Linear Optics from Closed Orbits (LOCO)

LOCO: S:\Private\XRay\LOCO\Mar24_03\set2\fixedP\xraylocooutput.mat

File - Inputs - Export to Workspace - Help

χ^2_{total} / D.O.F = 21911.029395

Model Beta Function (ν_y=0.2976)

Model-Measured+EnergyShifts Response Matrix
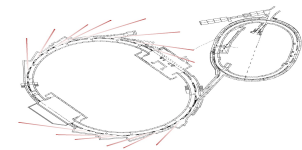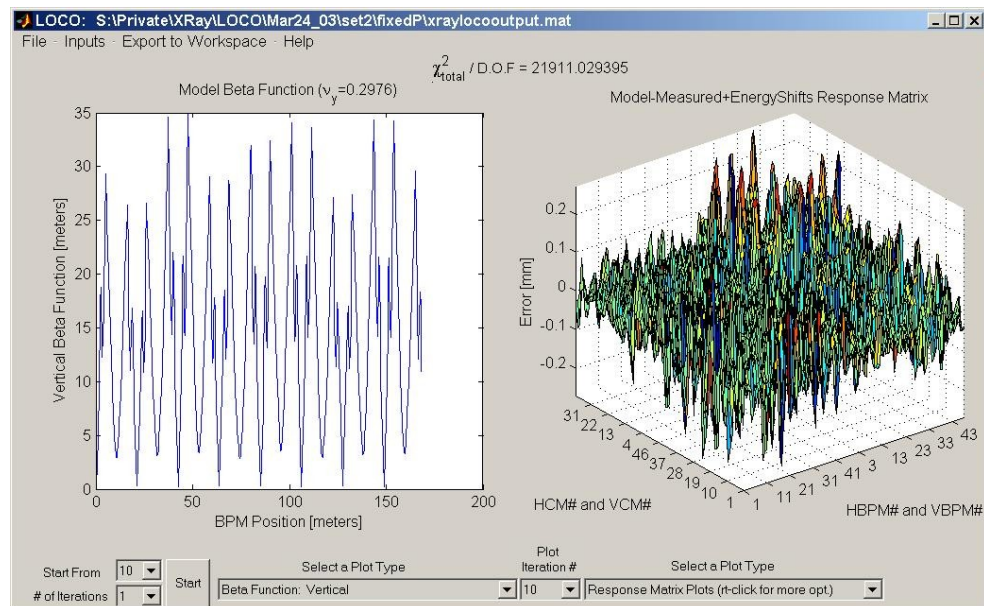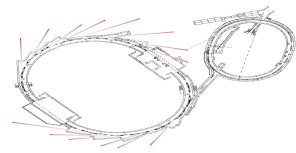
Linear optics from closed orbits (LOCO)

- **Given linear optics (quad. gradients), can calculate response matrix.**

- **Reverse is possible – calculate gradients from measured response matrix.**

- **Orbit response matrix has thousands or tens of thousands of highly accurate data points giving a measure of linear optics.**

- **The LOCO code uses this data to calibrate and correct linear optics.**
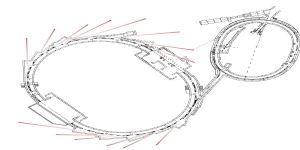
Lecture outline

- **LOCO method**

- **Error analysis**

- **Applications**

LOCO GUI

LOCO GUI main window

NSLS VUV ring example

The VUV ring optics were not well controlled. There was a problem with incorrect compensation for insertion device (ID) focusing. LOCO was used to calibrate the strength of the ID focusing and to find the changes the current to the quadrupoles that best restored the optics.

The results were

• **20% increase in lifetime**

• **Few percent decrease in both ε_x and ε_y**

VUV Ring before correction

Machine Functions (m)

Distance(m)

IDs

VUV Ring after correction

Machine Functions (m)

Distance(m)
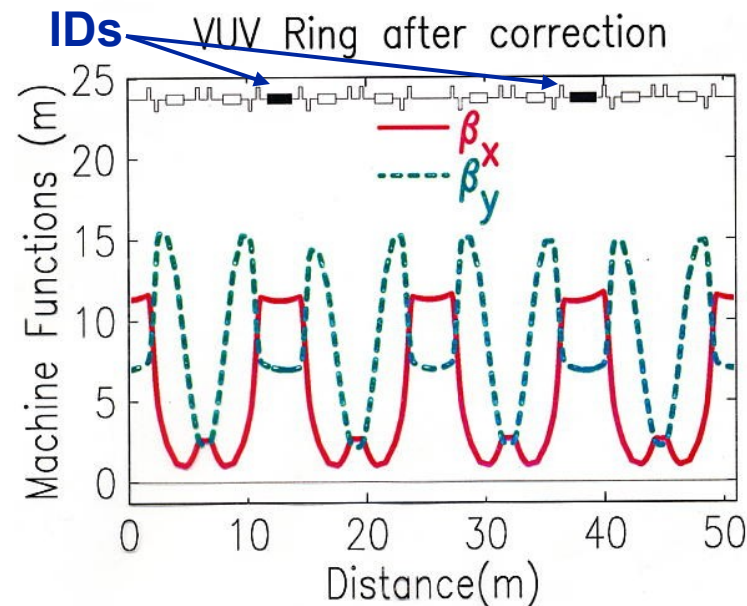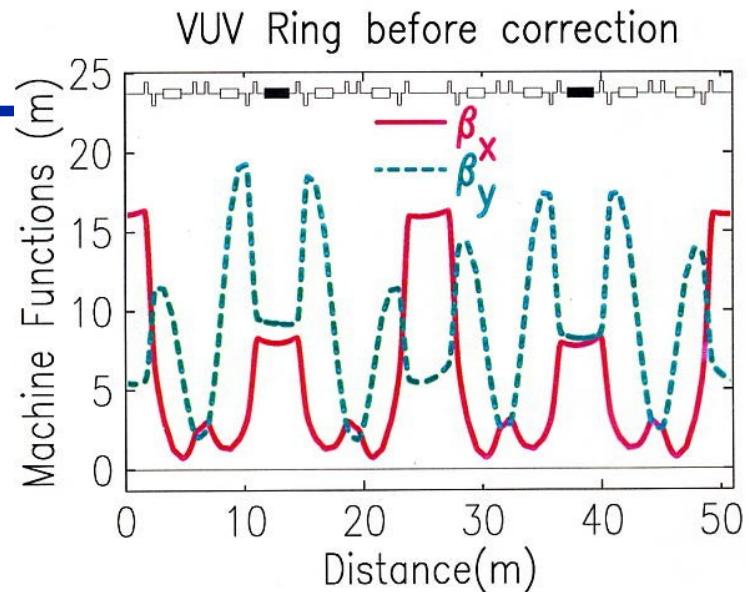
Method

The orbit response matrix is defined as

$$\begin{bmatrix} \vec{x} \\ \vec{y} \end{bmatrix} = M \begin{bmatrix} \vec{\theta}_x \\ \vec{\theta}_y \end{bmatrix}$$

The parameters in a computer model of a storage ring are varied to minimize the χ^2 deviation between the model and measured orbit response matrices (M_{mod} and M_{meas}).

$$\chi^2 = \sum_{i,j} \frac{(M_{ij}^{meas} - M_{ij}^{model})^2}{\sigma_i^2} \equiv \sum_{k=i,j} E_k^2$$

The σ_i are the measured noise levels for the BPMs; E is the error vector.

The χ^2 minimization is achieved by iteratively solving the linear equation

$$E_k^{new} = E_k + \frac{\partial E_k}{\partial K_l} \Delta K_l = 0$$

$$- E_k = \frac{\partial E_k}{\partial K_l} \Delta K_l$$

For the changes in the model parameters, K_l, that minimize $||E||^2 = \chi^2$.

Response matrix review

The response matrix is the shift in orbit at each BPM for a change in strength of each steering magnet.

Vertical response matrix, BPM i, steerer j:

$$M_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin \pi \nu} \cos(|\phi_i - \phi_j| - \pi \nu)$$

Horizontal response matrix:

$$M_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2 \sin \pi \nu} \cos(|\phi_i - \phi_j| - \pi \nu) + \frac{\eta_i \eta_j}{\alpha_c L_0}$$
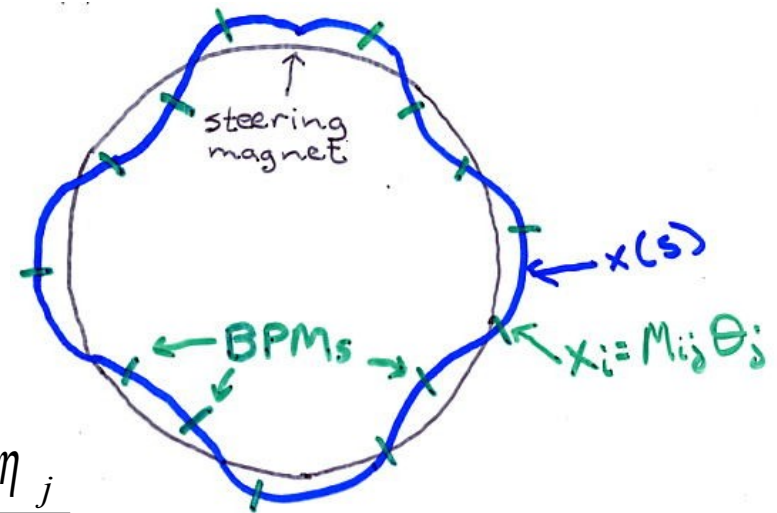
Additional η term keeps the path length constant (fixed rf frequency).

