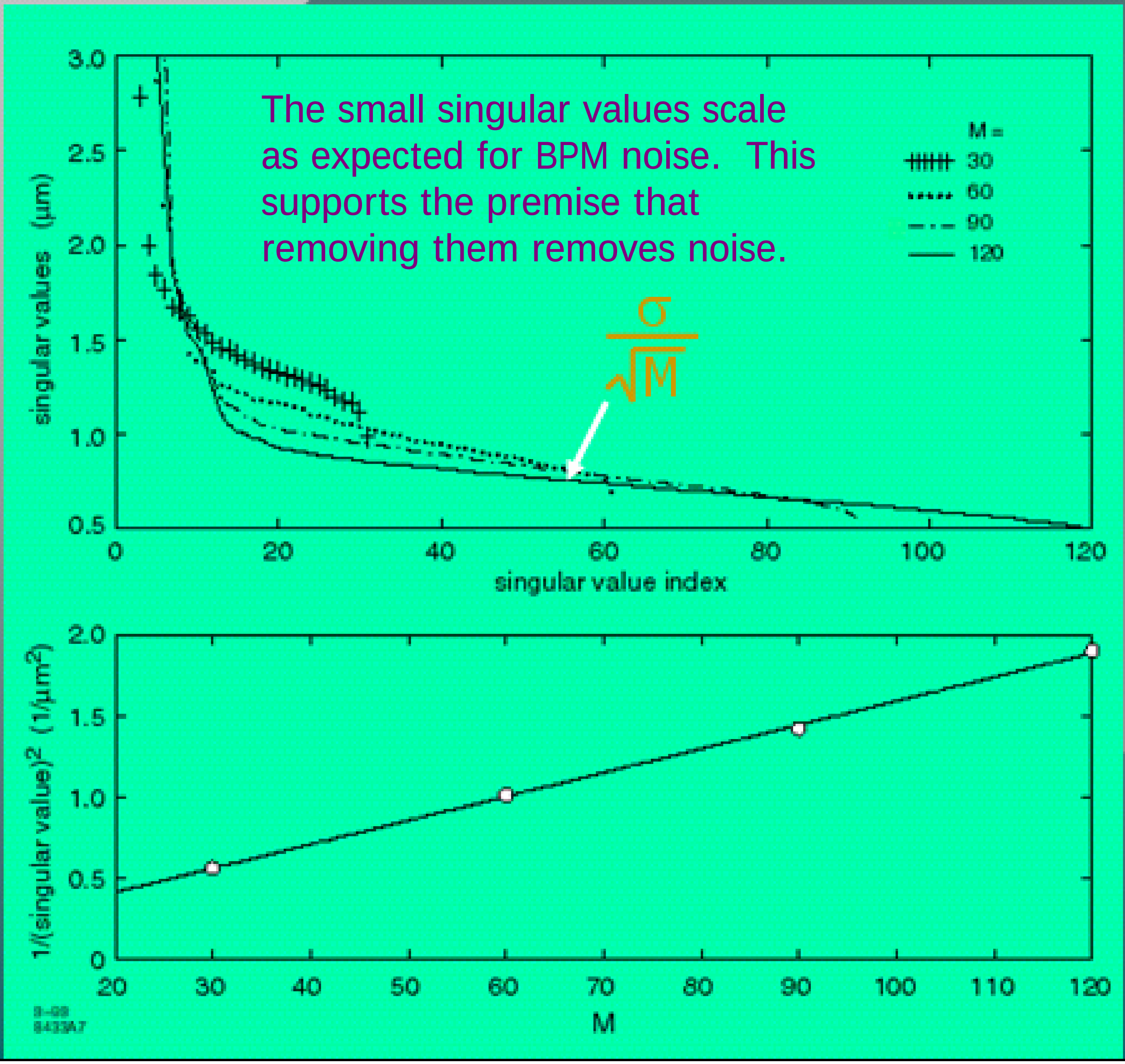
Noise floor dependency on the number of BPMs--M



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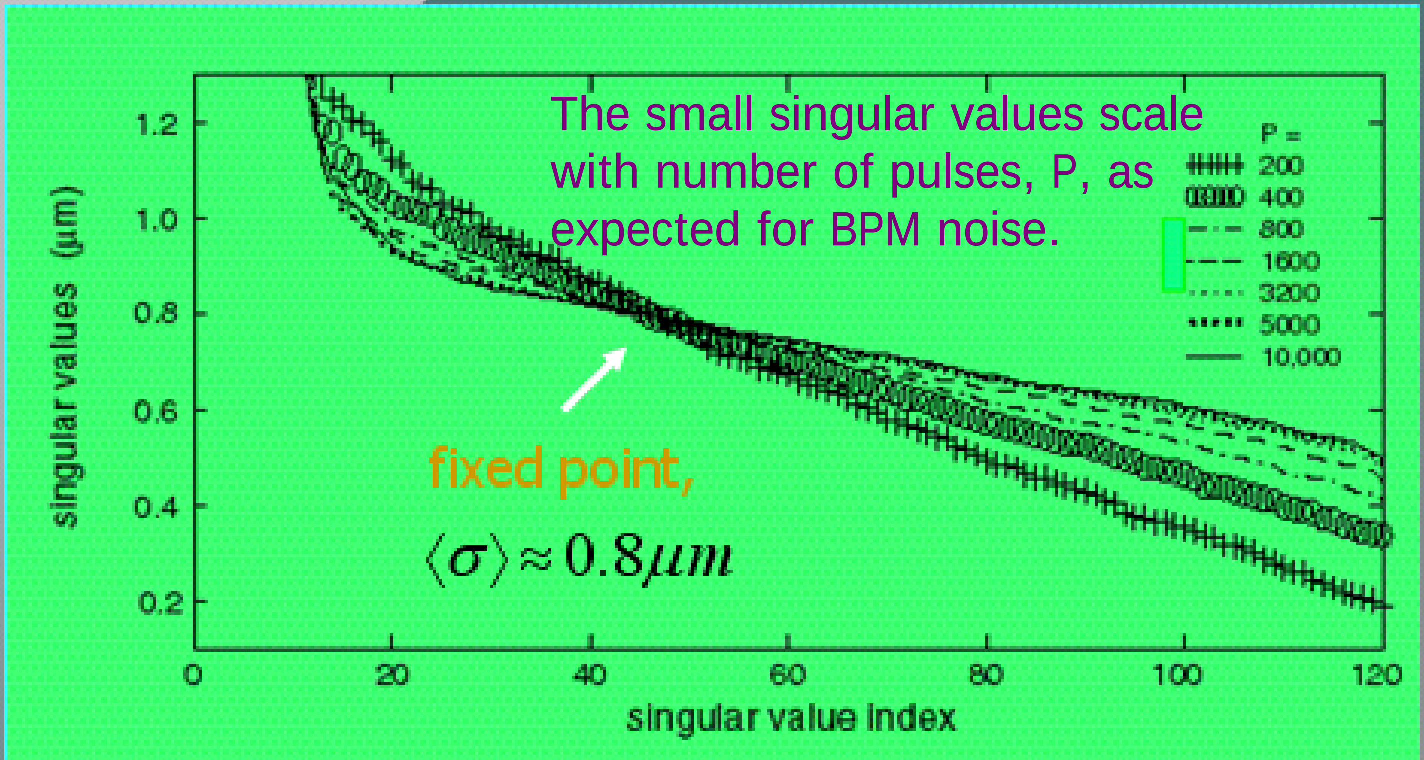
Noise floor dependency on the number of pulses--P



BPM resolution spectrum:

