

Coupling: Introduction, Diagnostics, Correction



How to minimize coupling in order to maximize machine performance

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- Outline:
 - Introduction/Motivation
 - Correction Methods
 - Measuring/How to get your model
 - Optimizing the correction algorithm
 - Experimental Results, Dynamics at very small emittance
 - Summary

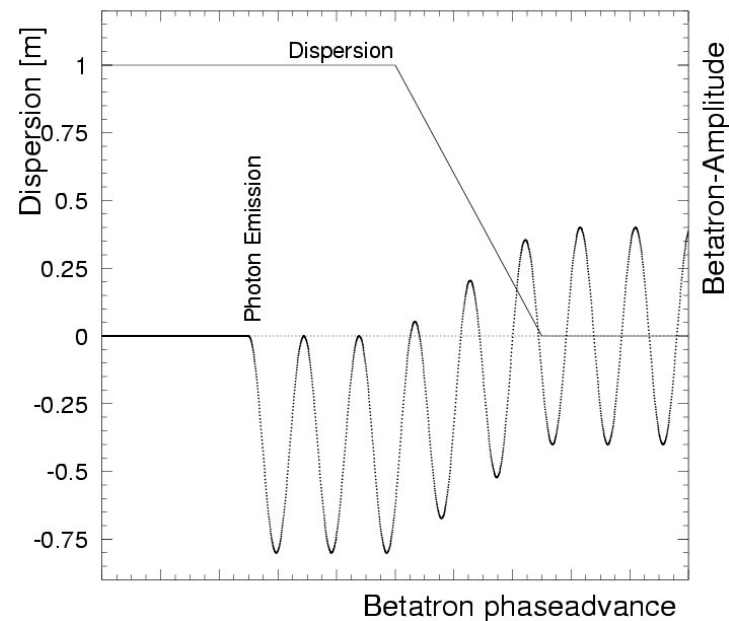


Motivation: Reducing vertical emittance

- ❖ Vertical **emittance of ideal, flat accelerator is very small** (for ALS of order of 0.5 pm) – correcting coupling errors can help to optimize brightness, luminosity, etc. by substantial amounts
- ❖ Simplest errors are **tilts of quadrupoles and offsets in sextupoles**
- ❖ Effects are:
 1. Global coupling
 2. Local coupling
 3. Vertical dispersion
- To optimize performance, **all three effects have to be corrected simultaneously**
- Methods include orbit manipulation, skew quadrupoles, moving of sextupoles, ...
- Most successful strategy at light sources: Do not target the three quantities individually, instead **use combined approach**

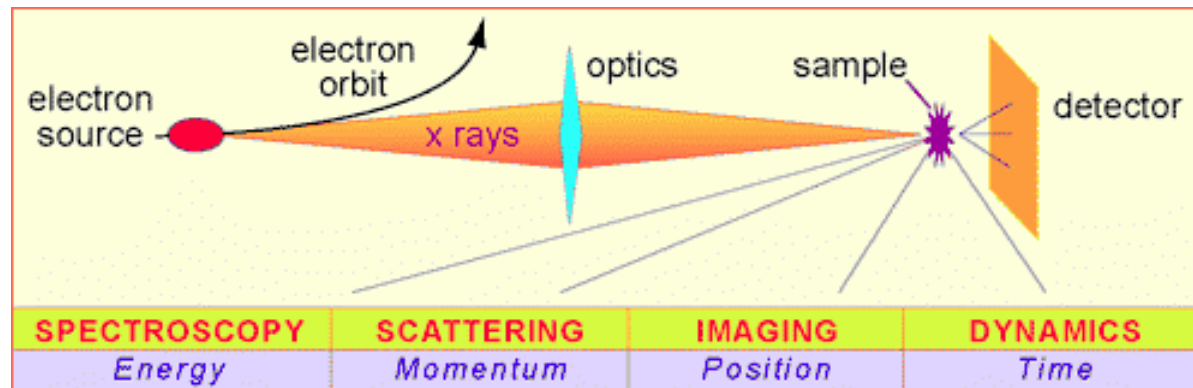
Reminder: Quantum excitation

Particles change their energy in a region of dispersion undergoes increase transverse oscillations. This balanced by damping gives the equilibrium emittances.



Experiments requiring small vertical emittance

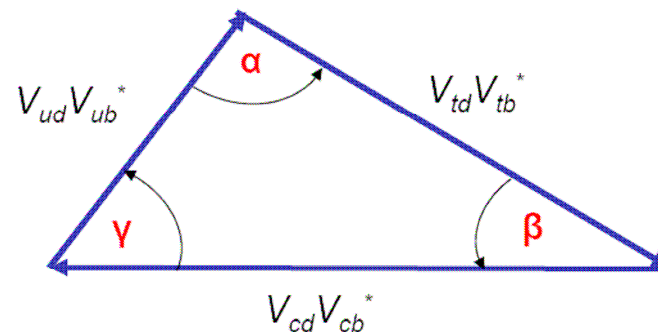
- ❖ Synchrotron light sources: High brightness (photon flux/sizes/divergences) enables high resolution experiments and provide partial transverse coherence



- ❖ Colliders: Particle physics experiments require high statistics – high luminosity – small vertical beamsizes at IP

$$L = \frac{f N_1 N_2}{4\pi \sigma_x \sigma_y}$$

$$R = L \sigma_{total}$$





Coupling

Skew quadrupole field errors generate betatron coupling between horizontal and vertical equations of motion.

4x4 transfer matrix for a quadrupole rotated by a small angle φ

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{\text{final}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -k & 1 & -2k\varphi & 0 \\ 0 & 0 & 1 & 0 \\ -2k\varphi & 0 & k & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_{\text{initial}}$$

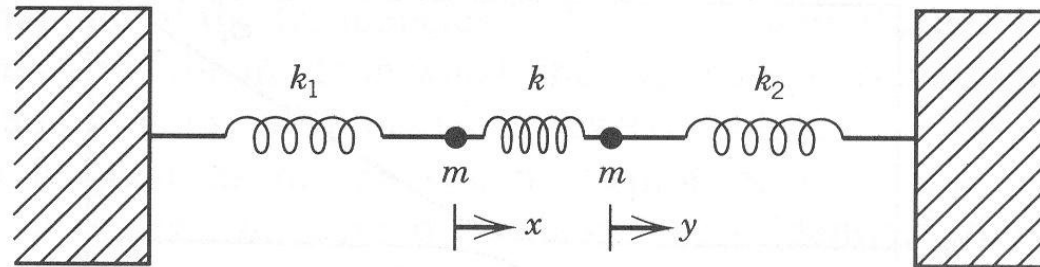
Local/Global Coupling, Vertical Dispersion

- ❖ Coupled (Hills) equations of motion :

$$x'' - Kx = -K_s y \quad y'' + Ky = -K_s x$$

- ❖ With
$$K = \frac{1}{B\rho} \frac{\partial B_y}{\partial x} \quad K_s = \frac{1}{B\rho} \frac{\partial B_x}{\partial x}$$

- ❖ Analogy with mechanical coupled harmonic oscillators (with springs)



$$m\ddot{x} + (k_1 + k)x - ky = 0,$$

$$m\ddot{y} + (k_2 + k)y - kx = 0,$$



Resonance Description of Global Coupling

- ❖ Global coupling is typically described using a resonance theory
- ❖ Difference coupling resonance

$$\kappa = \frac{1}{4\pi} \int ds K_s \sqrt{\beta_x \beta_y} e^{i\phi_D}$$

$$\frac{\phi_D}{2\pi} = \mu_x(s) - \mu_y(s) - \frac{s}{C} \Delta_r \quad \Delta_r = (v_x - v_y - N)$$

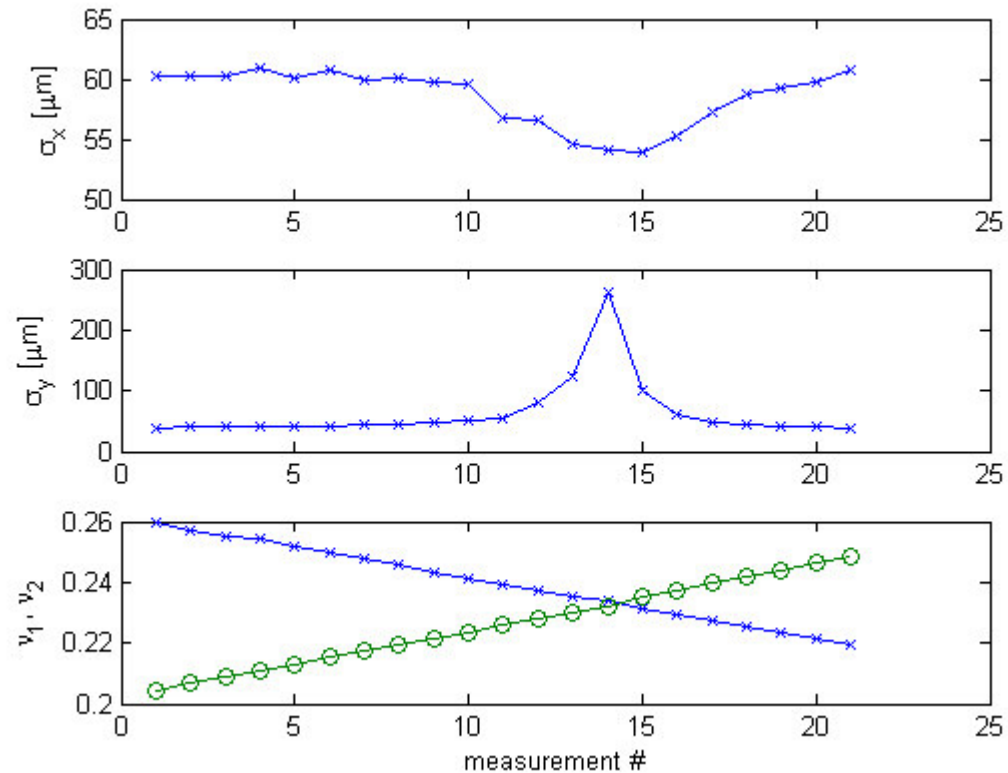
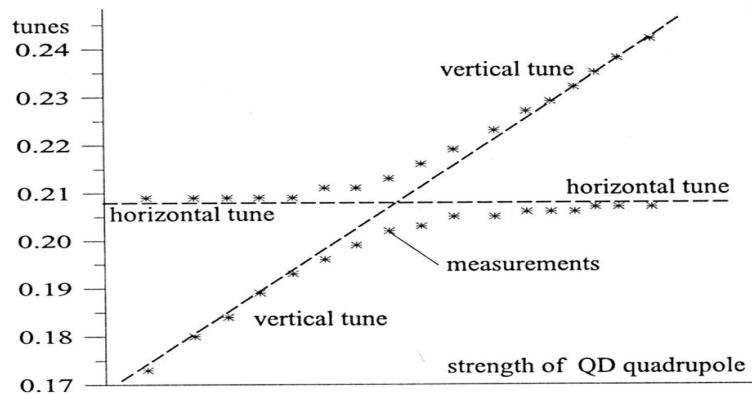
- Vertical emittance near difference resonance:

$$\frac{\mathcal{E}_y}{\mathcal{E}_x} = \frac{|\kappa|^2}{|\kappa|^2 + \Delta_r^2 / 2}$$

κ is resonance strength, Δ_r is distance from resonance.