LOCO option to use this linear form of the response matrix (faster) or can calculate response matrix including magnet nonlinearities and skew gradients (slower, more precise). First converge with linear response matrix, then use full response matrix.

Parameters varied to fit the orbit response matrix

NSLS XRay Ring fit parameters:

56 quadrupole gradients

48 BPM gains, horizontal

48 BPM gains, vertical

90 steering magnet kicks

=242 Total fit parameters

NSLS XRay data:

(48 BPMs)*(90 steering magnets)

=4320 data points

χ^2 fit becomes a minimization problem of a function of 242 variables.

Fit converted to linear algebra problem, minimize $||E||^2 = \chi^2$.

For larger rings, fit thousands of parameters to tens of thousands of data points. For APS, full $\partial \vec{E}/\partial \vec{K}$ matrix is ~9 Gbytes, so the size of the problem must be reduced by limiting the number of steering magnets in the response matrix. For rings the size of LEP, problem gets too large to solve all at once on existing computers. Need to divide ring into sections and analyze sections separately.

Recent results - Australia
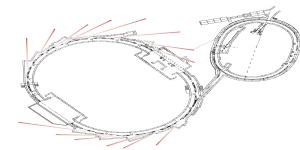
○ **BPM gains**

○ **Steering magnet calibration**

○ β_y **before & after correction**
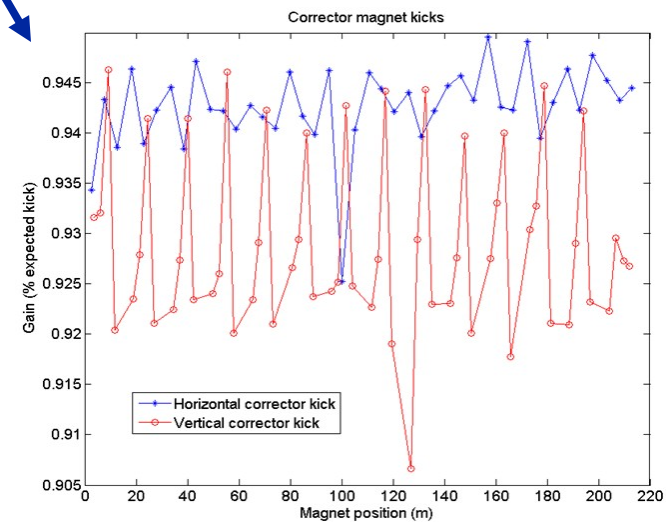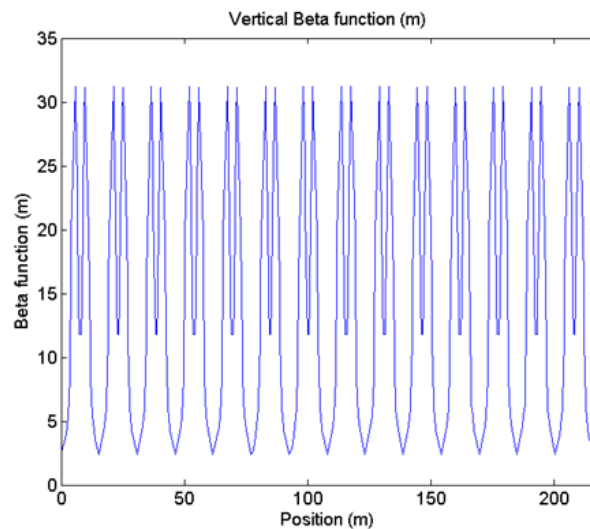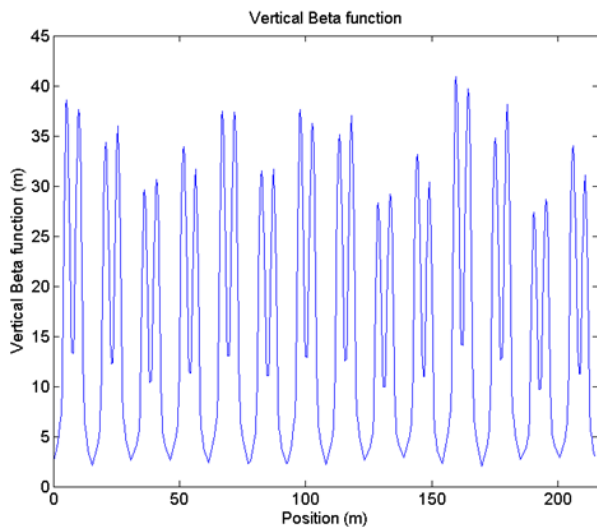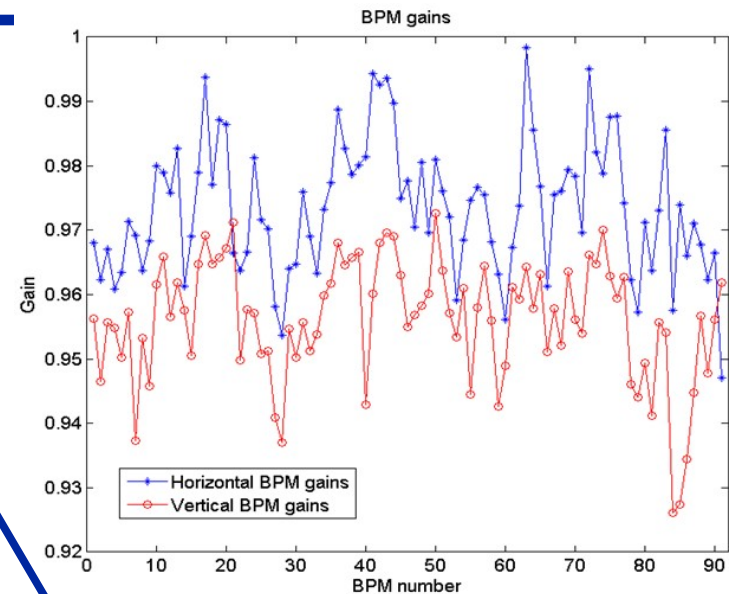
More fit parameters
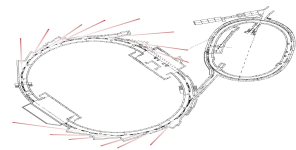
Why add BPM gains and steering magnet calibrations?

• **Adding more fit parameters increases error bars on fit gradients due to propagation of random measurement noise on BPMs. If you knew that all the BPMs were perfectly calibrated, it would be better not to vary the BPM gains in the fit.**

• **More fit parameters decreases error on fit gradients from systematic modeling errors. Not varying BPM gains introduces systematic error.**

• **As a rule, vary parameters that introduce 'significant' systematic error. This usually includes BPM gains and steering magnet kicks.**

Other parameters to vary:

• **Quadrupole roll (skew gradient)**

• **Steering magnet roll**

• **BPM coupling**

• **Steering magnet energy shifts**

• **Steering magnet longitudinal centers**

Parameters for coupled response matrix,

$$\begin{bmatrix} \vec{x} \\ \vec{y} \end{bmatrix} = \begin{bmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{bmatrix} \begin{bmatrix} \vec{\theta}_x \\ \vec{\theta}_y \end{bmatrix}$$

Horizontal response matrix:
$$M_{ij} = \frac{\sqrt{\beta_i \beta_j}}{2\sin\pi\nu}\cos(|\phi_i - \phi_j| - \pi\nu) + \frac{\eta_i \eta_j}{\alpha_c L_0}$$

Betatron amplitudes and phases depend only on storage ring gradients:
$$\frac{1}{2}\beta\beta'' - \frac{1}{4}\beta'^2 + \beta^2 K = 1 \qquad d\phi = \frac{ds}{\beta}$$

Dispersion depends both on gradients and dipole field distribution:
$$\eta'' + K\eta = \frac{1}{\rho}$$

If the goal is to find the gradient errors, then fitting the full response matrix, including the term with η, will be subject to systematic errors associated with dipole errors in the real ring not included in the model. This problem can be circumvented by using a "fixed momentum" model,
$$M_{ij}^{\text{fixed p}} = \frac{\sqrt{\beta_i \beta_j}}{2\sin\pi\nu}\cos(|\phi_i - \phi_j| - \pi\nu),$$

and adding a term to the model proportional to the measured dispersion
$$M_{ij}^{\text{mod}} = M_{ij}^{\text{fixed p}} + \frac{\Delta p_i}{p}\eta_j^{\text{meas}}$$

$\Delta p_i/p$ **is a fit parameter for each steering magnet. In this way the η^{model} is eliminated from the fit, along with systematic error from differences between η^{model} and η^{meas}.**