- Average resolution $\approx \sqrt{120}\langle\sigma\rangle \approx 9\mu m$ =0.8 μm
- Spread $\approx 2\mu m$

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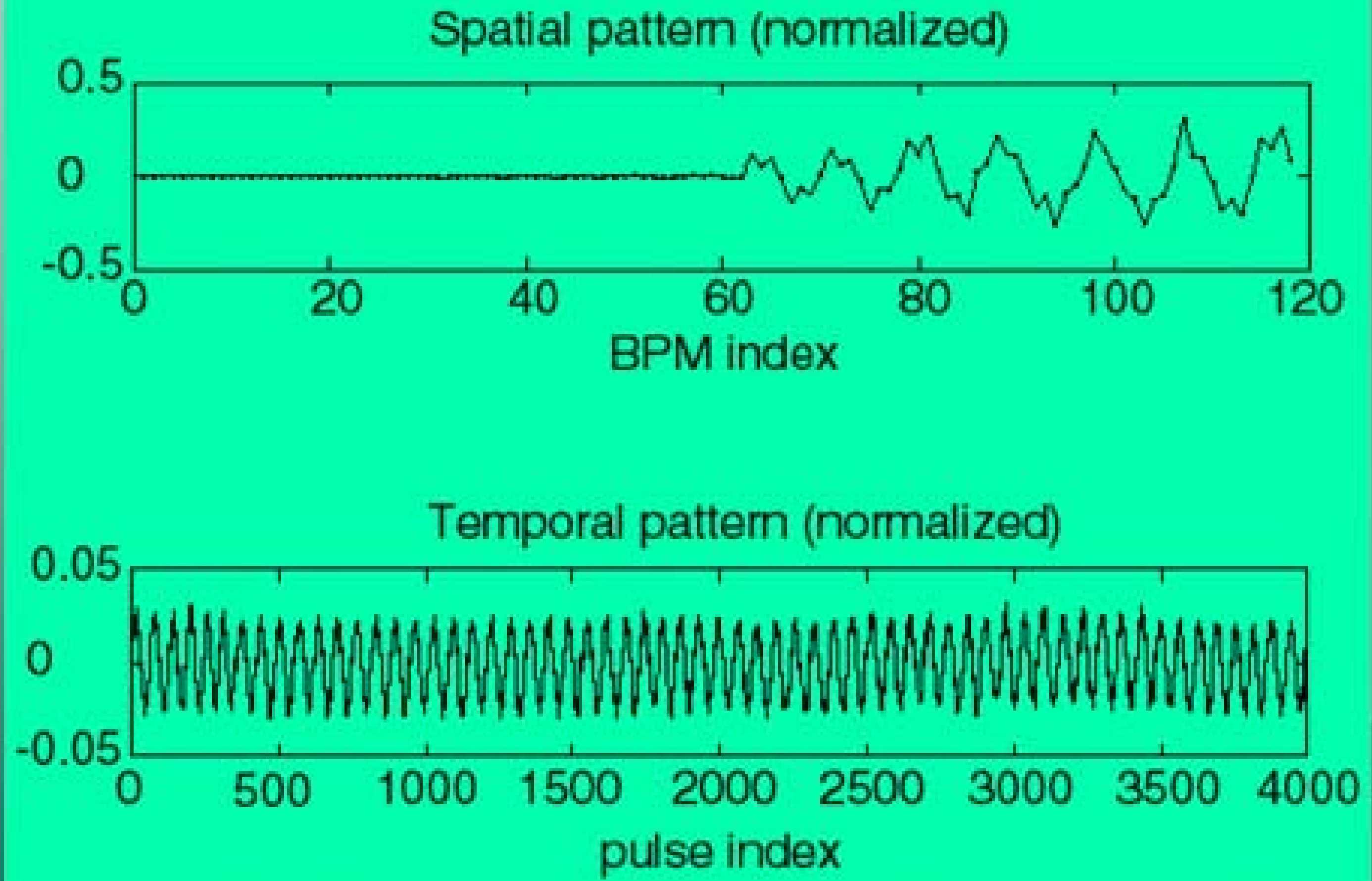
A physical mode due to a dithering corrector

When motion is induced while measuring data matrix, it shows up in one of the large singular value eigenvector pairs.



Experiment done at SLC

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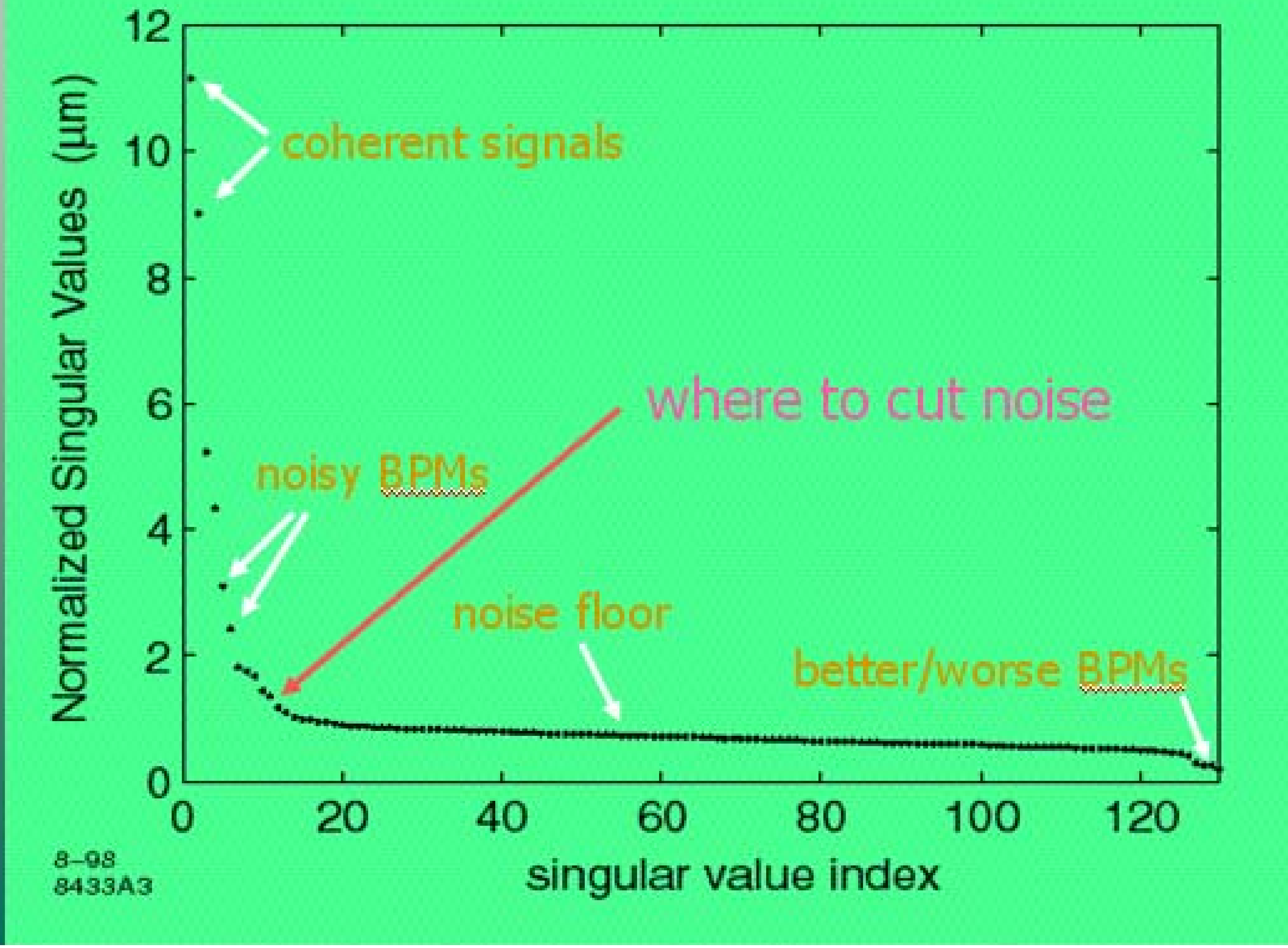
Singular-value Plot of SLC



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SVD noise reduction

- Singular-Value Decomposition (SVD)

$$B = U\,S\,V^T$$

- Identify the noise floor and zero the corresponding singular values

$$S \rightarrow \underline{S}$$

- Re-multiply the matrices to generate noise-cut $\underline{B}$

$$\underline{B} = U\,\underline{S}\,V^T$$

- Random noise is reduced by a factor of $\sqrt{\frac{d}{M}}$

C.X. Wang

Effects of noise-cut on the first 5 pulses of 5000

Simulations demonstrate noise reduction.