Scan of difference resonance

- ❖ There are sum resonances as well (phase advance proportional to sum of horizontal and vertical phase advance) and of course higher order resonances.
- ❖ One can create orthogonal knobs of skew quadrupoles directly acting on one of those coupling resonances



- ❖ Minimum tune split (on resonance):

$$(\nu_x - \nu_y)_{\min} = 2|\kappa|$$

Normal mode Analysis: C matrix

- ❖ On Monday, we only discussed the uncoupled case (and mostly looked at 2x2 matrices). If there are coupling errors, one can do a so-called normal mode analysis (diagonalizing matrix)
- ❖ Start with 4x4, one-turn matrix $R_{\text{one-turn}}$, which maps the 4 transverse coordinates $\mathbf{x}=(x,x',y,y')$. Normal mode form:

$$\mathbf{R}_{\text{one-turn}} = \mathbf{V}\mathbf{U}\mathbf{V}^{-1}, \text{ normal mode matrix } \mathbf{U} = \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{B} \end{pmatrix},$$

$$\text{with } \mathbf{A} = \begin{pmatrix} \cos \phi_a + \alpha_a \sin \phi_a & \beta_a \sin \phi_a \\ -\gamma_a \sin \phi_a & \cos \phi_a - \alpha_a \sin \phi_a \end{pmatrix},$$

\mathbf{V} is of the form (Edwards + Teng)

$$\mathbf{V} = \begin{pmatrix} \mathcal{N} & \mathbf{C} \\ -\mathbf{C}^+ & \mathcal{N} \end{pmatrix},$$

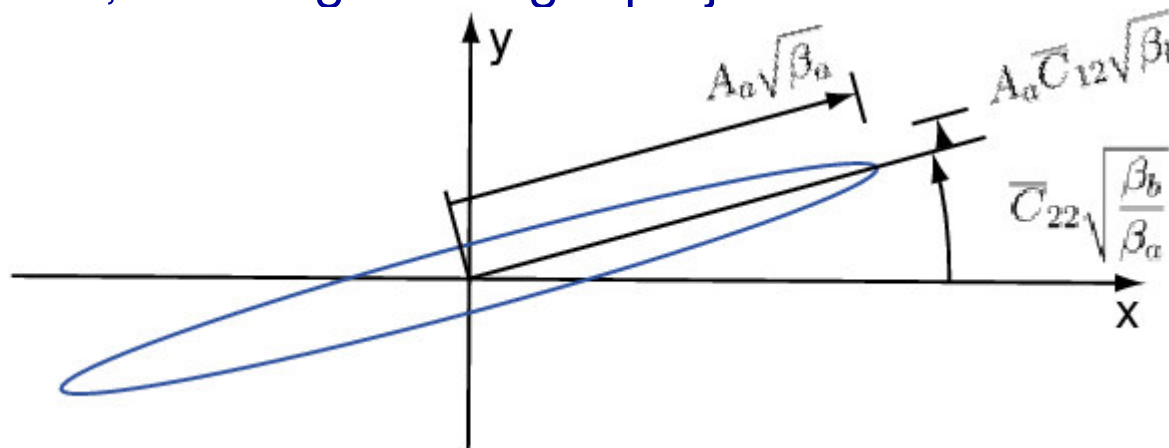
with $\gamma^2 + \|\overline{\mathbf{C}}\| = 1$. The magnitude of \mathbf{C} is a measure of local coupling.

Local Coupling

Often the normalized matrix $\bar{\mathbf{C}}$ is used :

$$\bar{\mathbf{C}} \equiv \mathbf{G}_a \mathbf{C} \mathbf{G}_b^{-1}, \text{ where } \mathbf{G}_a = \begin{pmatrix} 1 & 0 \\ \frac{\alpha_a}{\sqrt{\beta_a}} & \sqrt{\beta_a} \end{pmatrix}.$$

- ❖ Locally there is torsion in addition to the global invariant vertical emittance, resulting in a larger projected emittance:



- ❖ Again driving terms scale like the sqrt of the product of the beta functions at the location of the skew errors.



Vertical Dispersion

- ❖ There are two main terms that can create vertical dispersion:

$$\eta_y'' + K \eta_y = \frac{1}{\rho_y} - K_s \eta_x$$

- Dipole errors (steering magnets, misalignments, ...) or intentional vertical bending magnets
- Skew quadrupole fields at the location of horizontal dispersion (due to quadrupole tilts, or vertical offsets in sextupoles)

$$\kappa_{\eta_y} = \int ds K_s \eta_x \sqrt{\beta_y} e^{i\phi_{\eta_y}}$$

$$\frac{\phi_{\eta_y}}{2\pi} = \mu_y(s) - \frac{s}{C} (\nu_y - 5)$$

- ❖ Vertical dispersion directly causes increase of the vertical emittance by quantum excitation (compare my talk on Monday)



Correction Techniques

- ❖ One can correct the three coupling effects using skew quadrupoles, vertical offsets (movers or orbit bumps) in sextupoles, steering magnets, ...
- ❖ The corrections can either target global quantities, local quantities at individual points of the ring, or local quantities everywhere.
 - Coupling correction scales like sqrt of product of beta functions times skew strength.
 - Dispersion correction scales like product of horizontal dispersion times sqrt of vertical beta function time skew quadrupole strength
 - Dispersion' from steering magnets scales like the bending angle.
- ❖ Phase advance of coupling (dominant $\mu_x - \mu_y$) and dispersion (μ_y) are different!

Resonance correction of the sum and difference resonance (global)



To correct coupling, tweak orthogonal harmonic knobs for both difference resonance phases. Minimize tune split.

Sum resonance also generates linear coupling.

$$K_{sum} = \frac{1}{4\pi} \int ds K_s \sqrt{\beta_x \beta_y} e^{i\phi_s}$$

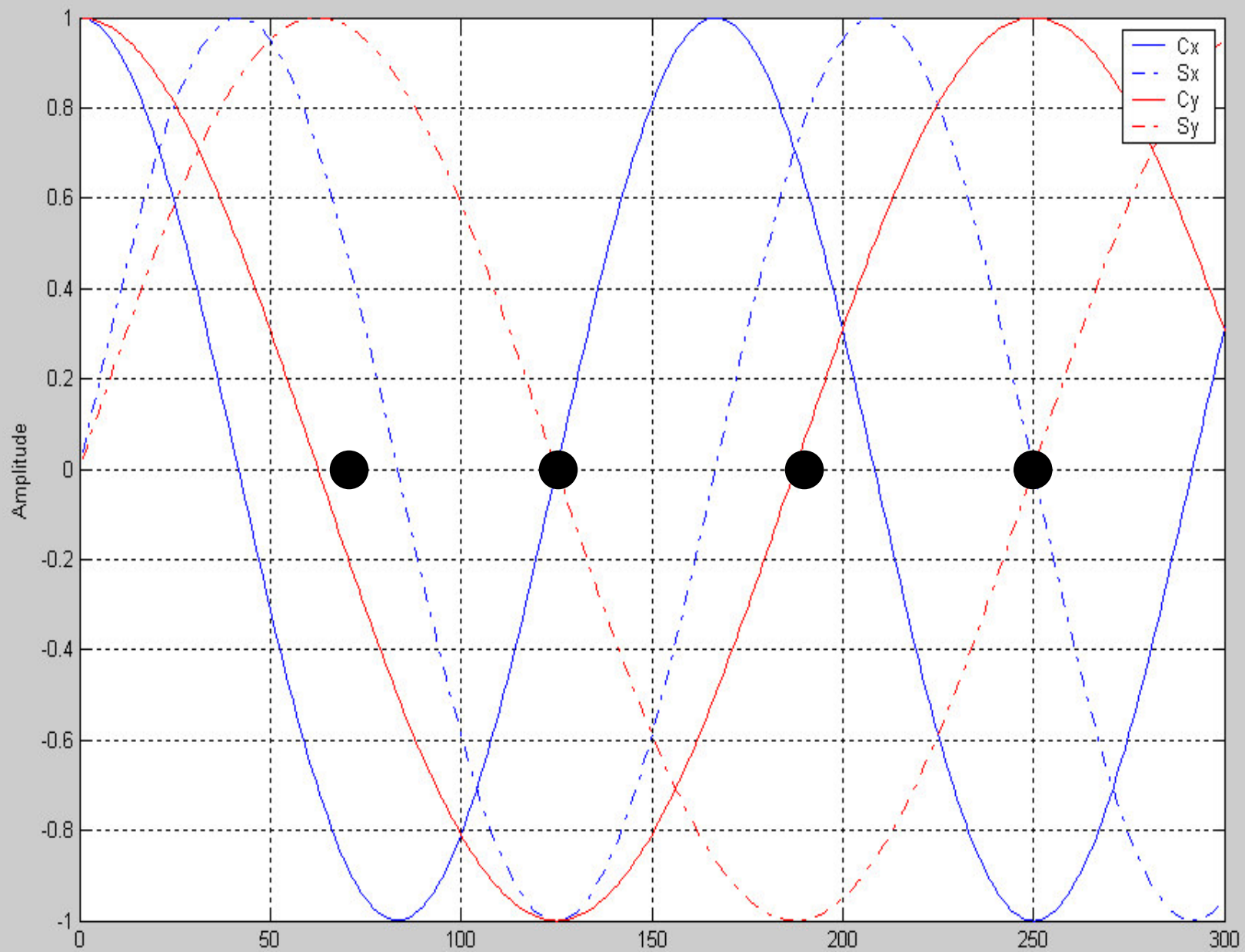
$$\frac{\phi_s}{2\pi} = \mu_x(s) + \mu_y(s) - \frac{s}{C} \Delta_r \quad \Delta_r = (\nu_x + \nu_y - N)$$

Coupling correction – minimize measured vertical beam size as a function of skew quad strengths:

$$\sigma_{y,meas}(K_{s,1}, K_{s,2}, \dots)$$

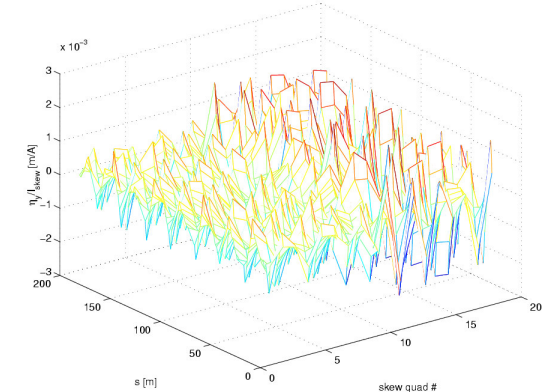
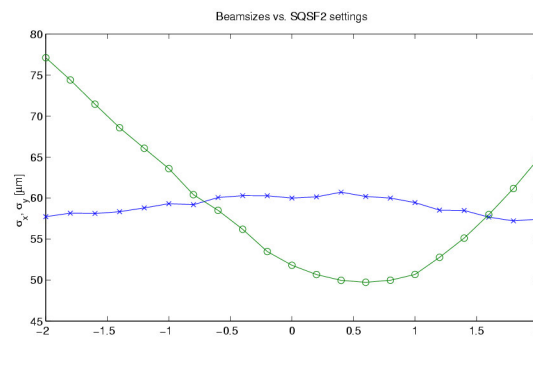
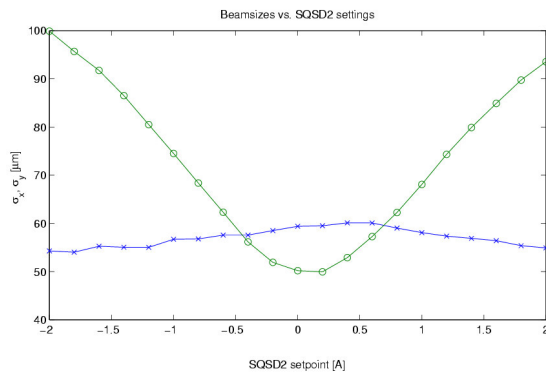
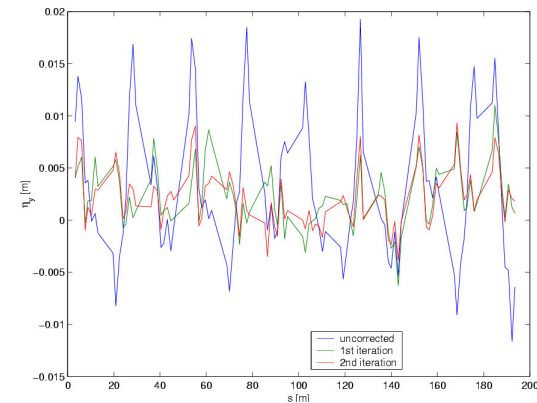
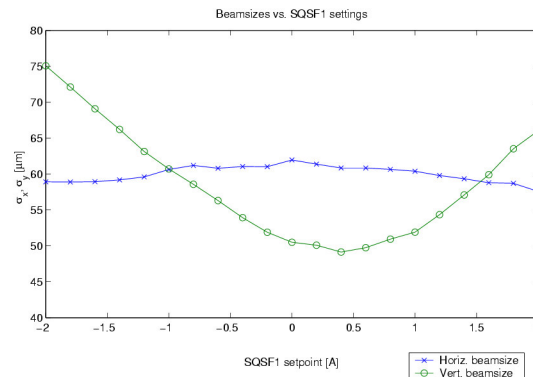
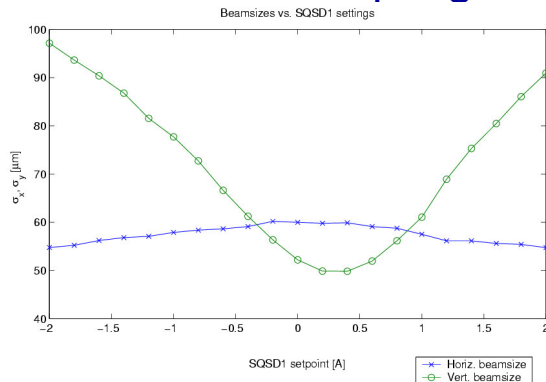
One possible method is to use orthogonal harmonic knobs:

$$\sigma_{y,meas}(K_{diff, \cos N}, K_{diff, \sin N}, K_{sum, \cos N}, K_{sum, \sin N}, K_{\eta_y, \cos N}, K_{\eta_y, \sin} \dots)$$



Unsuccessful Correction Attempts ... in ALS

- ❖ In the past, tried three different approaches:
 1. Coupling correction using chains of skew quadrupoles (single resonance)
 2. Dispersion correction using orbit correctors (TBA, chromaticity)
 3. Dispersion correction using skew quadrupoles without watching the coupling simultaneously



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Advanced Light Source



Separated approaches ...