Finding gradient errors at ALS

- **LOCO fit indicated gradient errors in ALS QD magnets making β_y distortion.**

- **Gradient errors subsequently confirmed with current measurements.**

- **LOCO used to fix β_y periodicity.**

- **Operational improvement (Thursday lecture).**

ALS before correction

ALS after correction

ALS QD gradient variations

Response matrix fit

Power supply current measurement
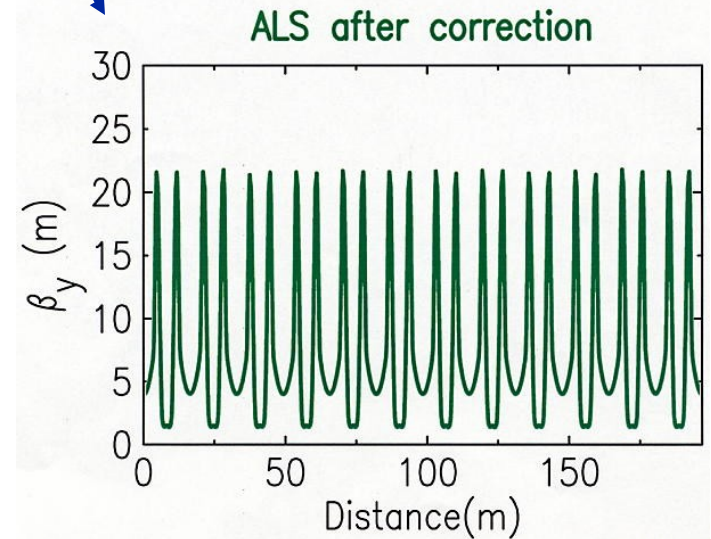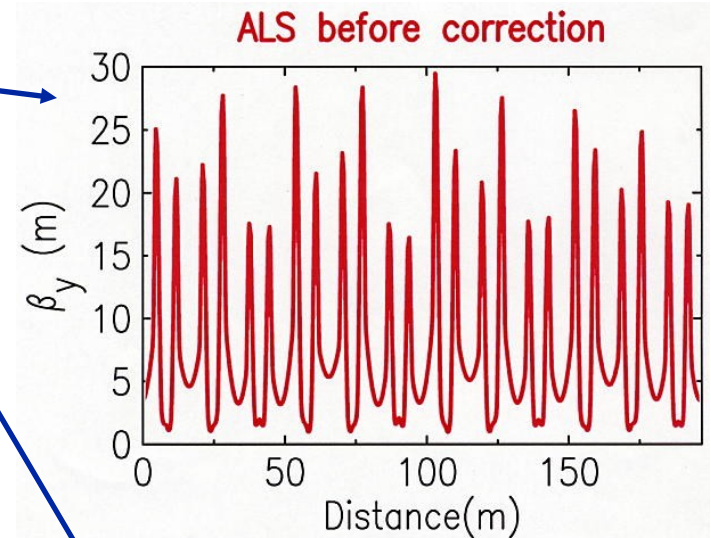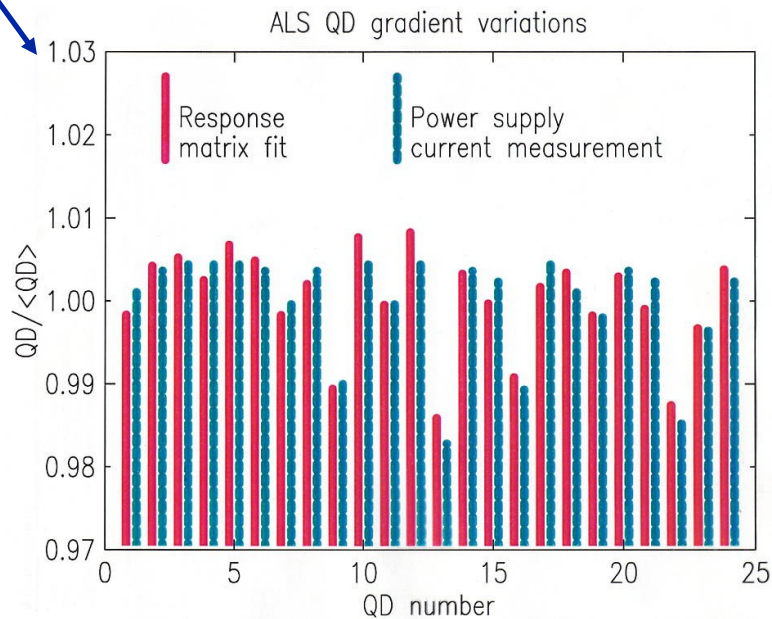
Different goals when applying LOCO
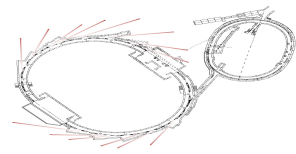
There are a variety of results that can be achieved with LOCO:

- **Finding actual gradient errors.**

- **Finding changes in gradients to correct betas.**

- **Finding changes in gradients to correct betas and dispersion.**

- **Finding changes in local gradients to correct ID focusing.**

- **Finding changes in skew gradients to correct coupling and η_y.**

- **Finding transverse impedance.**

The details of how to set up LOCO and the way the response matrix is measured differs depending on the goal.

In the previous example for the ALS, the goals were 1 and 2. LOCO is set up differently for each.

Finding gradient errors
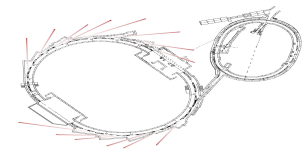
If possible, measure two response matrices – one with sextupoles off and one with sextupoles on.

- **Fit the first to find individual quadrupole gradients.**

- **Fit the second to find gradients in sextupoles.**

- **Fewer gradients are fit to each response matrix, increasing the accuracy.**

- **… Measure a 3rd response matrix with IDs closed.**

Vary all quadrupole gradients individually (maybe leave dipole gradient as a family).

Use either 1.) fixed-momentum response matrix and fit energy shifts or 2.) fixed-path-length depending on how well $1/\rho$ in the model agrees with $1/\rho$ in the ring (i.e. how well is the orbit known and controlled).

Get the model parameters to agree as best as possible with the real ring: model dipole field roll-off; check longitudinal positions of BPMs and steering magnets; compensate for known nonlinearities in BPMs.

Add more fitting parameters if necessary to reduce systematic error (for example, fit steering magnet longitudinal centers in X-Ray Ring.)
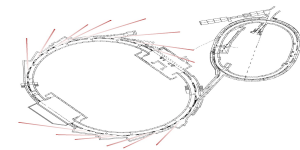
Correcting betas and dispersion

Measure response matrix with ring in configuration for delivered beam.

- **Sextupoles on**
- **Correct to golden orbit**
- **IDs closed (depending on how you want to deal with ID focusing)**

Fit only gradients that can be adjusted in real ring.

- **Do not fit gradients in sextupoles or ID gradients**
- **If a family of quadrupoles is in a string with a single power supply, constrain the gradients of the family to be the same.**

To correct betas only, use fixed-momentum model matrix and fit energy shifts, so dispersion is excluded from fit.

To correct betas and dispersion, use fixed-path length matrix and can use option of including η as an additional column in response matrix.

To implement correction, change quadrupole current of nth quadrupole or quad family:

$$\frac{\Delta I_n}{I_n} = - \frac{K_{\text{fit},n} - K_{\text{ideal},n}}{K_{\text{ideal},n}}$$
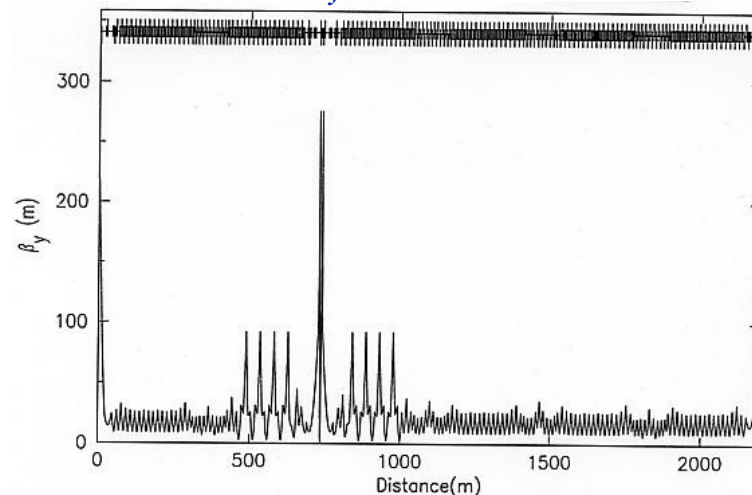
Correcting betas in PEPII

Often times, finding the quad changes required to correct the optics is easier than finding the exact source of all the gradient errors.