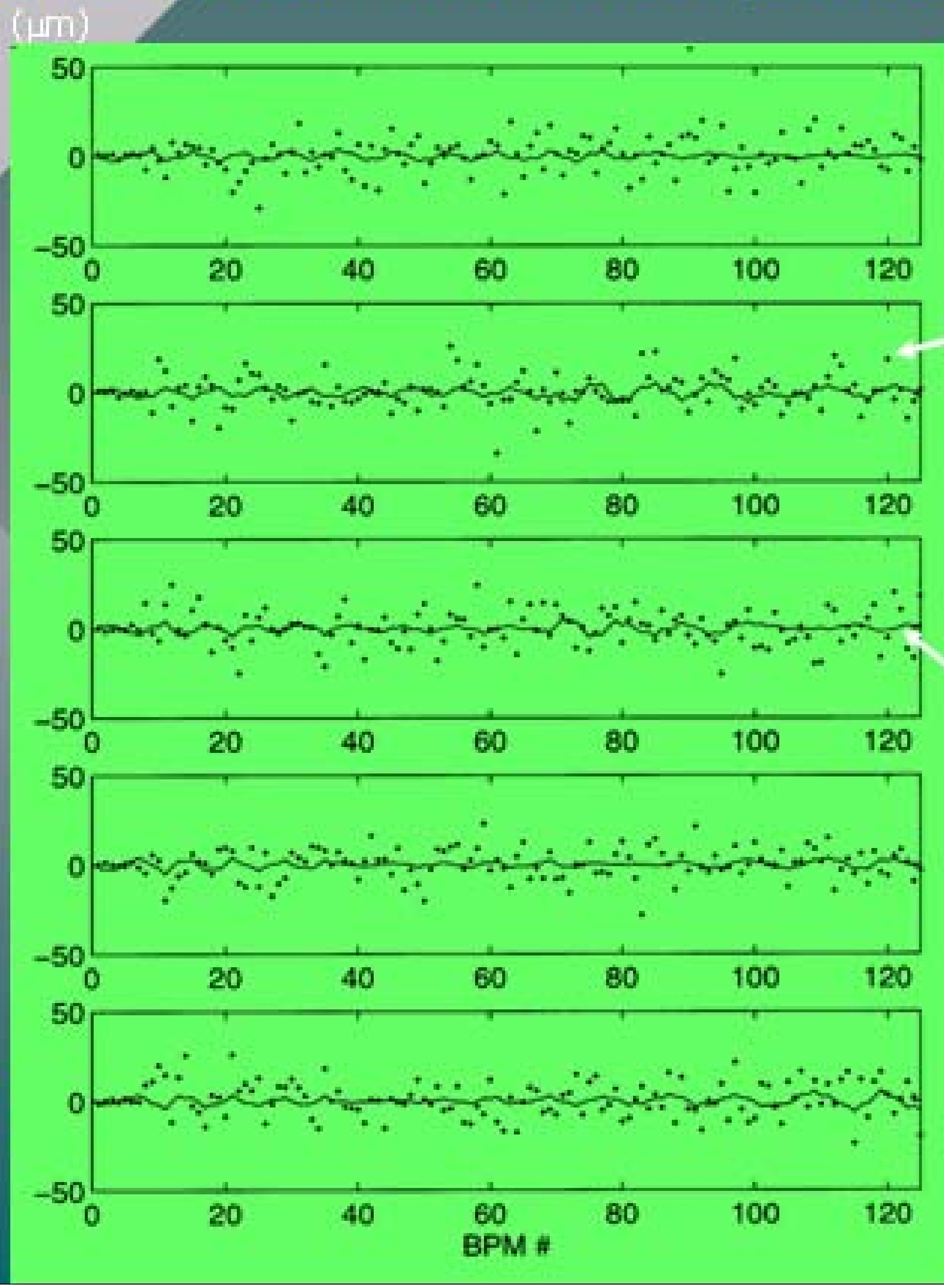
Pulse #1

#2

#3

#4

#5



Initial noise

Residual noise

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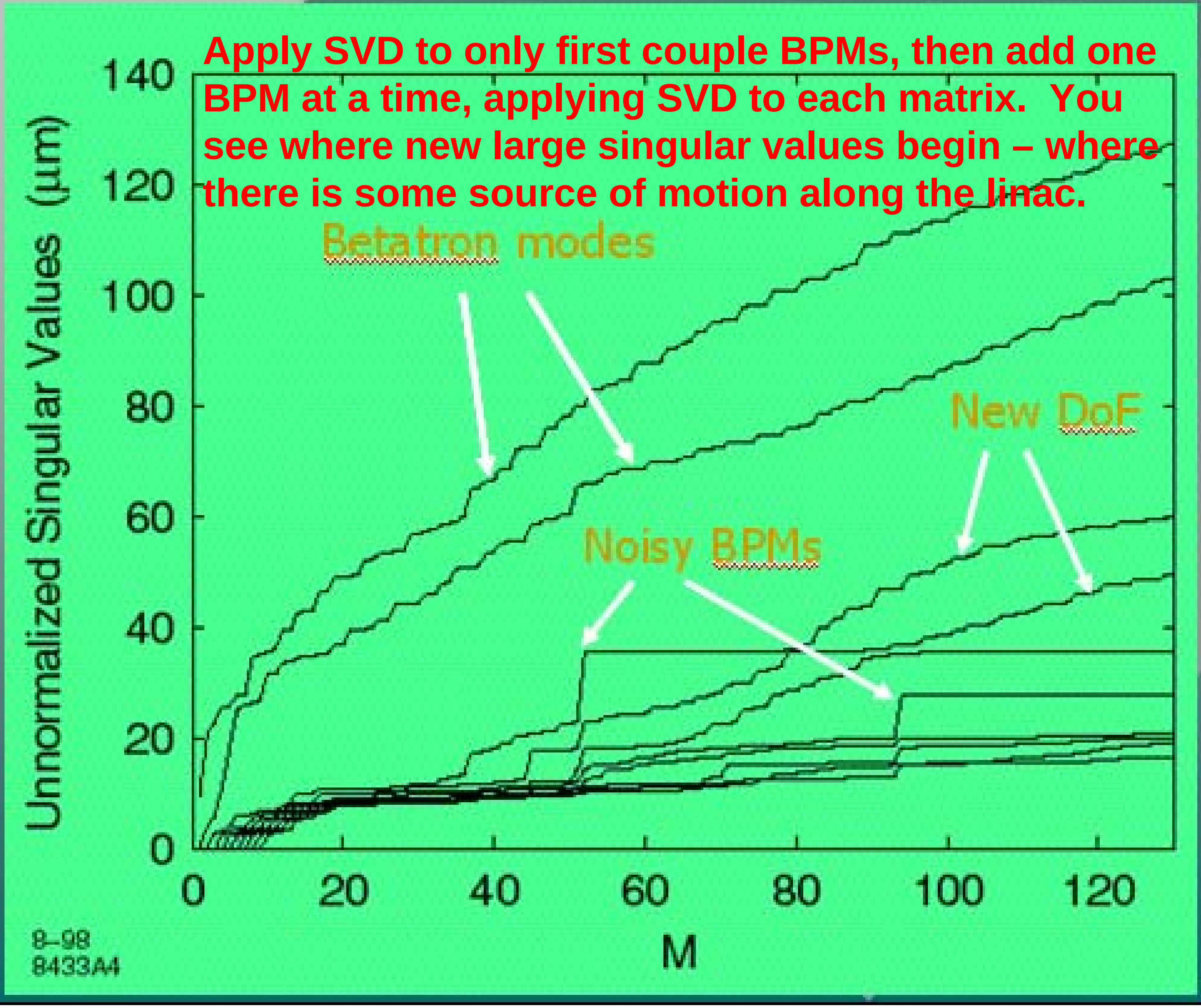
Degrees-of-Freedom Analysis