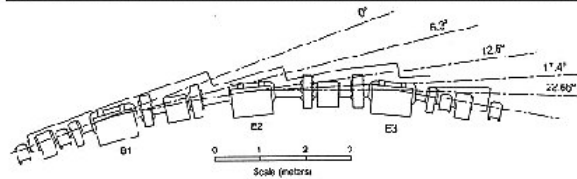
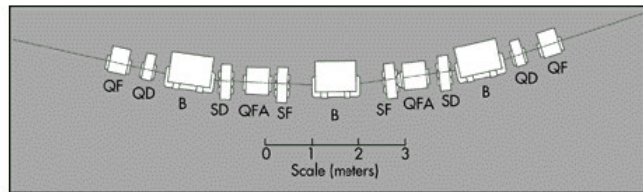
- ❖ In case you are dealing with a FODO lattice, or you do not have synchrotron radiation users and therefore can use your orbit as a free variable, the separated approach of coupling correction can actually work well (i.e. in colliders).
- ❖ FODO lattice is very simple and allows relatively dispersion correction via orbit correction/bumps. In addition/somewhat independently one can often minimize the global coupling with only four orthogonal skew (families). The local coupling is in most colliders only relevant at the interaction point and can be compensated there with few skew quadrupoles.



Simulation of coupling correction

- ❖ Use accelerator toolbox (Andrei Terebilo), Matlab and LOCO (James Safranek, Greg Portman) for simulations
- ❖ Use random skew error seeds
- ❖ Try to find effective skew corrector distributions and to optimize correction technique in simulation
- ❖ Used two correction approaches:
 1. Response Matrix fitting – ‘deterministic’, small number of iterations
 2. Direct minimization (nelder-simplex, ...) – easy to do on the model, difficult on real machine
- Surprisingly both approaches gave about the same performance in the model calculations
- For response matrix analysis you have to optimize several parameters of the code as well (weight of dispersion, number of SVs, use of effective model/full model ...)

Response Matrix/Machine Model



- fitting a machine model to the **response matrix** (SVD, LOCO)

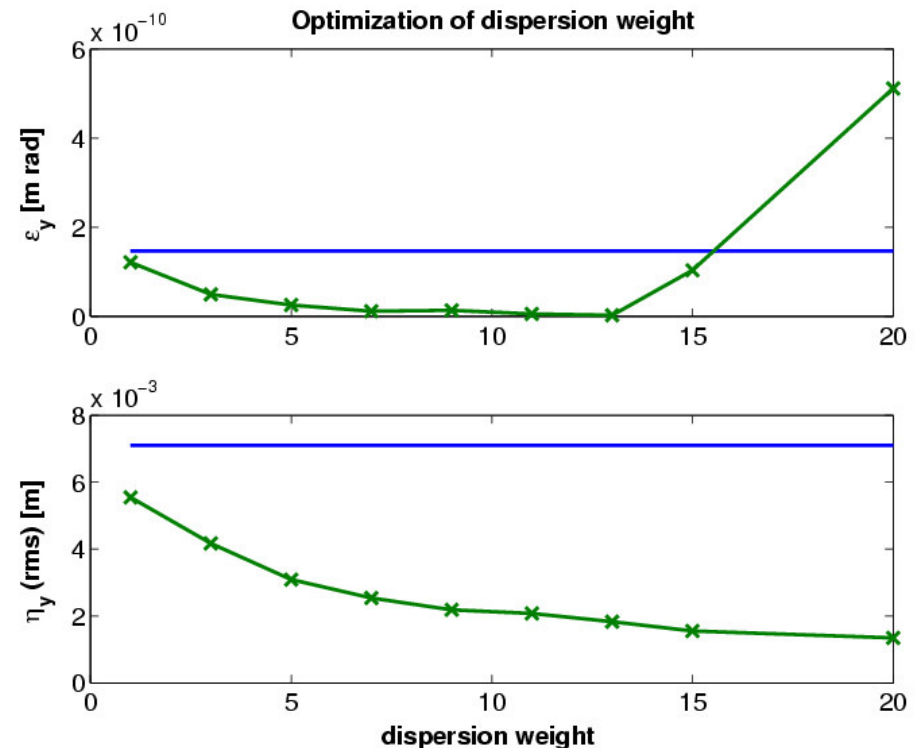
$$C_{12}^{ij} = \left[R^{ij} (1 - R^{jj})^{-1} \right]_{12} - \frac{\eta_i \eta_j}{\left(\alpha - \frac{1}{\gamma^2} \right) C}$$

$$\hat{C}^{ij} = C^{ij} + \sum_k \frac{\partial C^{ij}}{\partial g_k} \delta g_k + C^{ij} \Delta x^i - C^{ij} \Delta y^j$$

- can determine **individual quadrupole strengths**
- can determine **localized coupling strengths**

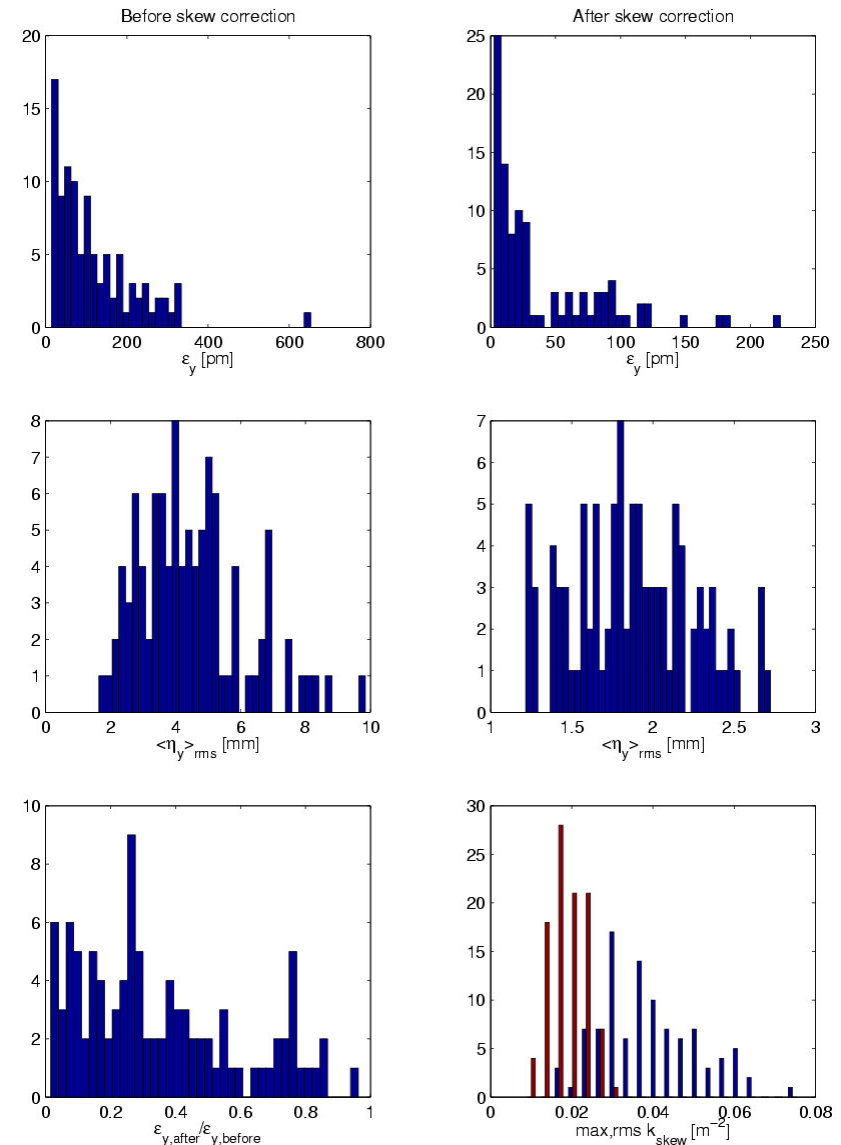
Weight of dispersion in LOCO fit

- ❖ The relative contribution of vertical dispersion and coupling to the vertical emittance depends on the particular lattice (and the particular error distribution).
- ❖ Therefore the optimum weight for the dispersion in the LOCO fit has to be determined (experimentally or in simulations).
- ❖ The larger the weight factor, the better the vertical dispersion gets corrected, but eventually the coupling ‘explodes’.
- ❖ Set weight to optimum somewhat below that point.
- ❖ Outlier rejection tolerance might be important parameter as well.