For example, in PEPII there are not enough BPMs to constrain a fit for each individual quadrupole gradient. The optics still could be corrected by fitting quadrupole families.
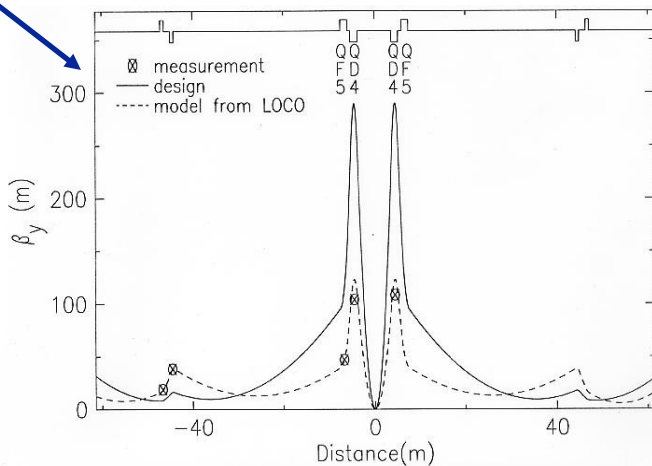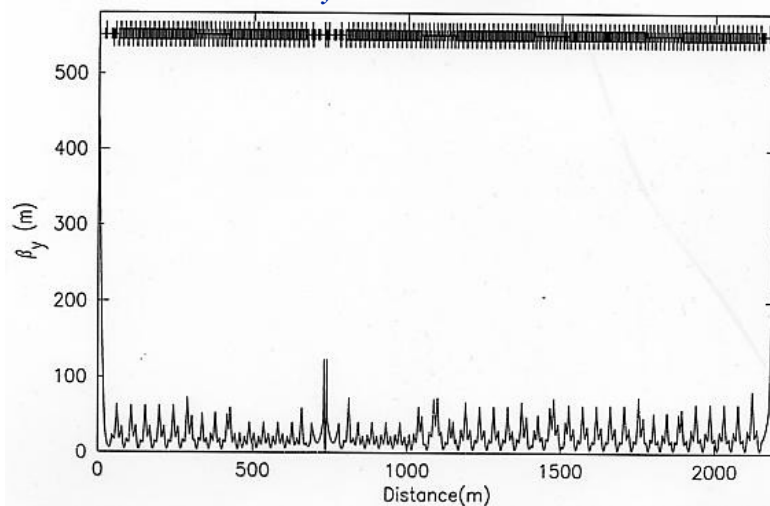
Independent β measurements confirmed that LOCO had found the real β's (x2.5 error!)

Quadrupole current changes according to fit gradients restored ring optics to the design.
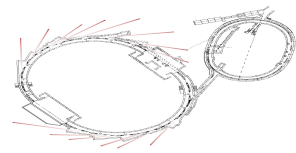
PEPII HER β_y, design

PEPII HER β_y, LOCO fit

Optics distortion from a single gradient error:

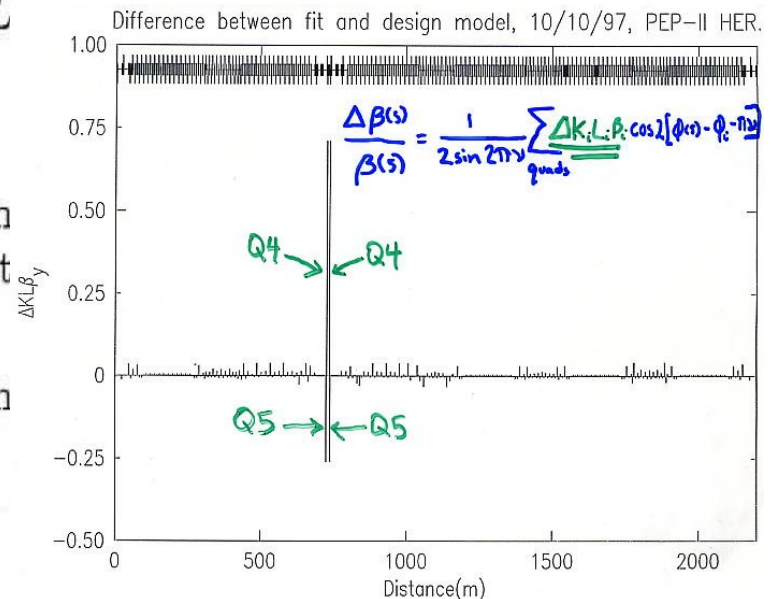The shift in tune from an integrated gradient error of ΔKL is

$$\Delta \nu = \frac{1}{4\pi}\beta_q \Delta KL.$$

(Recall that measuring the change in tune for a change in quadrupole gradient gives a measure of the β-function at the quadrupole, β_q.)

A gradient error also creates a distortion of the β-function around the ring.

$$\frac{\Delta\beta}{\beta} = \frac{1}{2\sin(2\pi\nu)}\beta_q \Delta KL \cos[2(\phi(s) - \phi_q) - 2\pi\nu],$$

so the distortion of the β-function from a gradient error scales as $\beta_q\Delta KL$. Looking at the fit value of $\beta_q\Delta KL$ for the different quadrupoles in the ring shows the source of the optics distortion.

Difference between fit and design model, 10/10/97, PEP–II HER.

$$\frac{\Delta\beta(s)}{\beta(s)} = \frac{1}{2\sin 2\pi\nu}\sum_{quads}\Delta K_i L_i \beta_i \cos 2[\phi(s) - \phi_i - \pi\nu]$$

Insertion device linear optics correction

The code LOCO can be used in a beam-based algorithm for correcting the linear optics distortion from IDs with the following procedure:
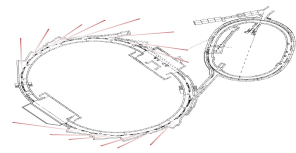
- Measure the response matrix with the ID gap open.

- Then the response matrix is measured with the gap closed.

- Fit the first response matrix to find a model of the optics without the ID distortion.

- Starting from this model, LOCO is used to fit a model of the optics including the ID. In this second fit, only a select set of quadrupoles in the vicinity of the ID are varied. The change in the quadrupole gradients between the 1rst and 2nd fit models gives a good correction for the ID optics distortion.

- Alternatively, LOCO can be used to accurately fit the gradient perturbation from the ID, and the best correction can be calculated in an optics modeling code.
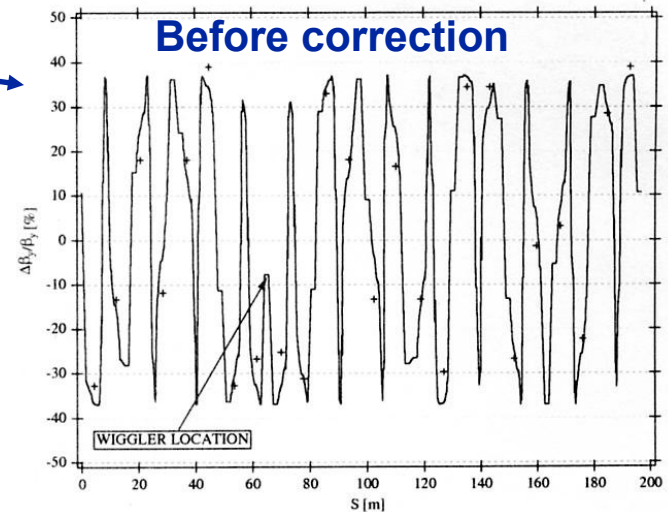
1.) L. Smith, LBNL, ESG Technical Note No. 24, 1986.
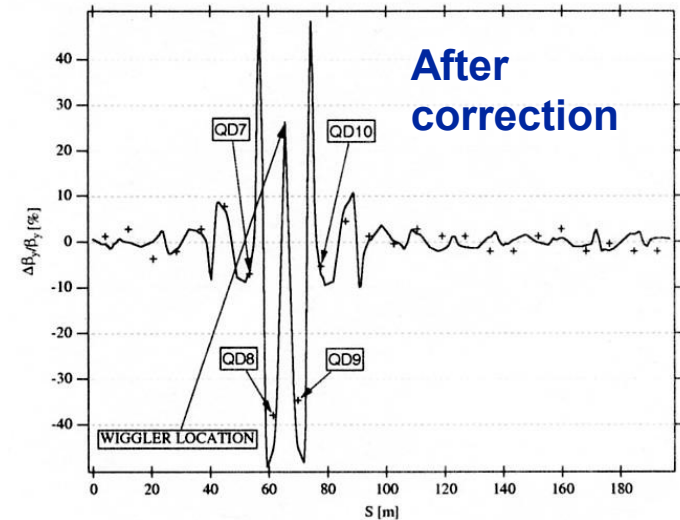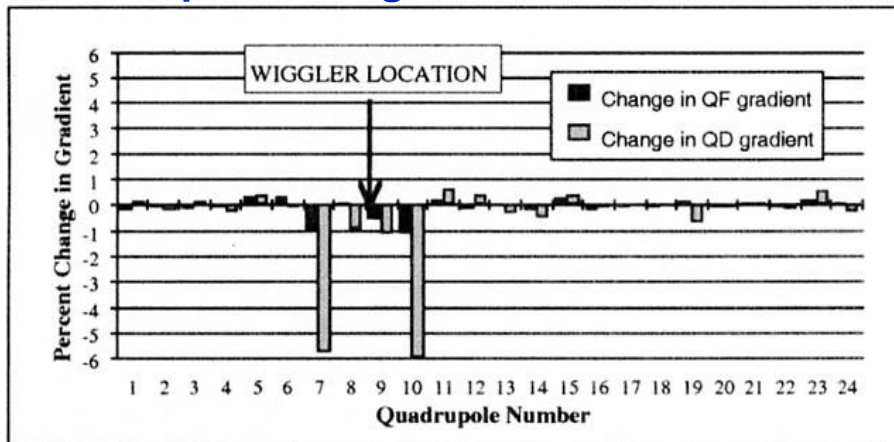
Linear optics correction at ALS
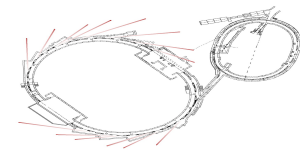
Beta function distortion from wiggler.