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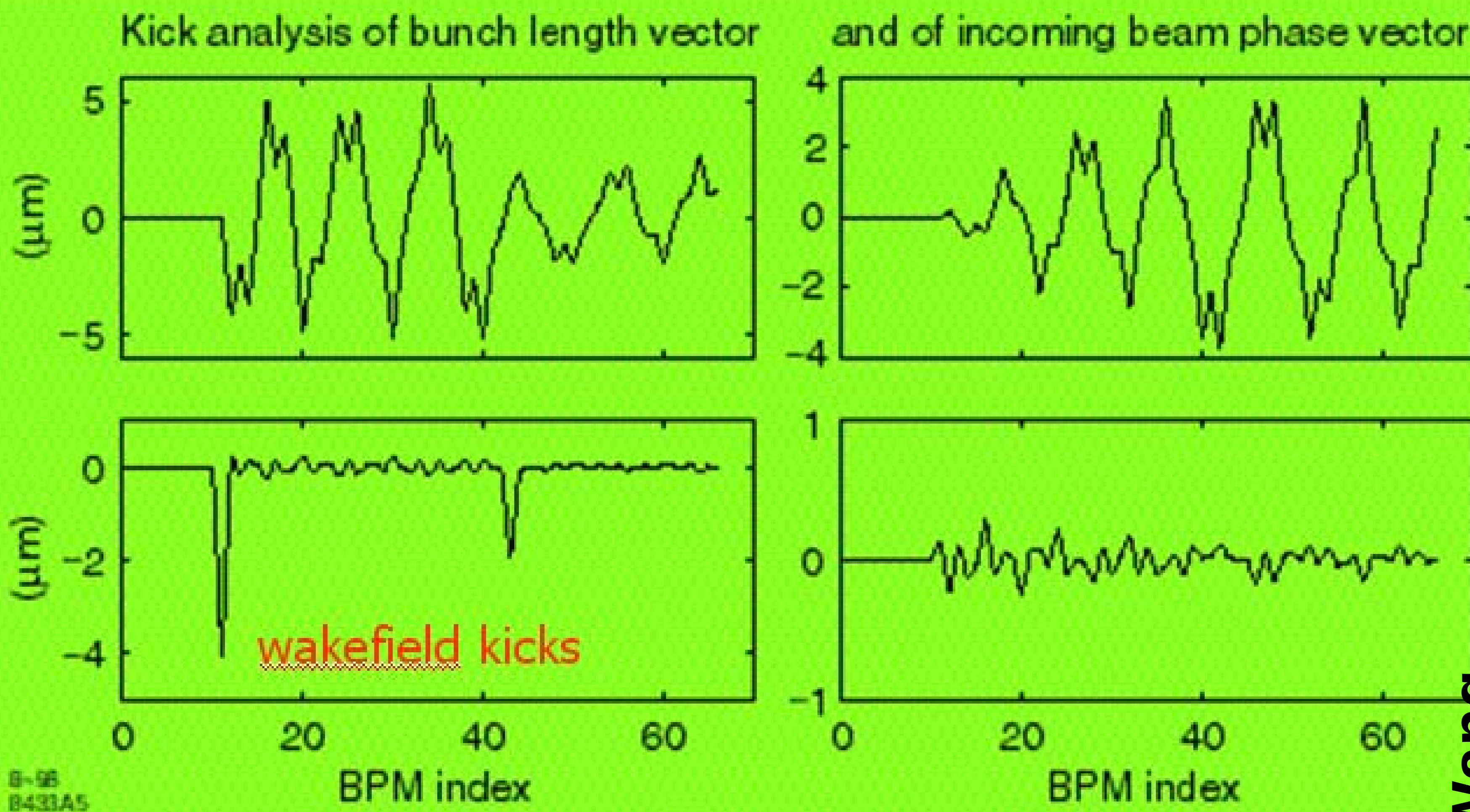
Kick analysis of spatial patterns



10% bunch length jitter

0.5° phase jitter

SLC simulation, 300 µm structure misalignments

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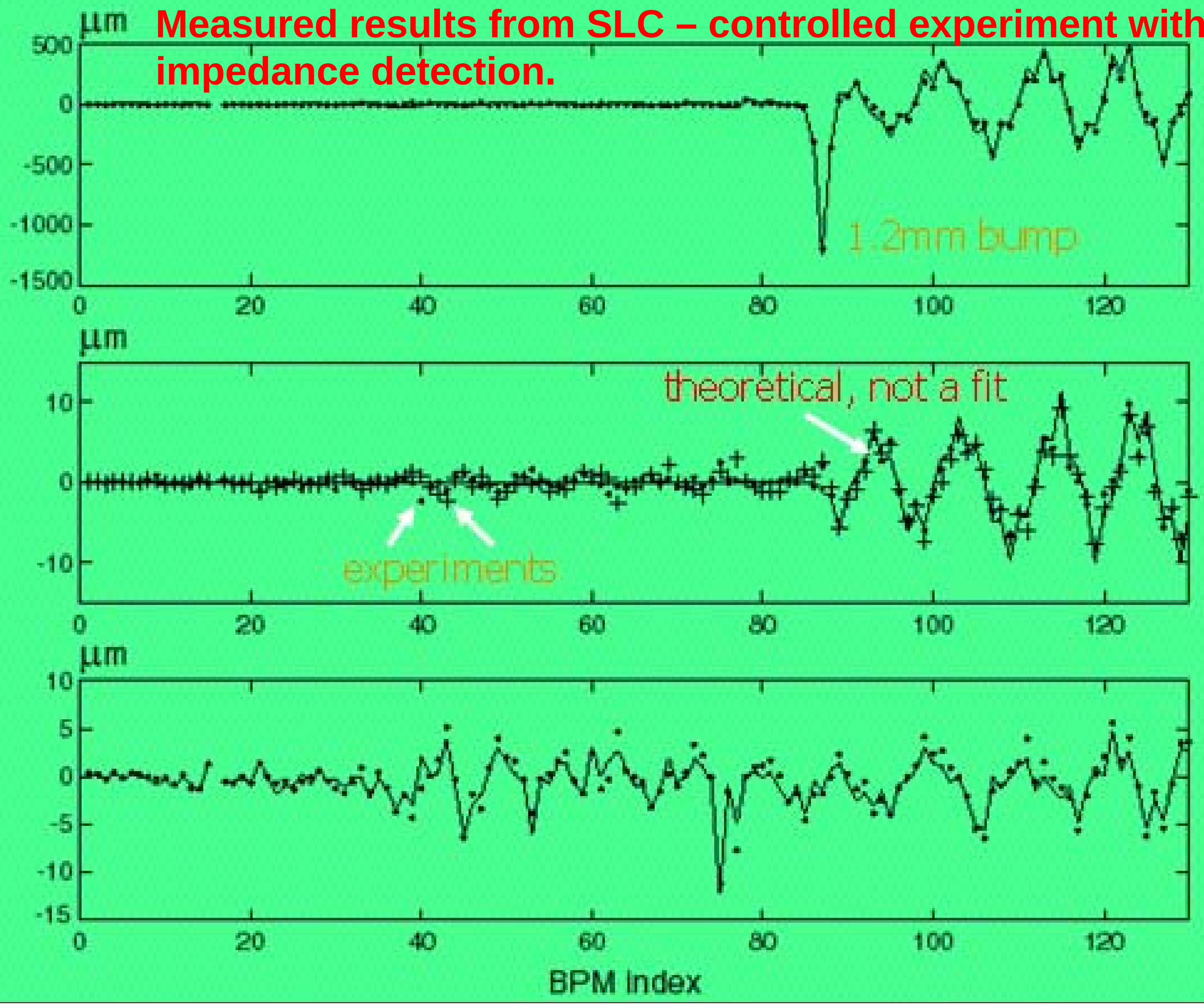
Transverse wakefield effect measurements