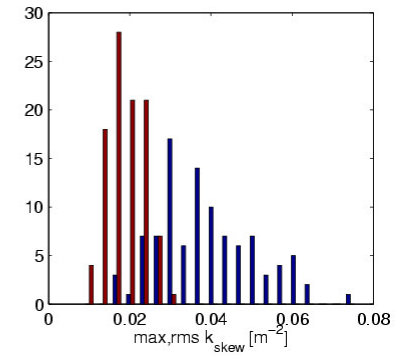
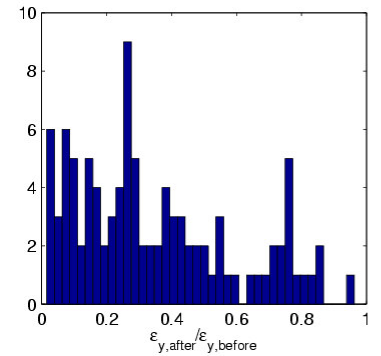
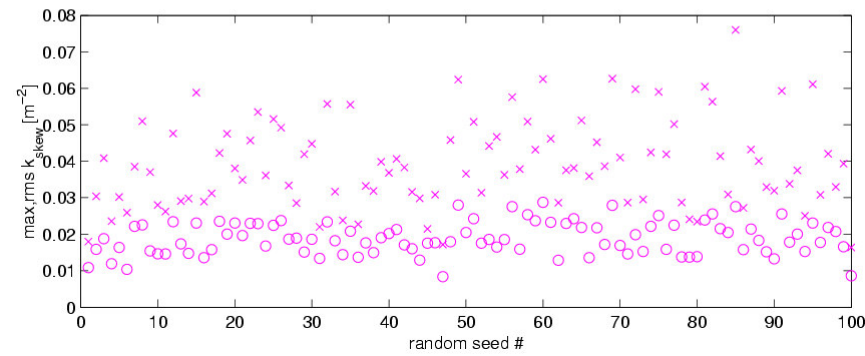
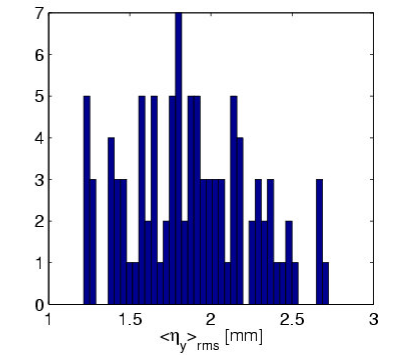
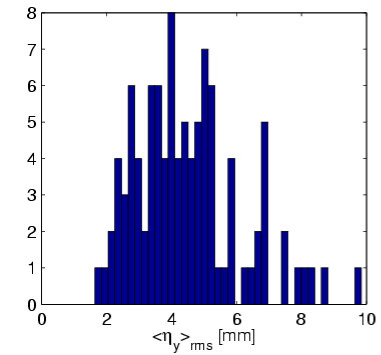
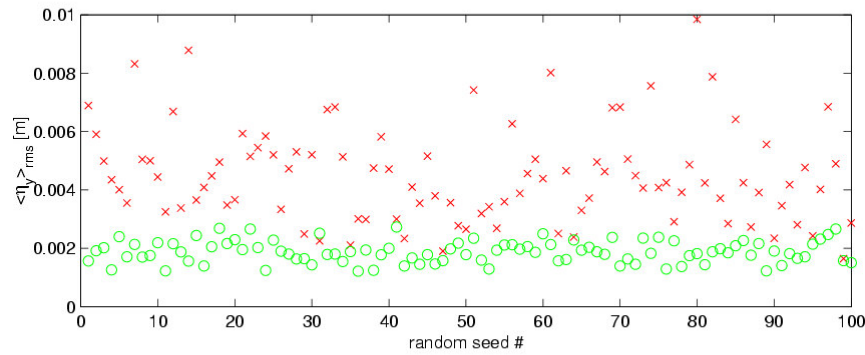
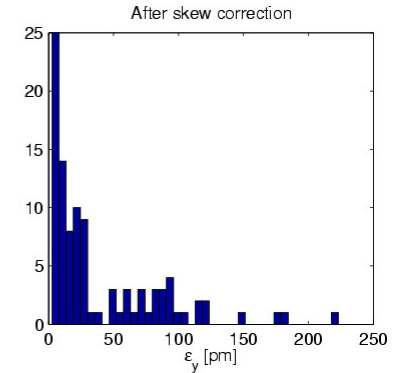
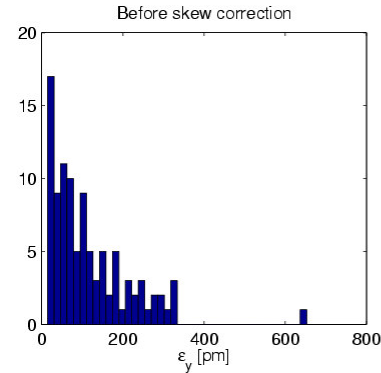
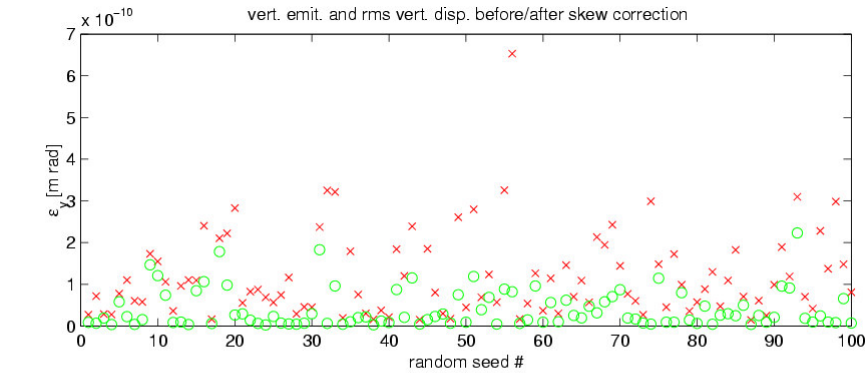


Finding an Effective Skew Quadrupole Set

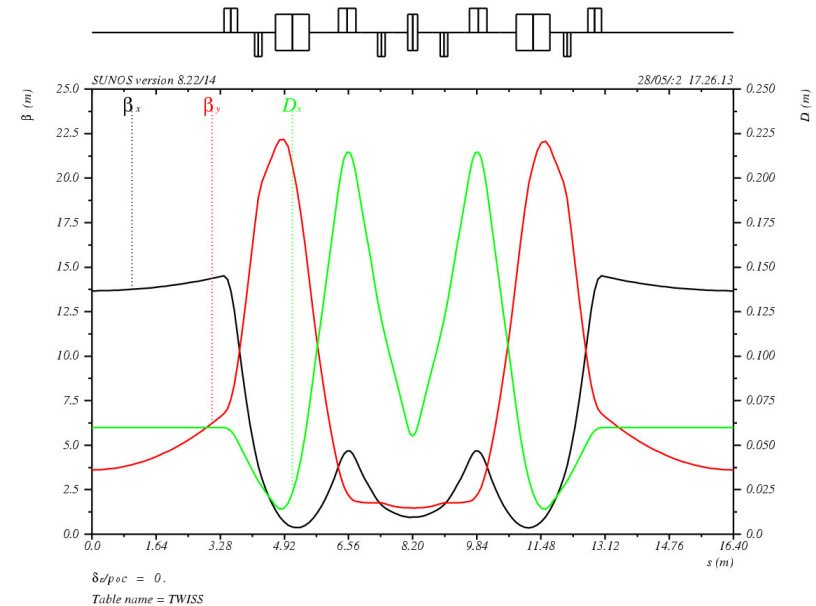
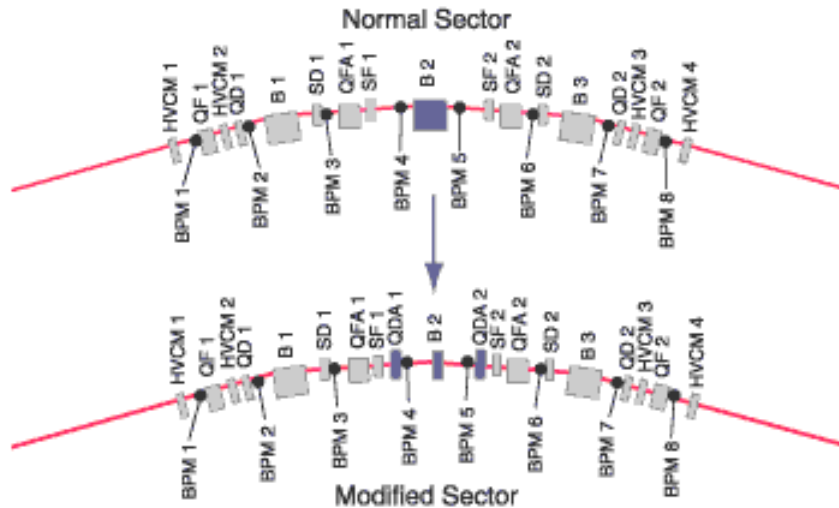
- ❖ To find an effective skew quadrupole distribution, we used several correction methods, first in simulations – best method was **orbit response matrix fitting (using LOCO)**
- ❖ **Predictive method, can be easily used on real machine**
- ❖ Issues are:
 - Cover set of phases relative to dominant coupling resonance(s)
 - Magnets should be distributed around the ring in order to avoid excessive local coupling/vertical dispersion
 - Need different values of dispersion/beta function to be effective both for coupling and vertical dispersion correction
- ❖ Set of **12 skew quadrupoles** was **reasonably efficient**



Finding an effective skew set II



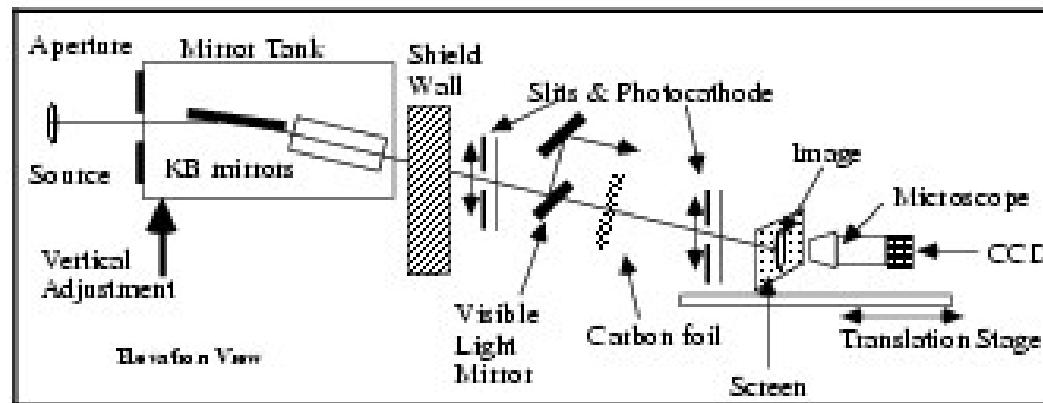
ALS Lattice – Location of Skew Quadrupoles



- 24 individual skew quadrupoles (integrated in sextupoles) serve two purposes:
 - global vertical emittance/dispersion control
 - local vertical dispersion bump

Ways to Measure Very Small Emittances

- ❖ The single photon emittance (diffraction) is $\lambda/4\pi$, which means that to measure emittances of a few pm, one has to use x-rays.
- ❖ Problem even with x-rays is **resolution of X-ray beamlines/optics**



- ❖ **Mirror roughness, aberrations, diffraction limit, misalignments, CCD resolution, BGO crystal (glow)**
- ❖ Fundamental limit (diffraction limit) is a few microns, i.e. sufficient for what we want to measure, but with real optics errors the resolution is often larger.

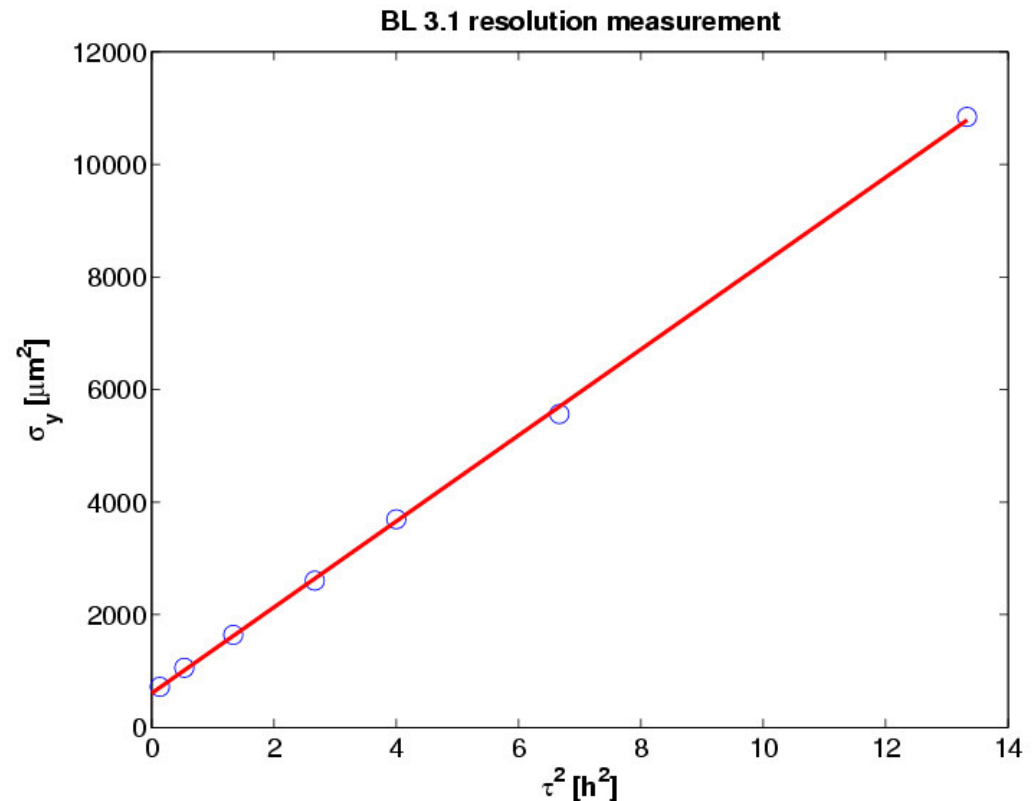


How to measure very small emittances II

- ❖ We used three different ways to verify vertical emittances around 5 μm :
 1. Resolution correction (resolution was about 25 μm , measured beamsizes got as small as about 27 μm)
 2. Analysis of orbit response matrix, using a sufficiently large number of skew gradient error fit parameters
 3. RF acceptance-lifetime scan. Quadratic part should scale just with the bunch volume. Therefore one can deduct the small emittance from a beamsize measurement at moderate coupling!
- All measurement methods gave vertical emittances between 4 and 7 μm in the best case.