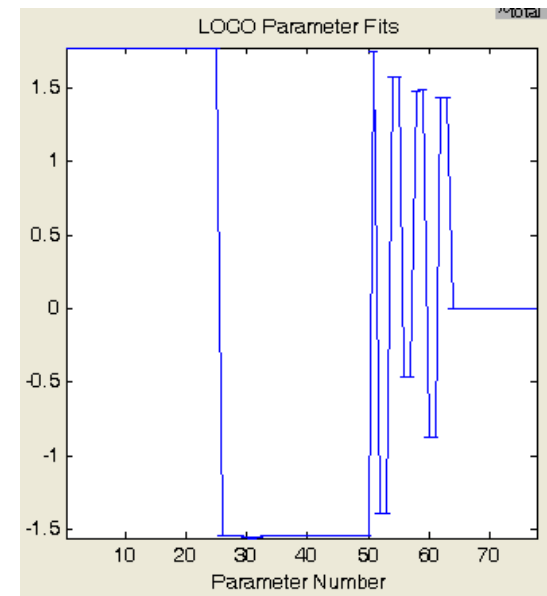At ALS the quadrupoles closest to the IDs are not at the proper phase to correct optics distortions, so the optics correction cannot be made entirely local.

Quadrupole changes used for correction

Before correction

After correction

D. Robin et al. PAC97

Optics correction at SPEAR3
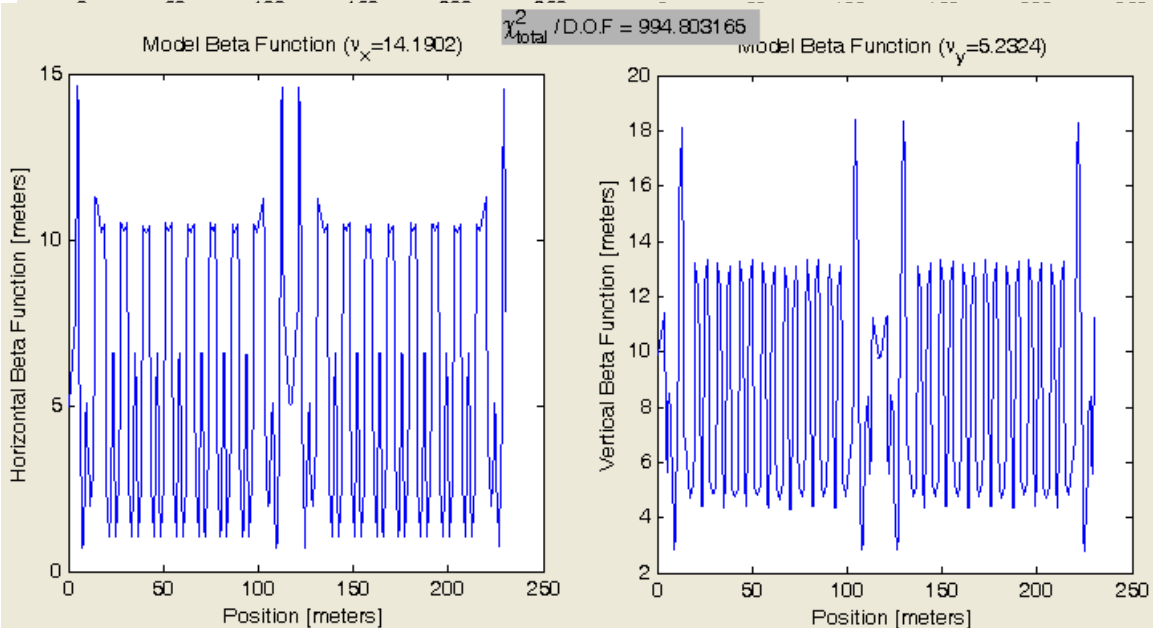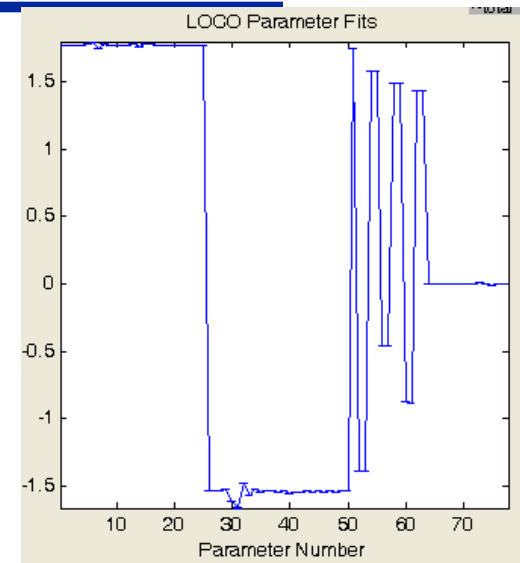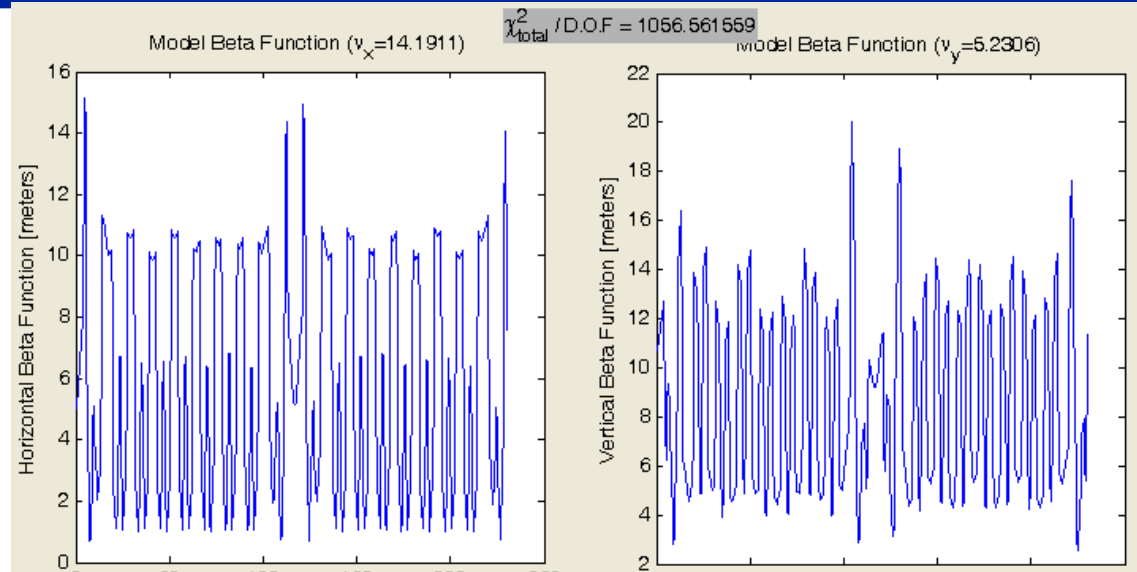
Skew quadrupole compensation for ALS EPUs
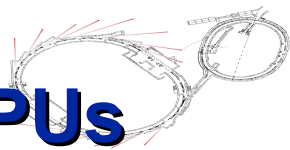
○ **Beamsize variation was solved in 2004: Installed correction coils for feedforward based compensation – routine use since June/September**

○ **Early 2005 we identified the root cause: 2-3 micron correlated motion of magnet modules due to magnetic forces**