C.X. Wang

Measured results from SLC – controlled experiment with impedance detection.



corrector bump

bump wake effect

SLC wake effect

MIA in rings, three techniques under development

- All three measure turn-by-turn driven oscillations to measure betatron phase advances.
- PEP-II (&ATF), Y. Yan, J. Irwin, Y. Cai et al.
  – Include measurement of transfer matrices between BPMs
- APS, C.X. Wang
  – Untangling mixed modes from PCA
- Fermilab, X. Huang
  – Applying ICA to untangle mixed modes.

MIA for PEP-II and ATF

- Has been used to correct beta beating at PEP-II and coupling at ATF.
- Identifies two principle modes for the two phases of horizontal and vertical betatron modes, including coupling.
- Use these modes to derive betatron phase advance and R12, R32, R14, R34 elements of the transfer matrix between BPMs
  - Fit model to above
  - Include BPM gains and coupling in fit
- Issues with coupling between measured modes; see both normal mode frequencies in both modes.

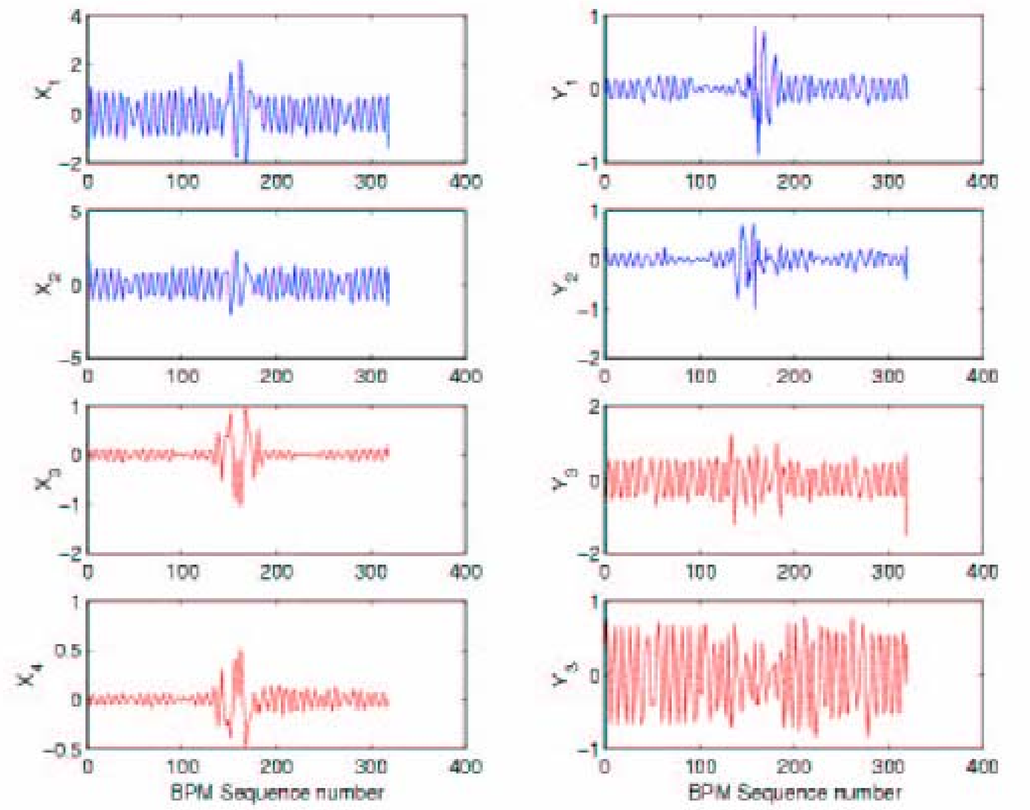


Figure 1: Four independent orbits extracted from PEP-II LER BPM buffer data. The first two orbits (x1, y1) and (x2, y2) are extracted from beam orbit excitation at the horizontal tune while the other two orbits (x3, y3) and (x4, y4) are from excitation at the vertical tune.

Summary of variables fit for PEP-II

- All quad families normal components.
- All skew (global and local) quad's skew components.
- All sextupole feed-downs' normal and skew components.
- All BPM gains and cross couplings.

- Additionally, all quad skew components and all skew quad normal components.

- Also form all these variables into a sequence (a single vector with data-structure pointers).

Yiton Yan

Enhanced SVD-enhanced Least Square fitting

we form all variables and all measured linear orbit derivatives into one-dimensional arrays, i.e. vectors, represented by $\vec{X}$ and $\vec{Y}m$ respectively. The corresponding derivatives from the model, which are implicit functions of all the fitting variables, are also formed into a vector functional form given by $\vec{Y}(\vec{X})$. Denoting the reasonably guessed variable values as $\vec{x}_o$, and letting $\vec{X} = \vec{x}_o + \vec{x}$, one has

$$\vec{Y}(\vec{x}_o + \vec{x}) = \vec{Y}(\vec{x}_o) + M\vec{x} + \vec{\eta}(\vec{x}) = \vec{Y}m,$$

where $\vec{\eta}(\vec{x})$ contains nonlinear terms of the the Taylor expansion of $\vec{Y}(\vec{x}_o + \vec{x})$, which is ignored in the iteration equation given by

$$M\vec{x} = \vec{Y}m - \vec{Y}(\vec{x}_o) = \vec{b}.$$

Yiton Yan

Need about two iterations of enhanced SVD-enhanced fittings.

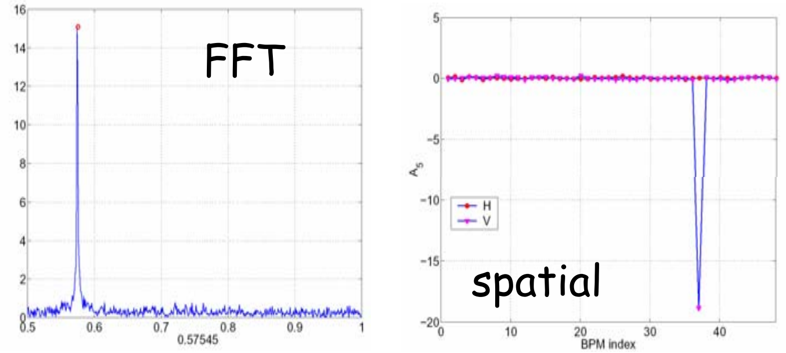
ICA method

- PCA uses SVD on matrix B, and find eigenvectors and eigenvalues of $B^T B$ and BB^T
- ICA also diagonalizes $B(t)^T B(t+n)$, the unequal time covariance matrices, with multiple choices of n, the number of turns of time inequality.
- In this way, it forces the principal components to have separate frequencies.