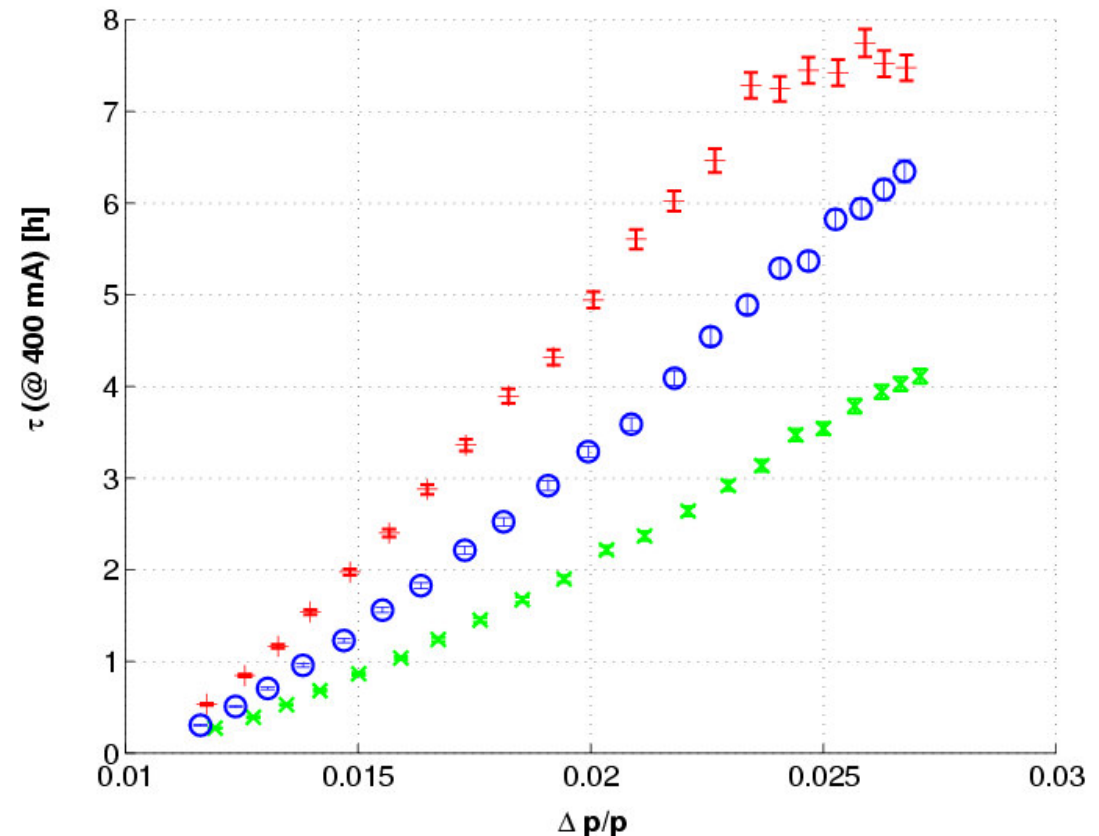
Resolution correction of SLM

- ❖ If the rf voltage is significantly reduced, changes in the beam dynamics/dynamic momentum aperture do not impact the total momentum aperture. Therefore the Touschek lifetime is only proportional to the rf-voltage and the bunch volume.
- ❖ If one now changes the vertical beamsize and plots the square of the measured beamsize as a function of the square of the measured Touschek lifetime, one can extrapolate the resolution limit of the profile monitor.
- ❖ Result with current optimization of BL 3.1 is about 25 μm



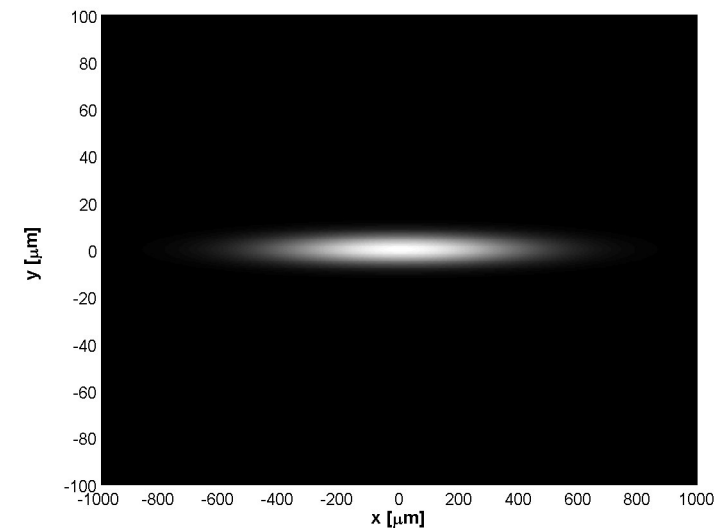
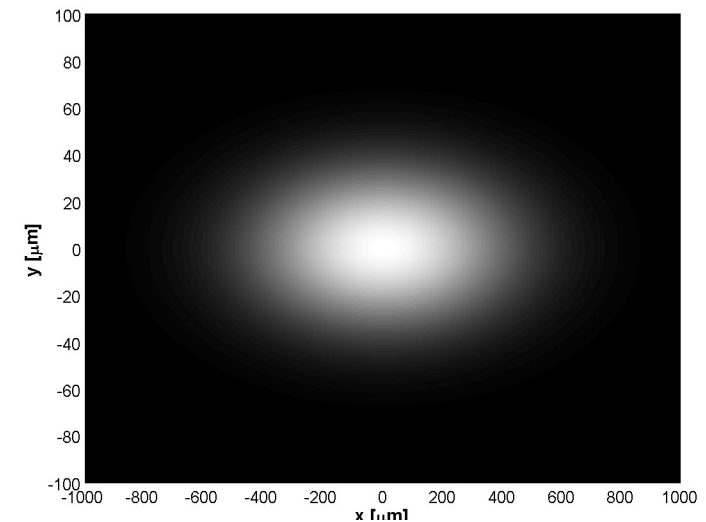
RF-acceptance/lifetime scan

- ❖ ‘Quadratic’ part of lifetime curve does not depend on single particle dynamics.
- ❖ Relative scaling of curves is determined by bunch volume only.
- ❖ Comparison gives an emittance scaling which combined with a direct emittance measurement at the ‘high’ emittance case allows a determination of the emittance in the small emittance case.

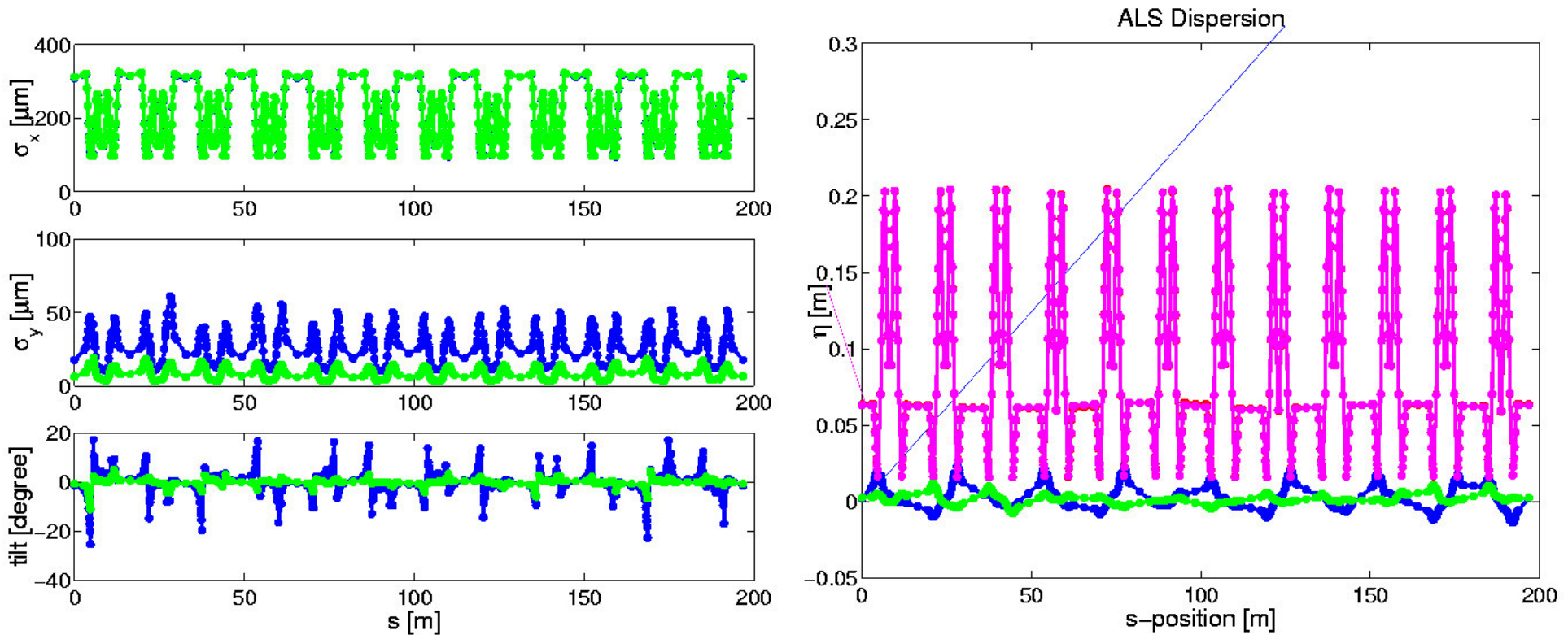


Achieved Emittance Reduction

- ❖ Achieved an **emittance reduction from 150 μm** (routine ALS operation) **to about 5 μm** (pictures on the right illustrate size reduction for insertion device straights)
- ❖ This was a world record in 2003 and about the NLC damping ring design value
- ❖ Correspondingly the **brightness increases by factor 30** (for hard x-rays – because of diffraction limit less for soft x-rays)



Example of Emittance and Dispersion Correction



- In this example vertical beamsize was reduced by factor of more than 4 (emittance by factor 20)
- Spurious vertical dispersion reduced from 7 mm rms to below 3 mm rms
- Tilt of phase space reduced significantly everywhere

Reminder: Phase Advance Measurement

- ❖ Use digitized position signal, then

The phase of the reference signal at turn n is used to construct sine cosine references

$$R_{\sin}(n) = \sin \phi_{ref}(n)$$

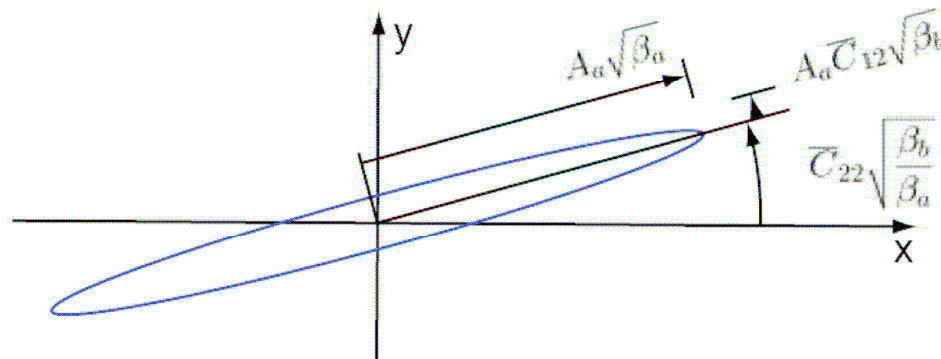
$$R_{\cos}(n) = \cos \phi_{ref}(n)$$

- ❖ Convolute the beam signal with these (digital lock-in) and integrate

Results are used to solve for the lattice functions

$$x = A_a \sqrt{\beta_a} \cos(n\omega_a + \phi_a),$$

$$y = -A_a \sqrt{\beta_b} (\bar{C}_{22} \cos(n\omega_a + \phi_a) + \bar{C}_{12} \sin(n\omega_a + \phi_a)).$$

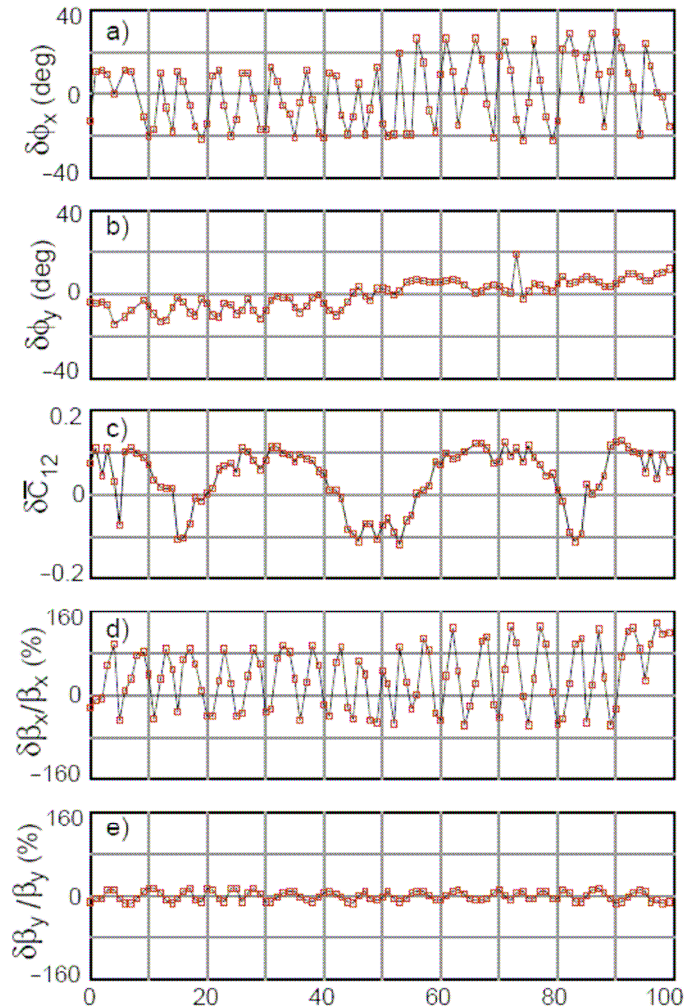


In practice assume $\beta = \beta(\text{design})$ and solve for ϕ and \bar{C}_{ij} .