○ **Will be able to modify design of future device such that active correction will not be necessary!**
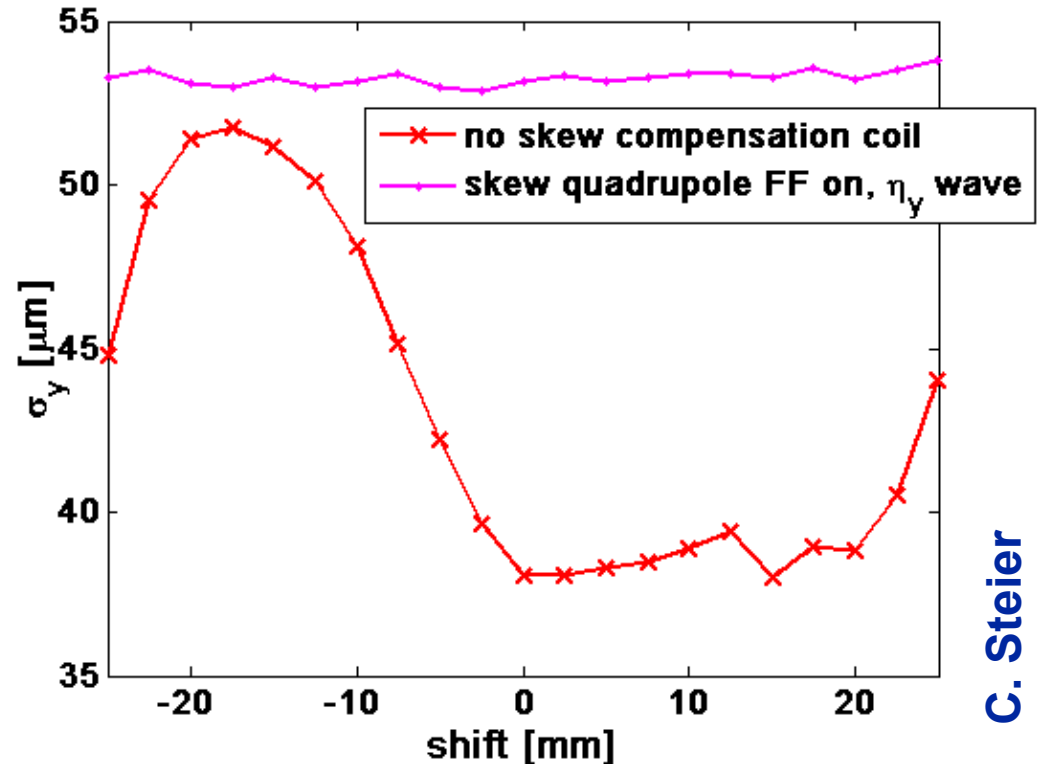
C. Steier

• **Just for reference: Whenever an undulator moves, about 120-150 magnets are changed to compensate for the effect (slow+fast feed-forward, slow+fast feedback)**
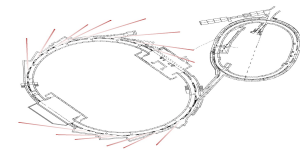
Coupling & η_y correction, LOCO

Minimize η_y and off-diagonal response matrix:
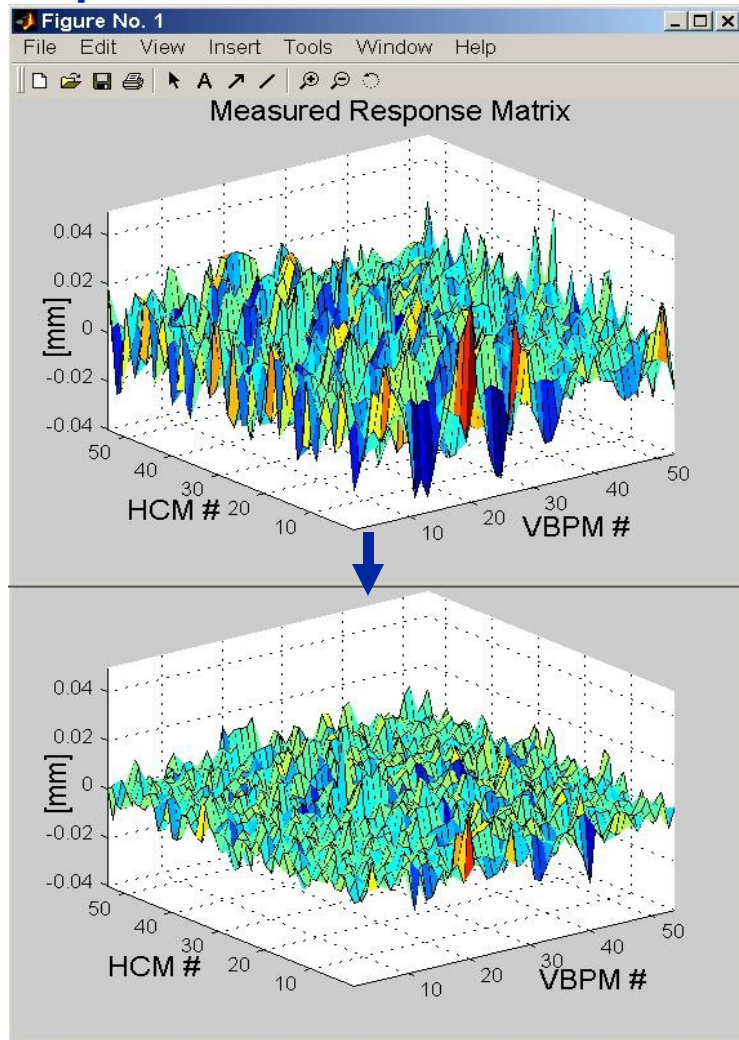
Lifetime, 19 mA, single bunch

4.5 hours

Correction off

Coupling correction on
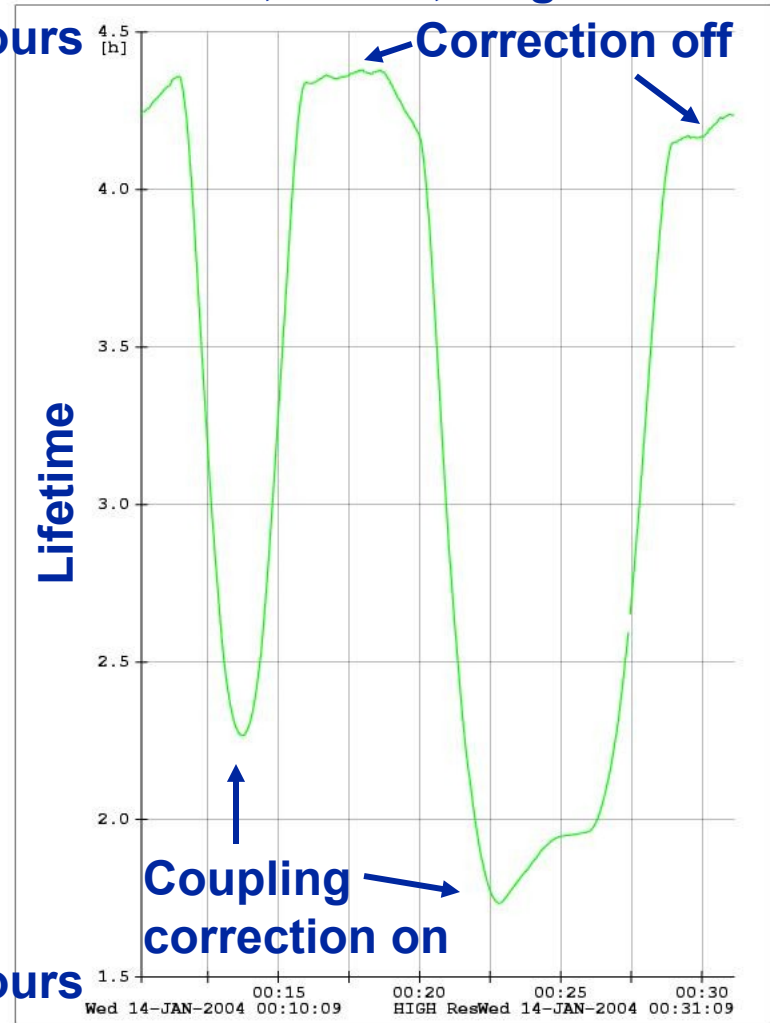
1.5 hours
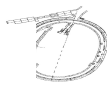
Simulation of coupling correction with LOCO

❖ Use accelerator toolbox (Andrei Terebilo), Matlab and LOCO (James Safranek, Greg Portman) for simulations

❖ Use random skew error seeds

❖ Try to find effective skew corrector distributions and to optimize correction technique in simulation

❖ Used two correction approaches:
1. Response Matrix fitting – 'deterministic', small number of iterations
2. Direct minimization (nelder-simplex, …) – easy to do on the model, difficult on real machine

– Surprisingly both approaches gave about the same performance in the model calculations

– For response matrix analysis you have to optimize several parameters of the code as well (weight of dispersion, number of SVs, use of effective model/full model …)

Weight of dispersion in LOCO fit

❖ The relative contribution of vertical dispersion and coupling to the vertical emittance depends on the particular lattice (and the particular error distribution).

❖ Therefore the optimum weight for the dispersion in the LOCO fit has to be determined (experimentally or in smulations).

❖ The larger the weight factor, the better the vertical dispersion gets corrected, but eventually the coupling 'explodes'.

❖ Set weight to optimum somewhat below that point.

❖ Outlier rejection tolerance might be important parameter as well.