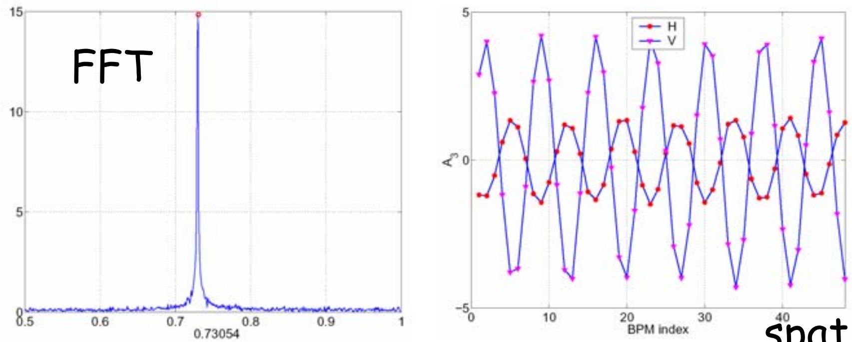
Decoupling mixed modes of PCA MIA with ICA

Comparison of the ICA and PCA

In this simulation, one sinusoidal signal is inserted to one of the BPMs to simulate a bad BPM. Then both ICA and PCA methods are applied to separate the modes.

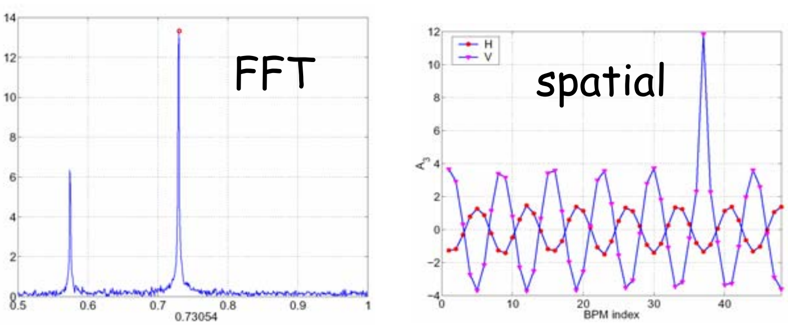


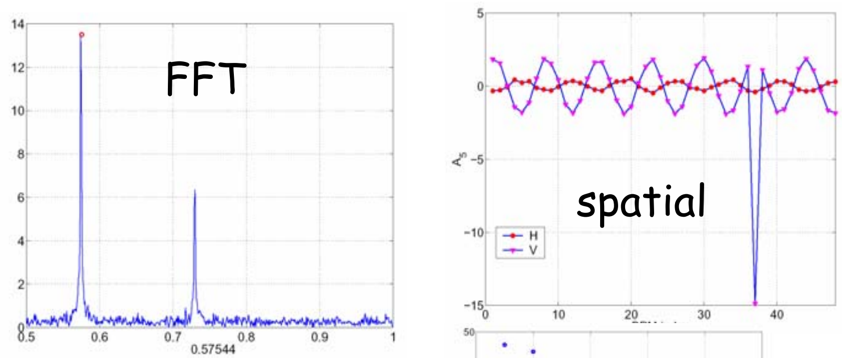
The bad-BPM mode

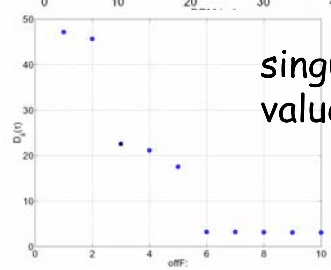


One of the betatron modes

ICA The ICA method always separates the inserted signal.





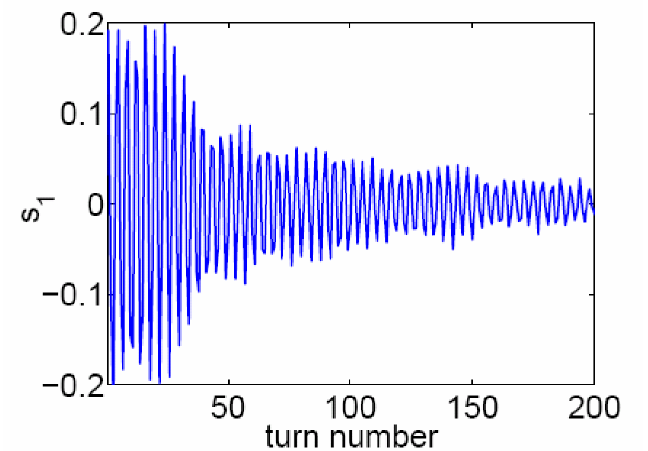


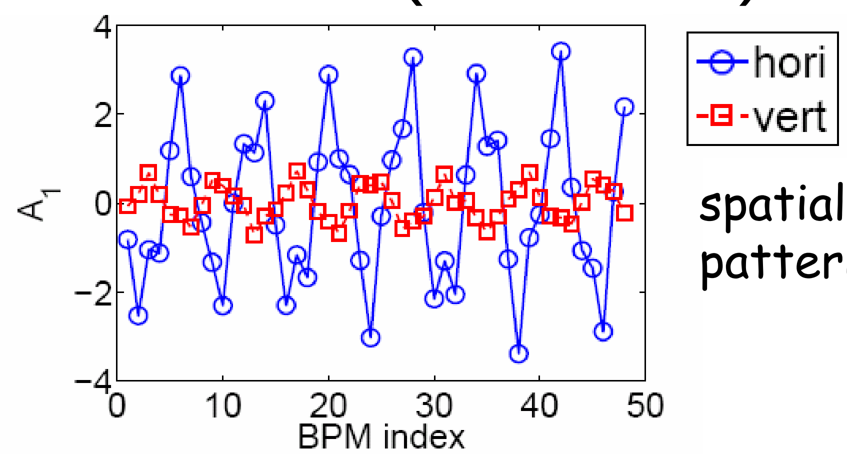
PCA However, the PCA method mixes the inserted signal with the betatron modes when the singular values are close.

Linear lattice function measurements

- Beta function and phase advance (DC beam)