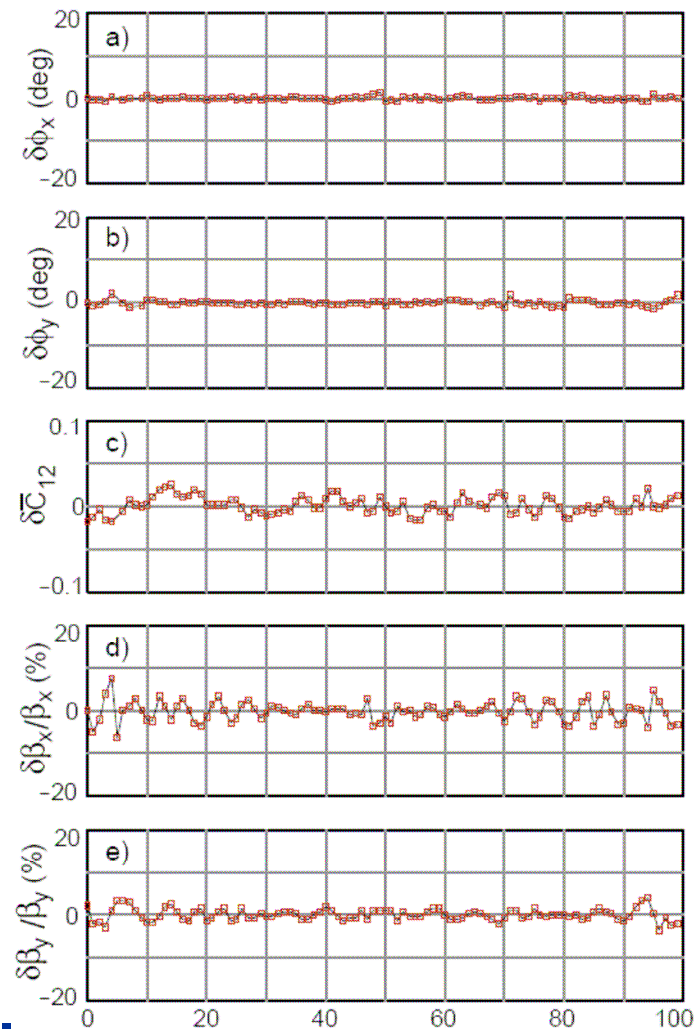
Coupling Correction using turn-by-turn data



Before



After



Note the change in scale

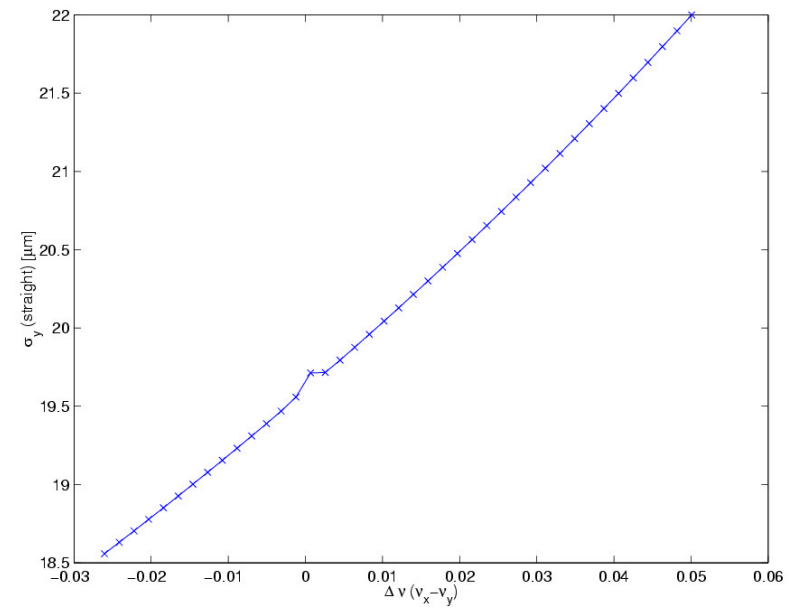
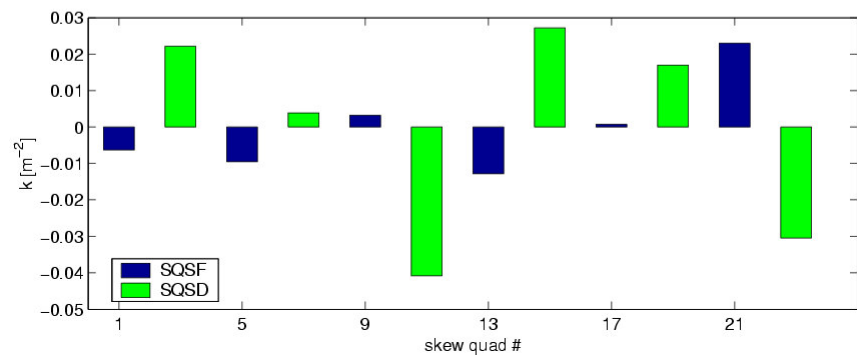
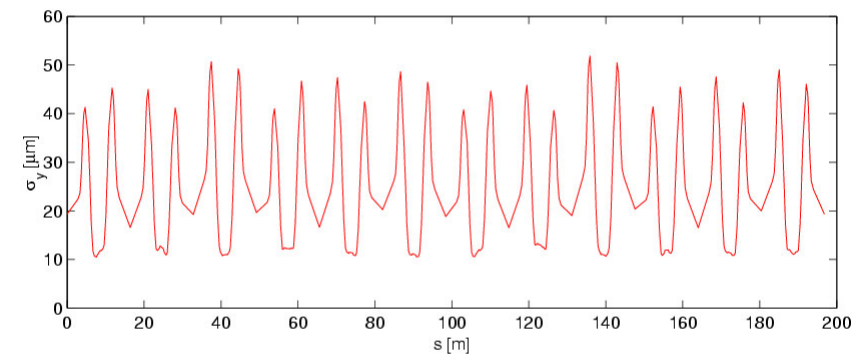
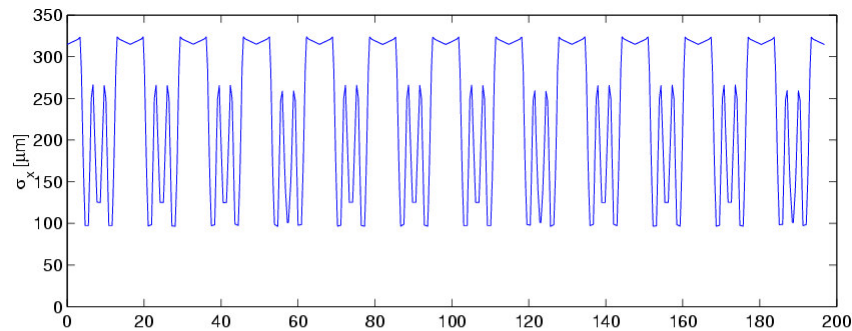
Advanced Light Source



Ways to Increase the Vertical Emittance ...

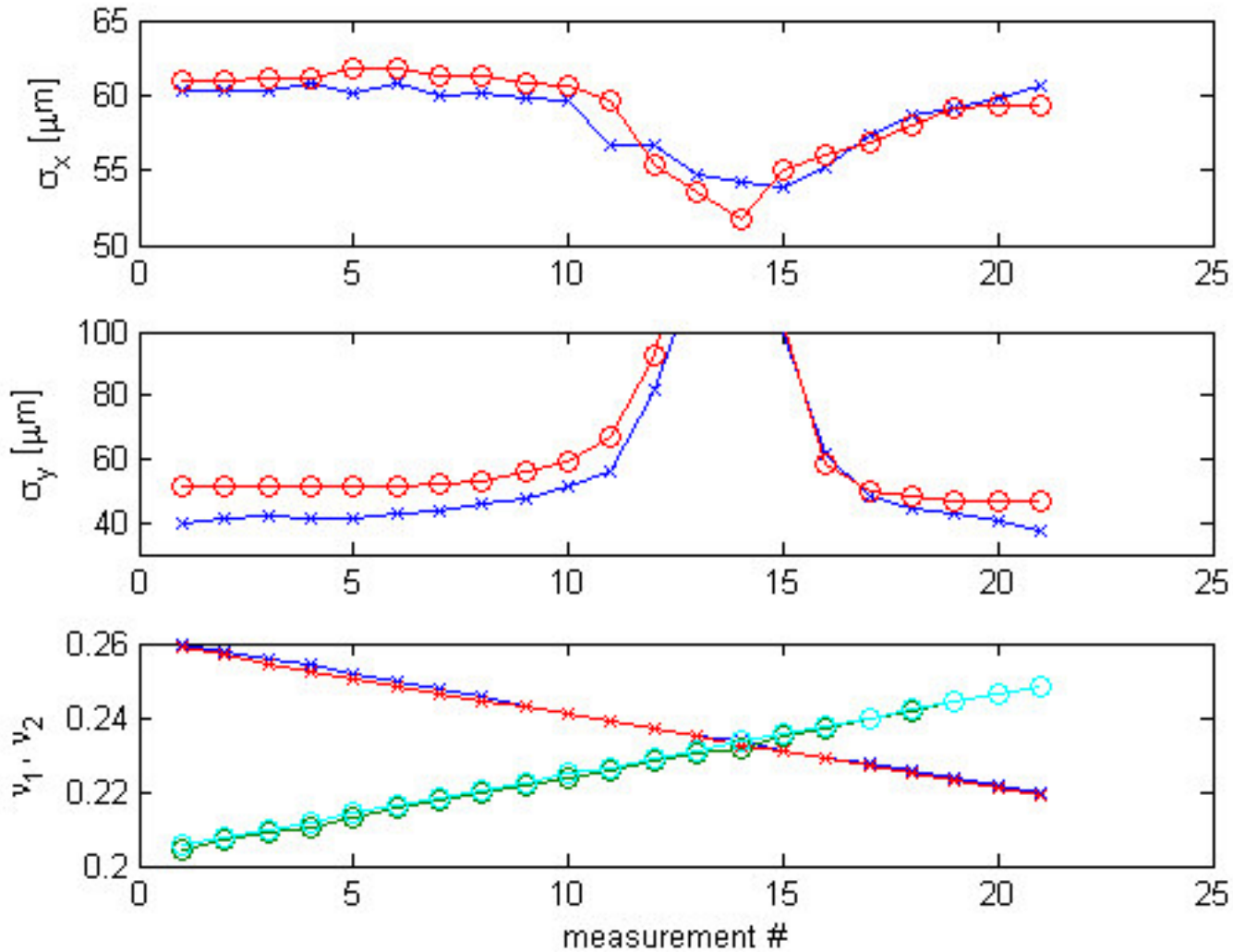
- ❖ Low energy third generation light sources usually **increase the vertical emittance intentionally to achieve acceptable lifetime.**
- ❖ Historically at the ALS we used a family of skew quadrupoles to excite linear coupling resonance.
- ❖ Recently switched to a mode where we correct the coupling and dispersion as well as possible and then blow up the vertical emittance using a **global vertical dispersion wave.**
- ❖ Method has many advantages (**beamsize stability, dynamic momentum aperture, ...**)