Optimization of dispersion weight

Advanced Light Source

LOCO   James Safranek, USPAS, Annapolis, June 16-20, 2008
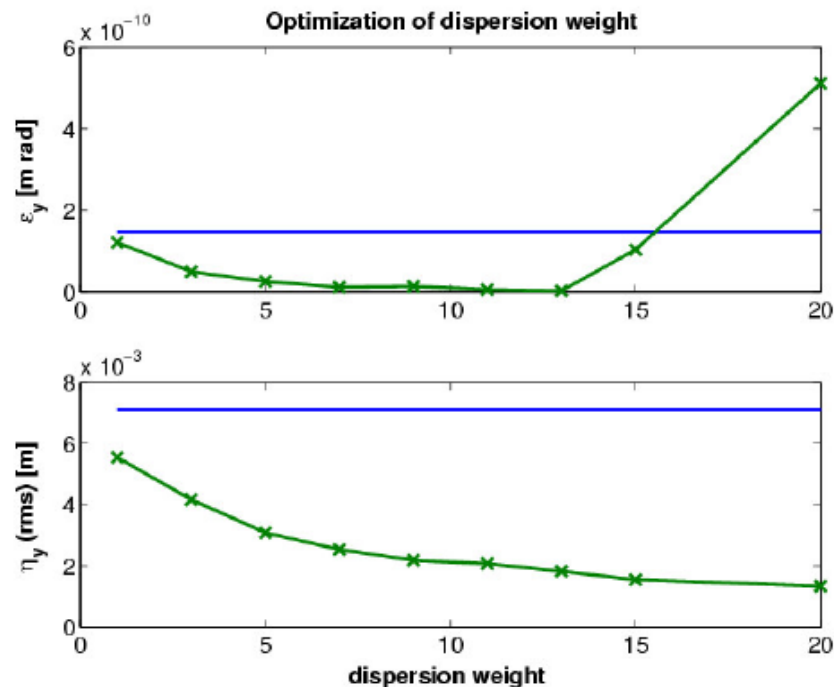
Finding an Effective Skew Quadrupole Set

❖ To find an effective skew quadrupole distribution, we used several correction methods, first in simulations – best method was orbit response matrix fitting (using LOCO)

❖ Predictive method, can be easily used on real machine

❖ Issues are:
  - Cover set of phases relative to dominant coupling resonance(s)
  - Magnets should be distributed around the ring in order to avoid excessive local coupling/vertical dispersion
  - Need different values of dispersion/beta function to be effective both for coupling and vertical dispersion correction

❖ Set of 12 skew quadrupoles was reasonably efficient

Before skew correction / After skew correction

James Safranek, USPAS, Annapolis, June 16-20, 2008
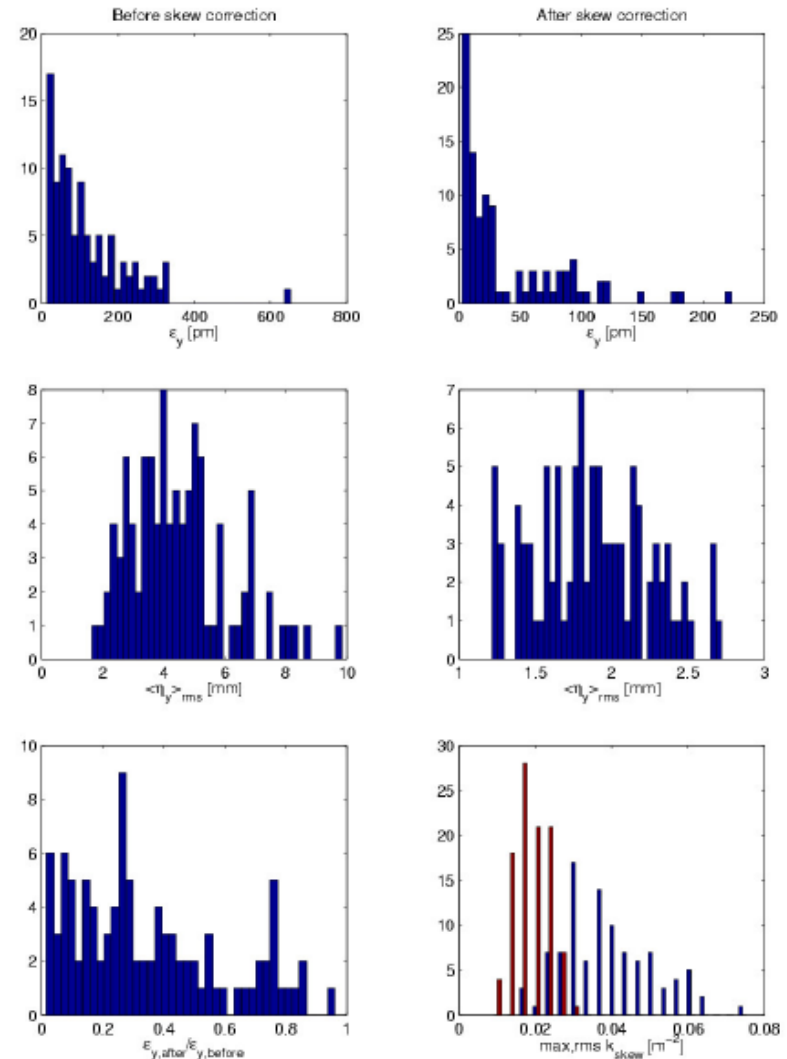
Advanced Light Source

Skew quadrupole corrector distribution

○ **Distribute in difference coupling resonance phase**

$$\kappa = \frac{1}{4}\pi \int ds K_s \sqrt{\beta_x \beta_y}\, e^{i\Phi_D} \qquad \frac{\Phi_D(s)}{2\pi} = (\mu_x(s) - \mu_y(s)) - \frac{s}{C}(\nu_x - \nu_y - N)$$

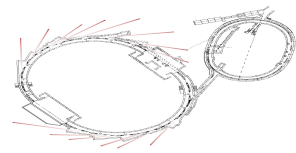○ **In sum coupling resonance phase**

$$\frac{1}{4}\pi \int ds K_s \sqrt{\beta_x \beta_y}\, e^{i\Phi_S} \qquad \frac{\Phi_S(s)}{2\pi} = (\mu_x(s) + \mu_y(s)) - \frac{s}{C}(\nu_x + \nu_y - M)$$

○ **And in η_y phase**

$$\eta_x \sqrt{\beta_y}\, e^{i\Phi_{\eta_y}} \qquad \frac{\Phi_{\eta_y}(s)}{2\pi} = \mu_y(s) - \frac{s}{C}(\nu_y - N)$$

⮱ **Need some skew quadrupoles at non-zero η_x**

LOCO GUI fitting options menu

Remove bad BPMs or steerers from fit.

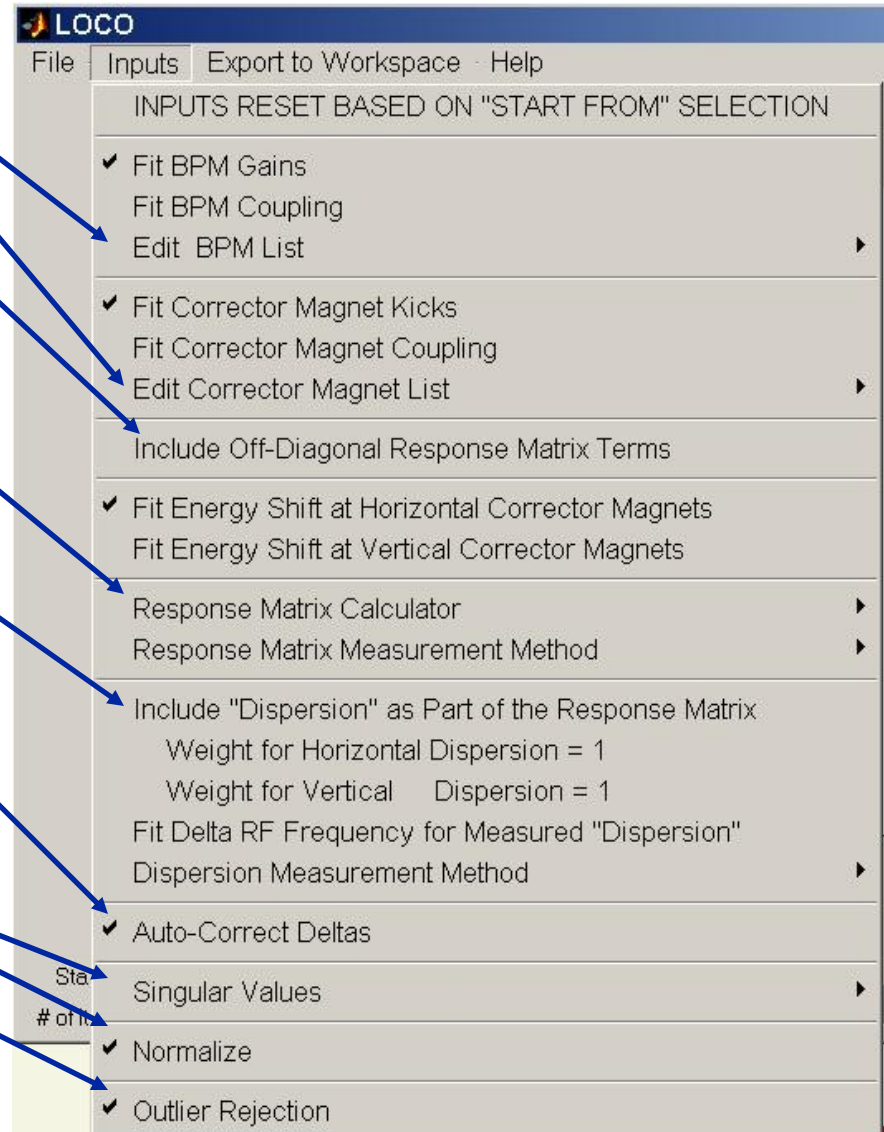**Include coupling terms (M_{xy}, M_{yx})**

Model response matrix: linear or full non-linear; fixed-momentum or fixed-path-length
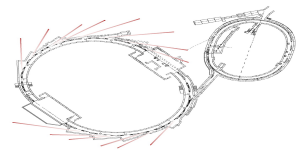
Include η as extra column of M

Let program choose Δs when calculating numerical derivatives of M with quadrupole gradients.