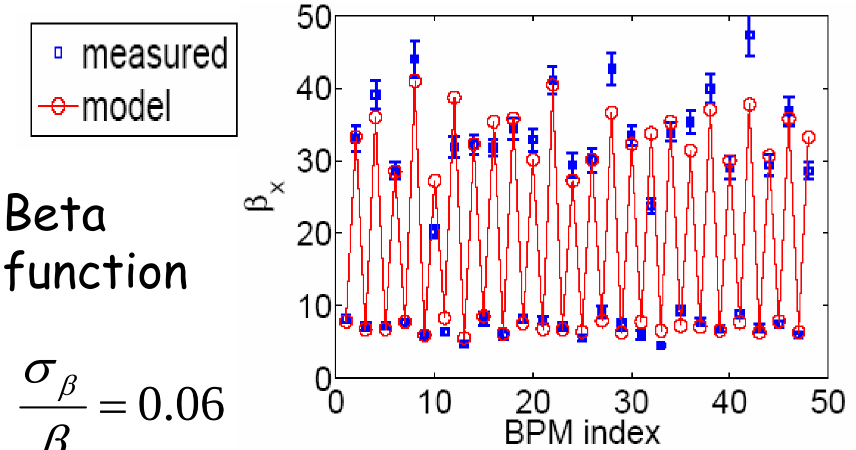
Temporal pattern



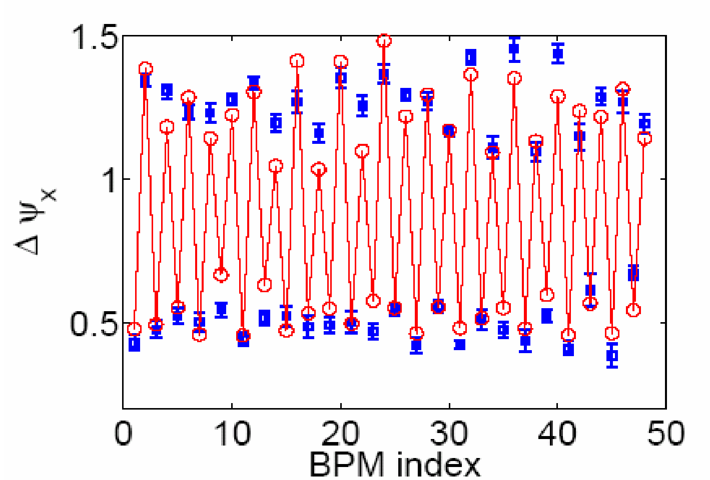


spatial pattern

Beta function



$$\frac{\sigma_\beta}{\beta} = 0.06$$



Phase advance

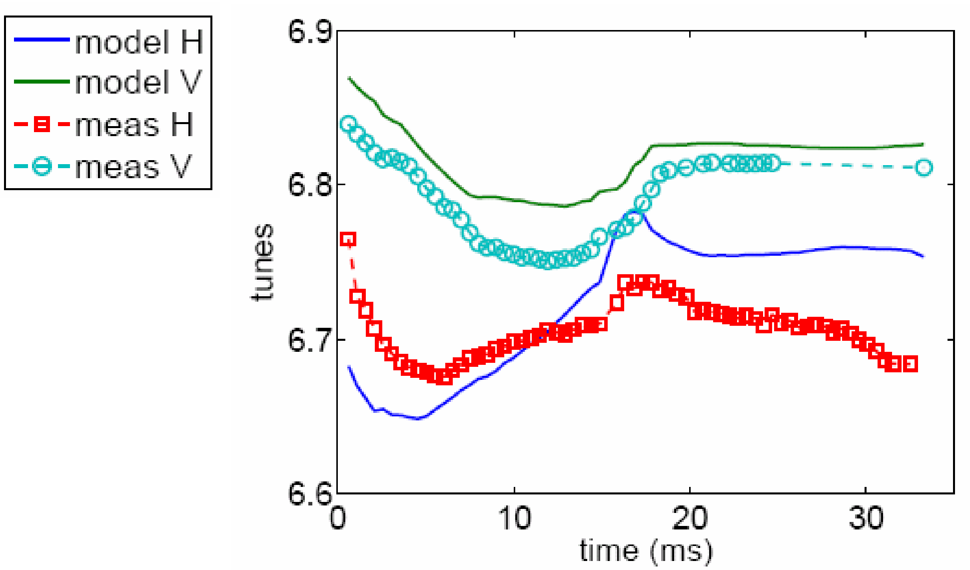
$$\sigma_{\Delta\psi} = 0.03$$

Horizontal beta function and phase advance

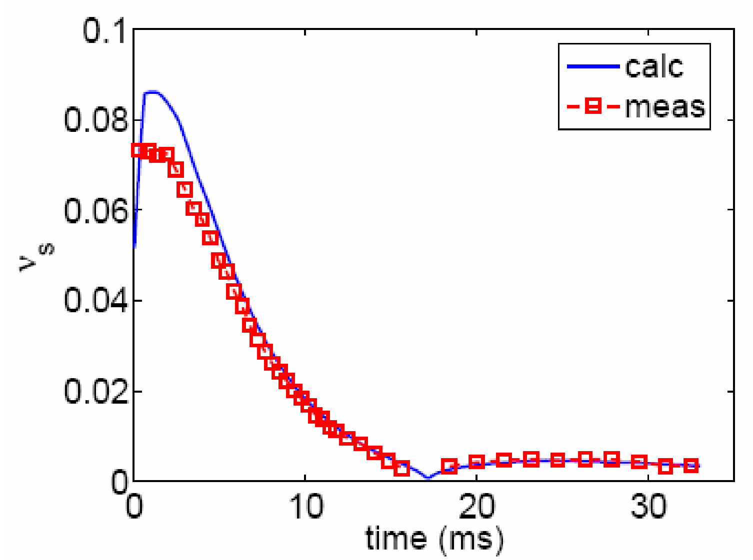
X. Huang

Betatron and Synchrotron Tunes

- Betatron and synchrotron tune measurements.



The betatron tunes



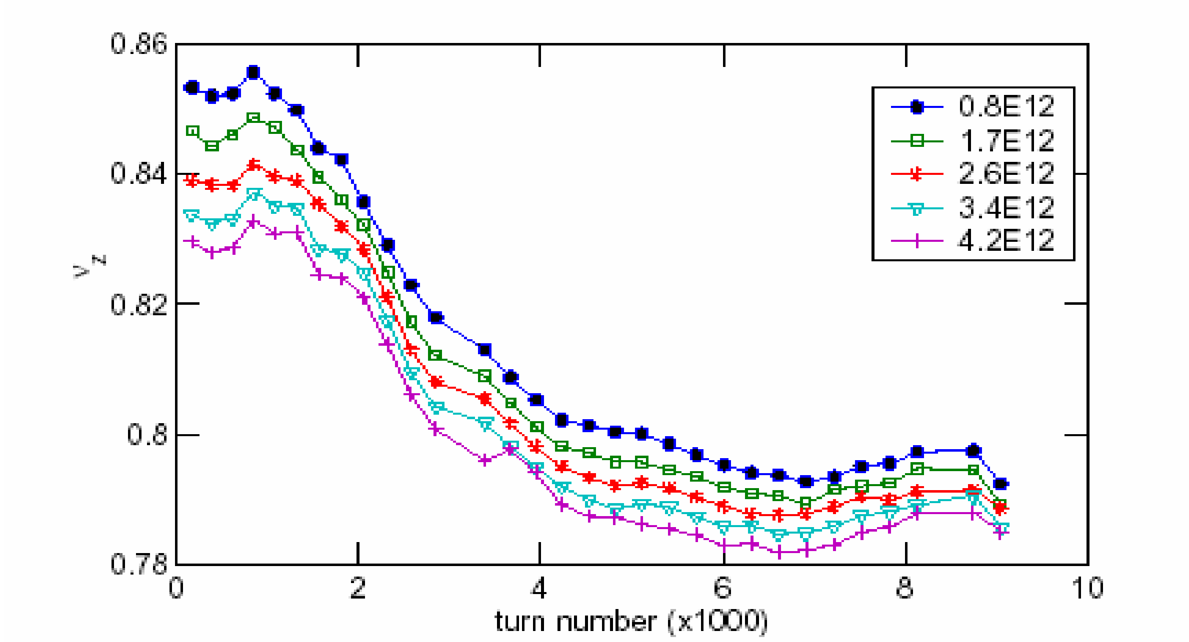
The synchrotron tunes

The clean coherent betatron modes and the interpolated FFT allows betatron tune measurements to high accuracy: <=0.0005 with 250 turns.