Vertical Dispersion Wave



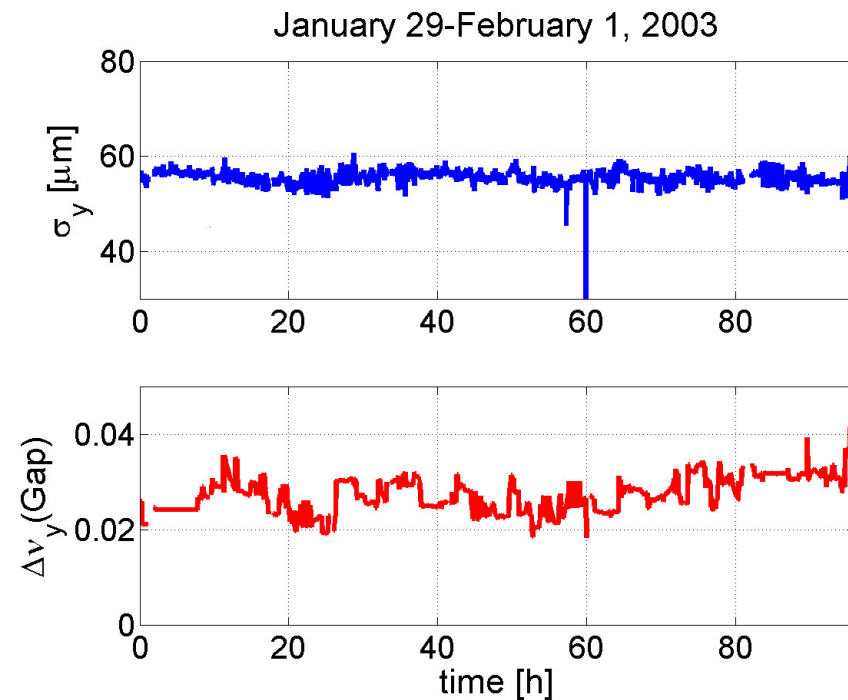
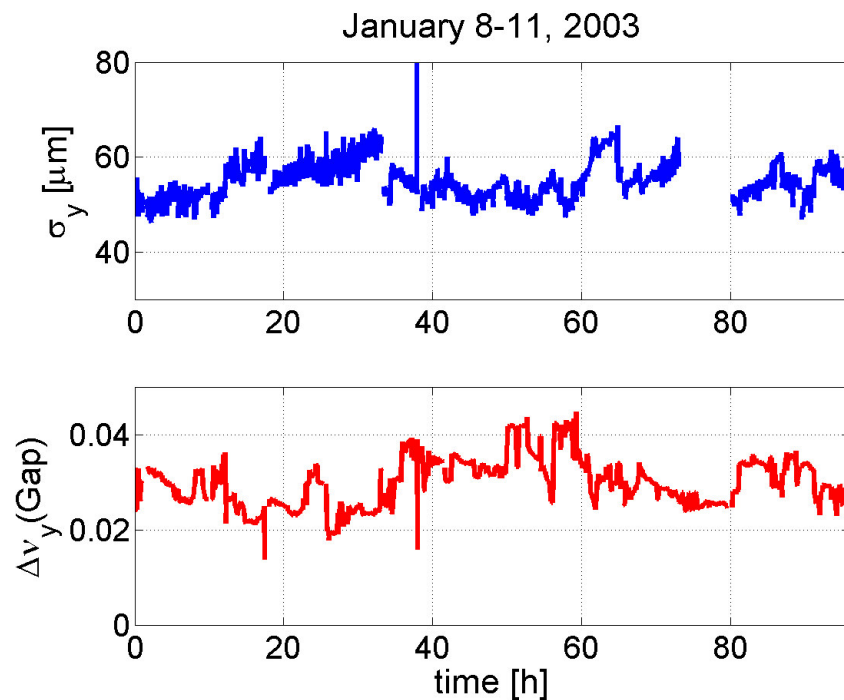
- 12-20 skew quadrupoles are used such, as to generate a **global vertical dispersion wave, without exciting nearby coupling resonances**
- Vertical emittance is directly generated by **quantum excitation**
- Local emittance ratio around the ring is fairly constant, local tilt angles are **small**

Vertical Dispersion Wave II

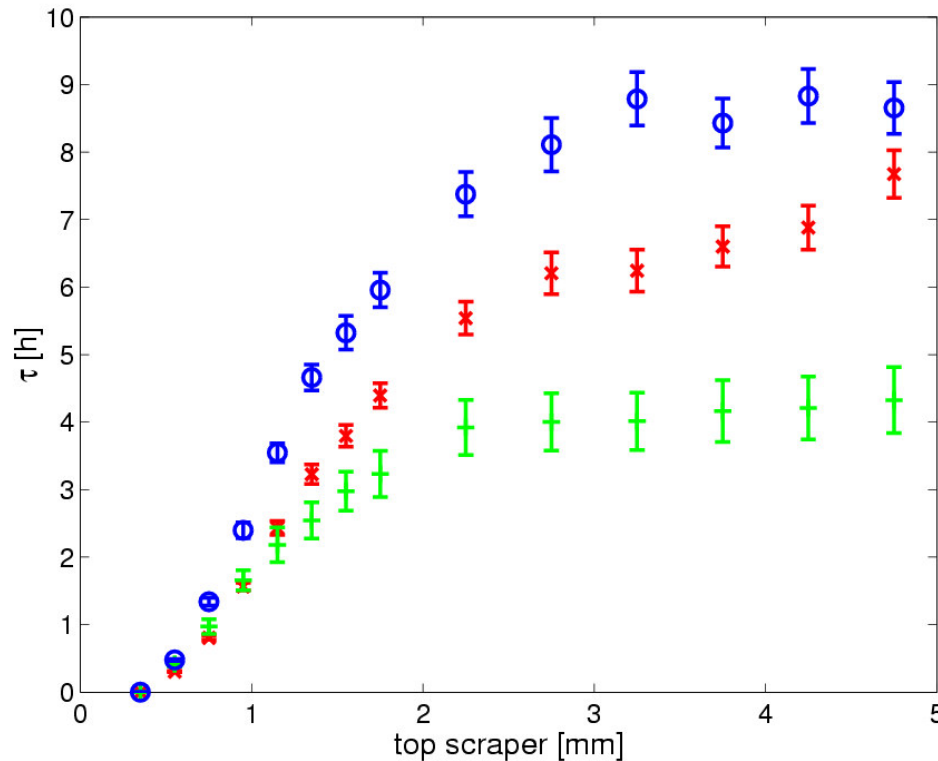


Vertical Beamsize Stability

- ❖ The stability of the (vertical) beamsize is important for users (not all effects of varying beamsize can be normalized out)
 - Main issues affecting the beamsize are **residual tuneshifts** (after feedforward compensation) when scanning undulators or **skew errors inside those undulators** (especially EPUs)
- ❖ Using **dispersion wave** instead of coupling resonance to increase vertical emittance **improves beamsize stability**



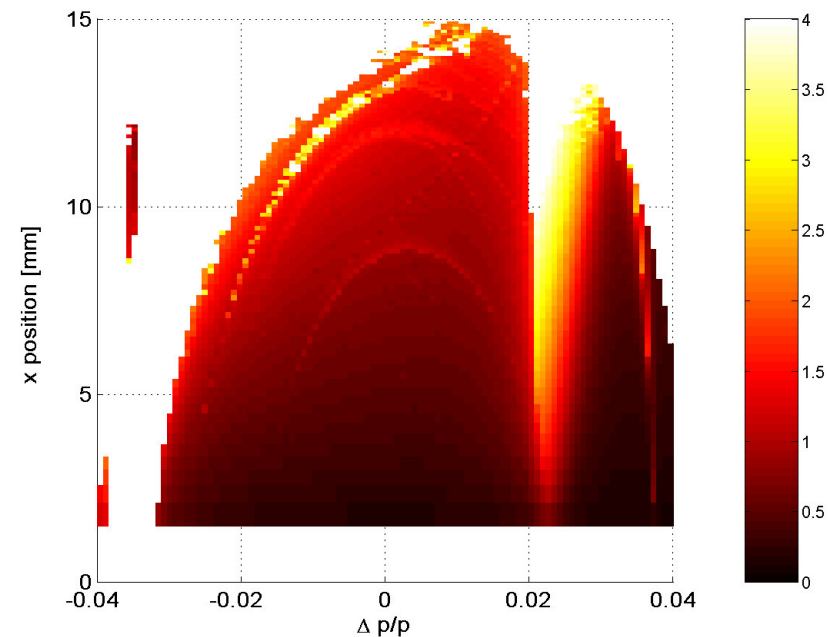
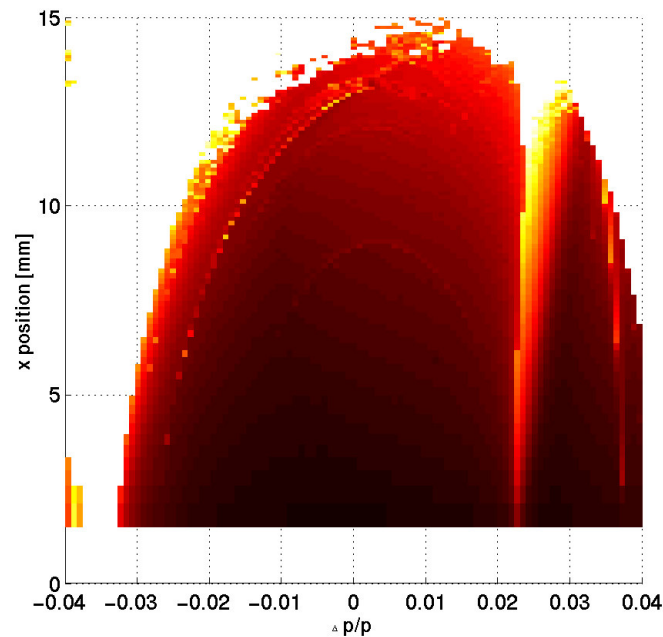
Lifetime vs. Vertical Physical Aperture



- Performance (**Brightness**) of undulators/wigglers (both permanent magnet and SC) depends on **magnetic gap**
- Strong incentive to push physical aperture as low as possible
- The vertical physical aperture at which the lifetime starts to get smaller depends strongly on how well global and local coupling is corrected!

Simulation Results (Momentum Aperture – Gap)

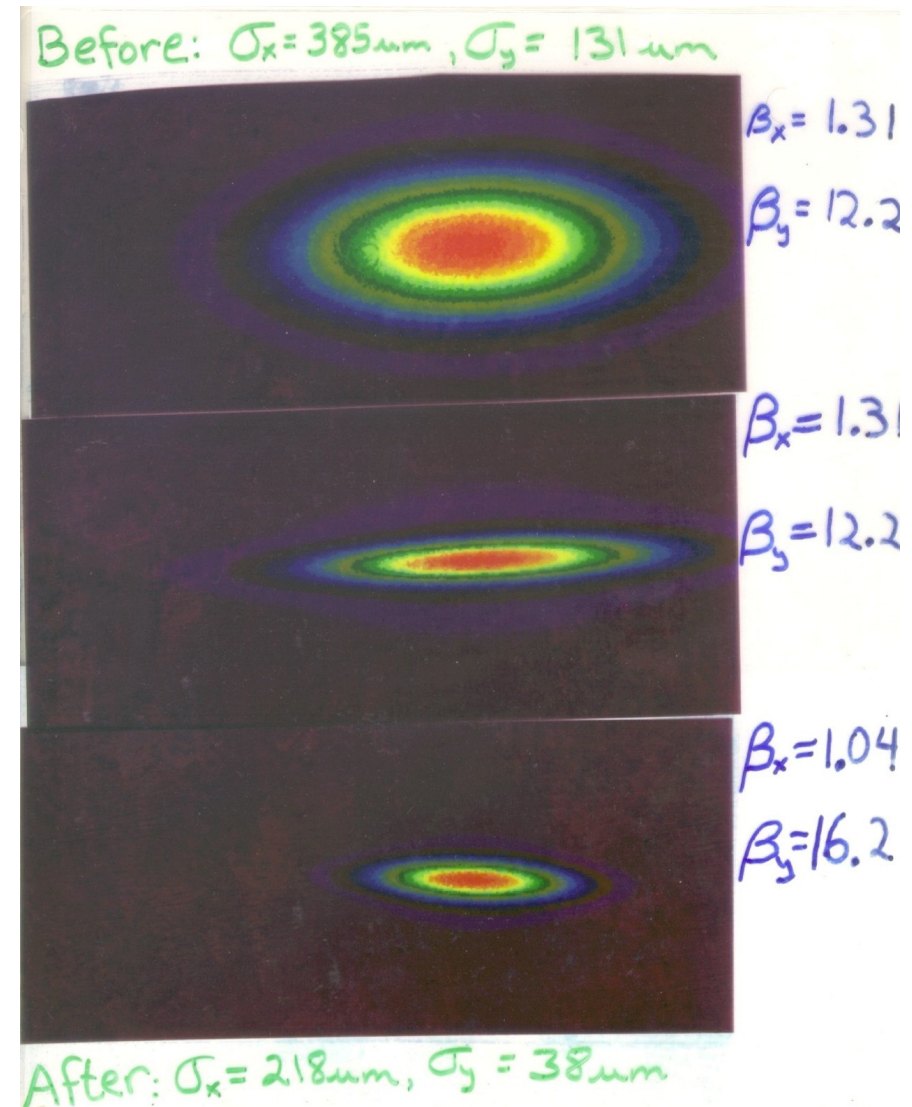
Emittance increased using vertical dispersion wave ... using excitation of coupling resonance



- ❖ Tracking results are in good agreement with measured effects, i.e. case with dispersion wave has less yellow and orange areas than the one with excited coupling resonance, indicating less sensitivity to reduced vertical aperture

Other Examples: NSLS

- ❖ James was (to my knowledge) the first to use response matrix based fitting to correct coupling.
- ❖ Applied it very successful at the NSLS, achieving less than 0.1% emittance ratio. Still about the best emittance ratio reached anywhere, though the absolute vertical emittance was somewhat larger than in ALS, because of much larger natural emittance of X-ray ring.
- ❖ (Data from James Safranek)



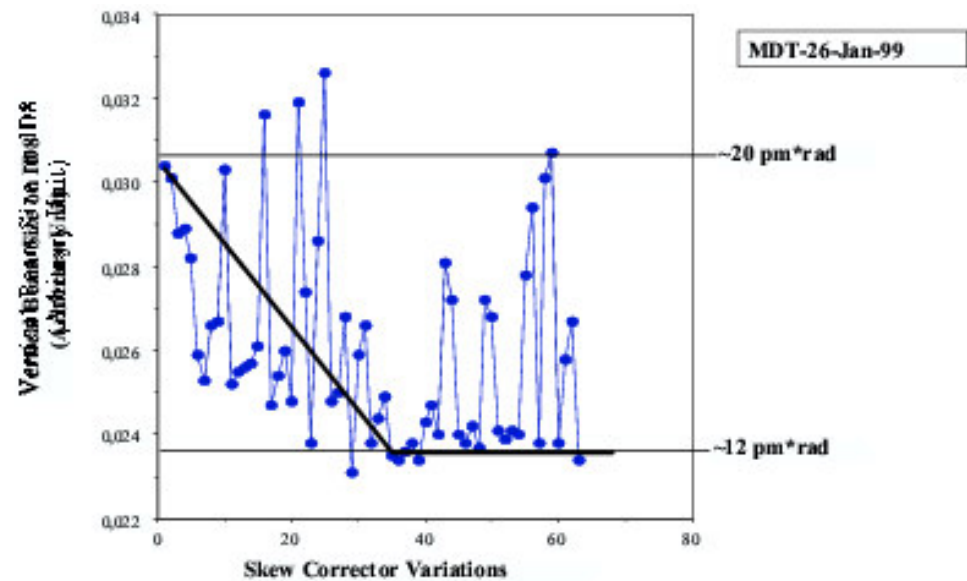
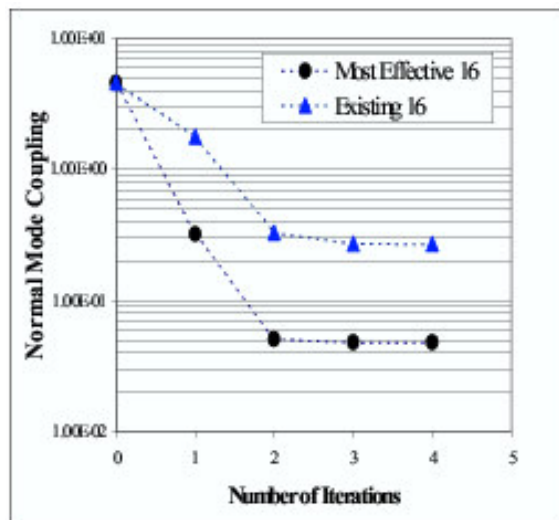
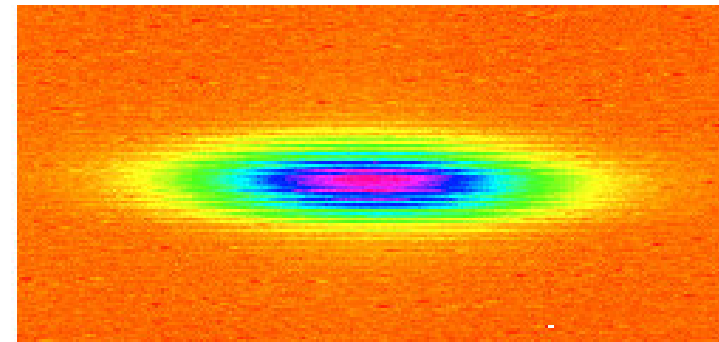


Other Example: ESRF

- ❖ Phi Nghiem, R. Nagaoka and Tordeux carried out very nice work at ESRF using a method similar to LOCO.
- ❖ Problem is the large number of elements in ESRF, order of magnitude is 400 correctors and 400 BPMs and similar number of quadrupoles, sextupoles.
- ❖ Could only use partial response matrix in analysis. Averaged over several of those matrices.
- ❖ Did not fit tilt errors of individual magnets, but effective skew distribution (enough to describe the local coupling structure, but few enough to not get strong degeneracies)
- ❖ It was important to study precisely what singular values to keep in inversion and which ones to neglect.
- ❖ Had to iterate with empirical correction on top of the LOCO predicted correction – reason seems to be relatively small number of skew quadrupoles (16).

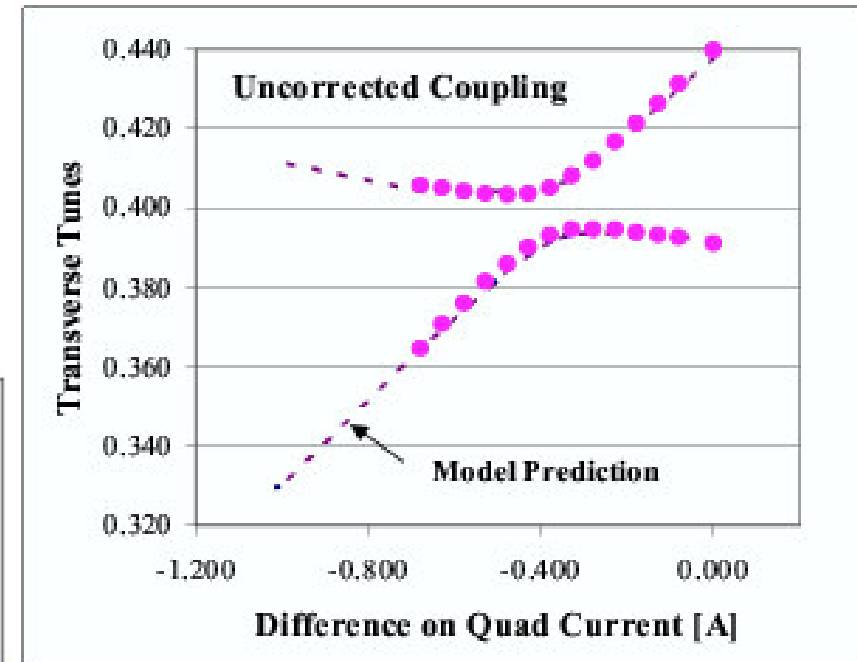
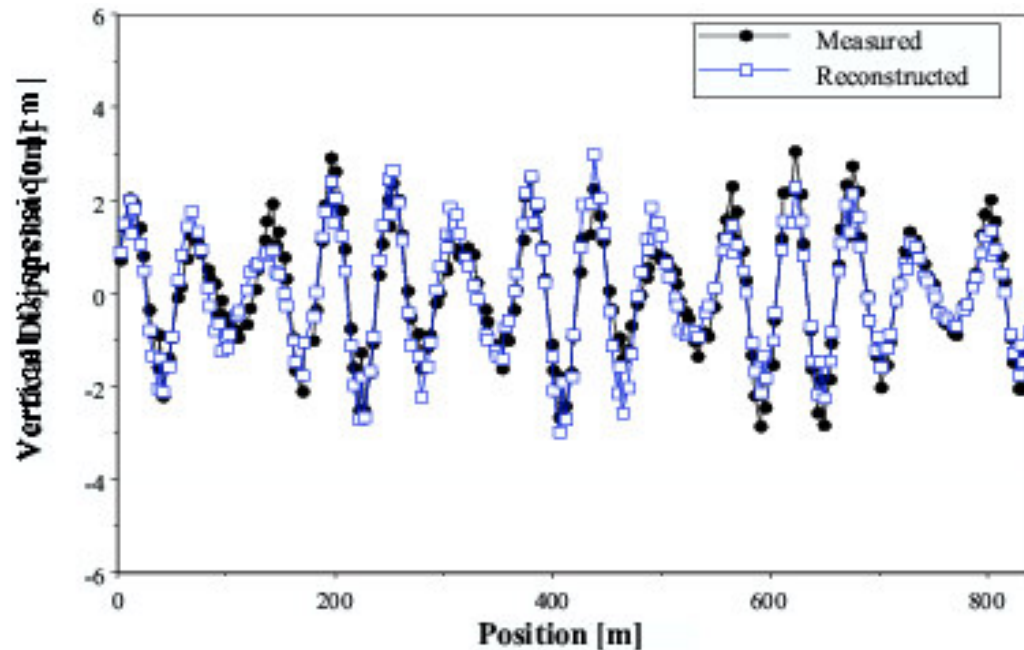
Other Example: ESRF

- ❖ Reached about 10 pm emittance.
- ❖ Predictions from model (tune scan, ...) agree very well with independent measurements.
- ❖ (All plots courtesy of R. Nagaoka ESRF/Soleil)



Other Example: ESRF

❖ (Plots courtesy of R. Nagaoka)





Summary

- ❖ Coupling correction is important to optimize the performance of an accelerator. Direct benefits are increased brightness or increased luminosity. More indirect improvements are dynamic (momentum) aperture and therefore injection efficiency and lifetime.
- ❖ There are several correction methods. At light sources a combined approach targeting local coupling, global coupling and vertical dispersion simultaneously has been most successful.
- ❖ Using orbit response matrix analysis (LOCO), emittance ratios below 0.1% have been achieved. For the ALS that corresponds to a vertical emittance of about 5 pm rad, which is within a factor of ten of the theoretical limit due to the finite opening angle ($1/\gamma$) of the synchrotron radiation!



Further Reading

- ❖ Guignard, CERN 76-06 1976
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- ❖ K. Ohmi et al., PRE 49, No 1, 1994
- ❖ D. Sagan and D. Rubin, PRST-AB, Vol 2, 1999
- ❖ D. Sagan et al. PRST-AB, Vol. 3, 2000.
- ❖ J. Safranek, and S. Krinsky, PAC'93 and AIP Proc. 315, 1993.
- ❖ J. Safranek, NIM A 388, p 27, 1997.
- ❖ C. Steier, and D. Robin, EPAC'00.
- ❖ P. Nghiem, and Tordeux, Coupling correction for the ESRF, SOLEIL internal report, 1999.
- ❖ R. Nagaoka, EPAC'00.
- ❖ R. Nagaoka, and L. Farvacque, PAC'01.
- ❖ K. Kubo, et al., Phys. Rev. Lett. 88:194891 (2002)
- ❖ C. Steier, et al., 'Coupling Correction and ...', PAC 2003