X. Huang

Betatron Tune Shifts and Transverse Impedance

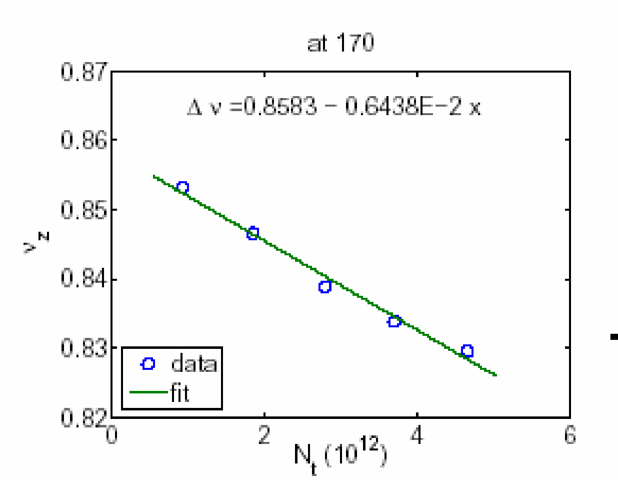
Vertical betatron tune is measured under different intensity levels.



The tune depression due to increased intensity comes from the imaginary part of transverse impedance.

X. Huang

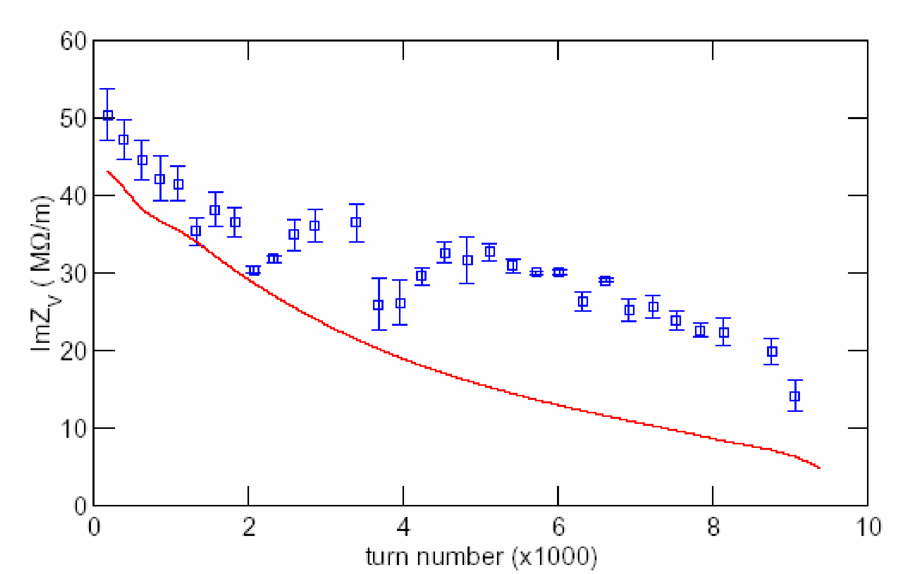
Transverse Impedance



Fitting tune vs. N at different time in the cycle.

$$\nu = \nu_0 - \frac{\mathrm{Im} Z_\perp}{Z_0} \frac{N r_0}{2\pi \gamma \nu_0} \frac{B_f}{\sqrt{2}}$$

Vertical transverse impedance



The red curve is calculated by assuming image currents in main magnets and image charge in vacuum pipe. Resistive wall impedance is estimated to be less important.

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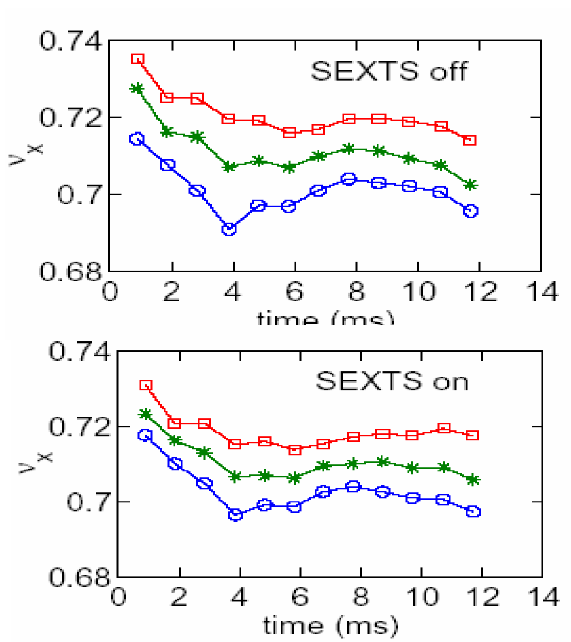
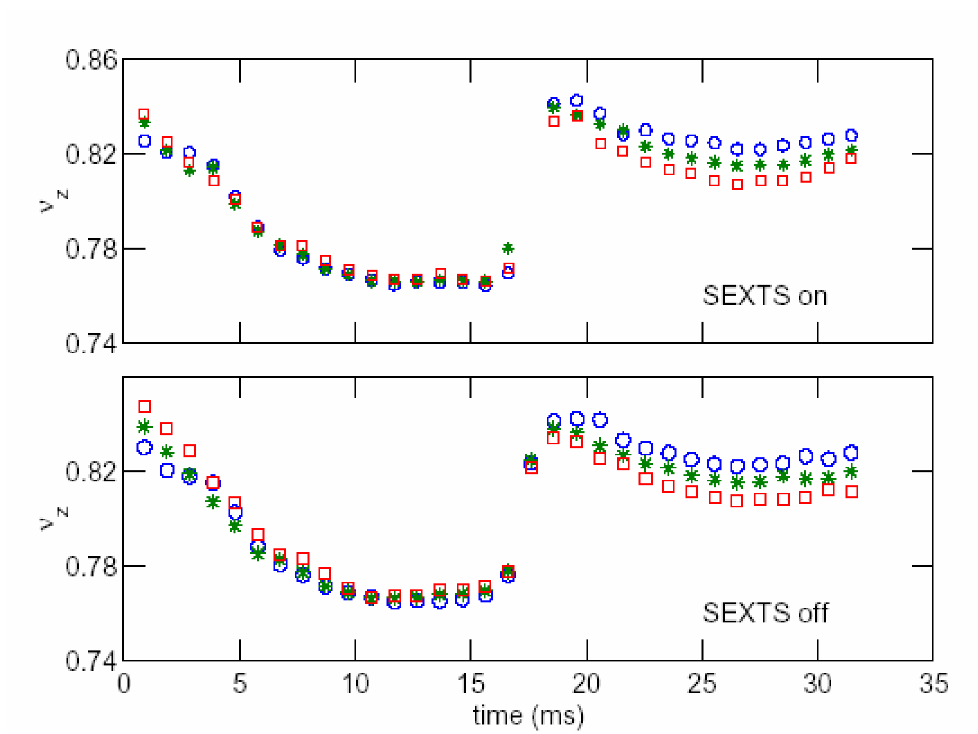
Chromaticity Measurement

Chromaticity is measured by monitoring the tune shifts while the beam orbit is swept from one side of the vacuum pipe to the other side.

The total change of momentum deviation is about 2.5E-3



Horizontal tune



Vertical tune

X. Huang

Summary of ICA at Fermilab

- The ICA method is an useful tool for turn-by-turn data analysis which should find application in other synchrotrons.
- For the Fermilab Booster, the lattice functions are measured from turn-by-turn data.
- The lattice model is verified with different setting (one or two re-positioned doglegs)
- We measured transverse impedance and chromaticity in the cycle through betatron tune shifts.
- Weak synchrotron motion is observed in BPM turn-by-turn data.
- The improved understanding of the Booster will help future upgrades.

X. Huang

References

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