More on these coming.

Reject outlier data points.

LOCO
File | Inputs | Export to Workspace | Help

INPUTS RESET BASED ON "START FROM" SELECTION

✔ Fit BPM Gains
Fit BPM Coupling
Edit BPM List ▶

✔ Fit Corrector Magnet Kicks
Fit Corrector Magnet Coupling
Edit Corrector Magnet List ▶

Include Off-Diagonal Response Matrix Terms

✔ Fit Energy Shift at Horizontal Corrector Magnets
Fit Energy Shift at Vertical Corrector Magnets

Response Matrix Calculator ▶
Response Matrix Measurement Method ▶

Include "Dispersion" as Part of the Response Matrix
   Weight for Horizontal Dispersion = 1
   Weight for Vertical   Dispersion = 1
Fit Delta RF Frequency for Measured "Dispersion"
Dispersion Measurement Method ▶

✔ Auto-Correct Deltas

Singular Values ▶

✔ Normalize

✔ Outlier Rejection

Error bars from BPM measurement noise

LOCO calculates the error bars on the fit parameters according to the measured noise levels of the BPMs. LOCO uses singular value decomposition (SVD) to invert $\partial E_k / \partial K_l$ and solve for fit parameters.

$$- E_k = \frac{\partial E_k}{\partial K_l} \Delta K_l \qquad\qquad E_k = \frac{M_{ij}^{\mathrm{meas}} - M_{ij}^{\mathrm{model}}}{\sigma_i}$$

The results from SVD are useful in calculating and understanding the error bars.

$$\frac{\partial \vec{E}}{\partial \vec{K}} = USV^T = \sum \vec{u}_l w_l \vec{v}_l^T$$

SVD reduces the matrix to a sum of a product of eigenvectors of parameter changes, v, times eigenvectors, u, which give the changes in the error vector, E, corresponding to v. The singular values, w_l, give a measure of how much a change of parameters in the direction of v in the multidimensional parameter space changes the error vector.

(For a more detailed discussion see Numerical Recipes, Cambridge Press.)
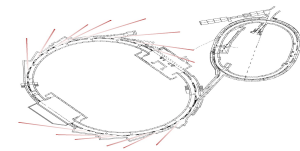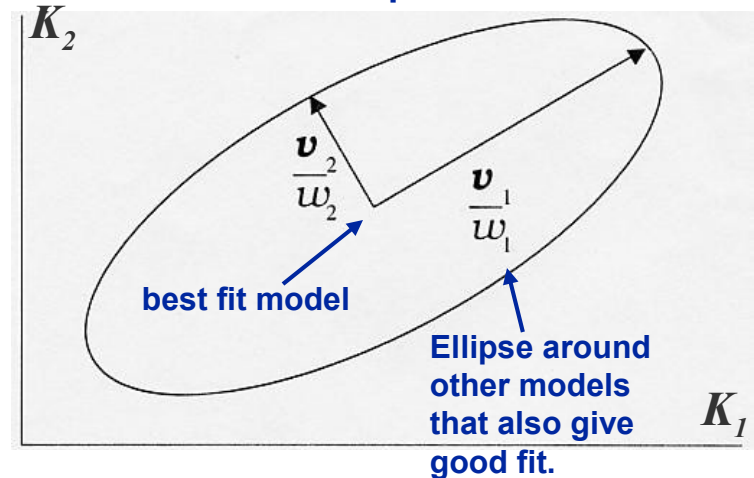
SVD and error bars

$$\frac{\partial \vec{E}}{\partial \vec{K}} = USV^T = \sum \vec{u}_l w_l \vec{v}_l^T$$

A small singular value, w_l, means changes of fit parameters in the direction v_l make very little change in the error vector. The measured data does not constrain the fit parameters well in the direction of v_l; there is relatively large uncertainty in the fit parameters in the direction of v_l. The uncertainty in fit parameter K_l is given by

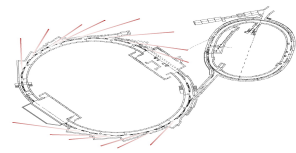$$\sigma^2(K_m) = \sum_l \frac{1}{\sigma_l^2}[\vec{v}_l]_m^2$$

Together the v_l and w_l pairs define an ellipse of variances and covariances in parameter space. LOCO converges to the center of the ellipse. Any model within the ellipse fits the data as well, within the BPM noise error bars.

Illustration for 2 parameter fit:

K_2

$\frac{v_2}{w_2}$

$\frac{v_1}{w_1}$

best fit model

Ellipse around other models that also give good fit.
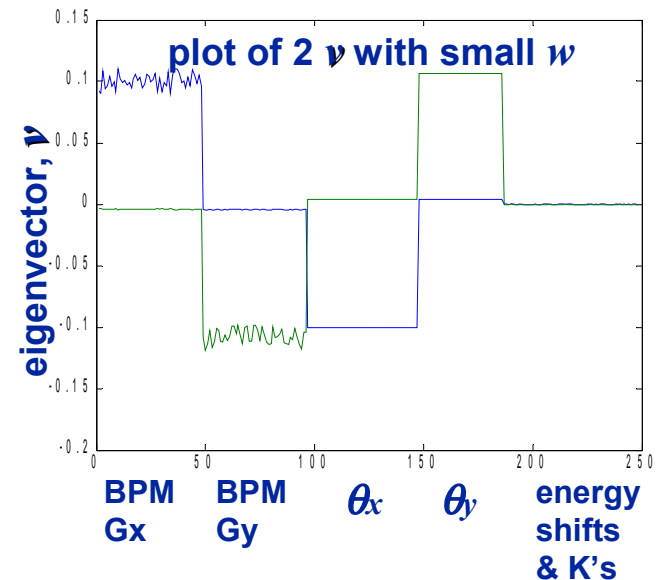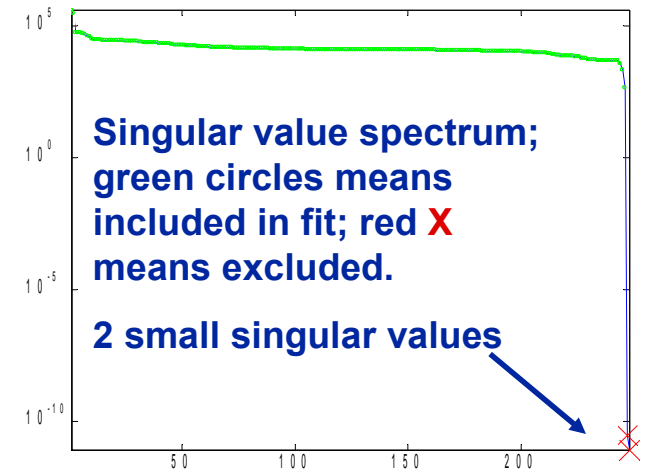
K_1

SVD and error bars, II

Eigenvectors with small singular values indicate a direction in parameter space for which the measured data does not constrain well the fit parameters.
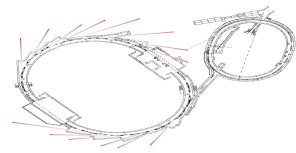
The two small singular values in this example are associated with a degeneracy between fit BPM gains and steering magnet kicks. If all BPM gains are increased and kicks decreased by a single factor, the response matrix does not change.

There two small singular values – horizontal and vertical plane.

This problem can be eliminated by including coupling terms in the fit and including the dispersion as a column of the response matrix (without fitting the rf frequency change).

Singular value spectrum; green circles means included in fit; red X means excluded.

2 small singular values

plot of 2 v with small w

eigenvector, v

BPM Gx   BPM Gy θ_x θ_y energy shifts & K's

SVD and error bars, III

LOCO throws out the small singular values when inverting $\partial \vec{E}/\partial \vec{K}$ **and when calculating error bars. This results in small error bars calculated for BPM gains and steering magnet kicks. The error bars should be interpreted as the error in the relative gain of one BPM compared to the next. The error in absolute gain is much greater.**

If other small singular values arise in a fit, they need to be understood.


vertical BPM number
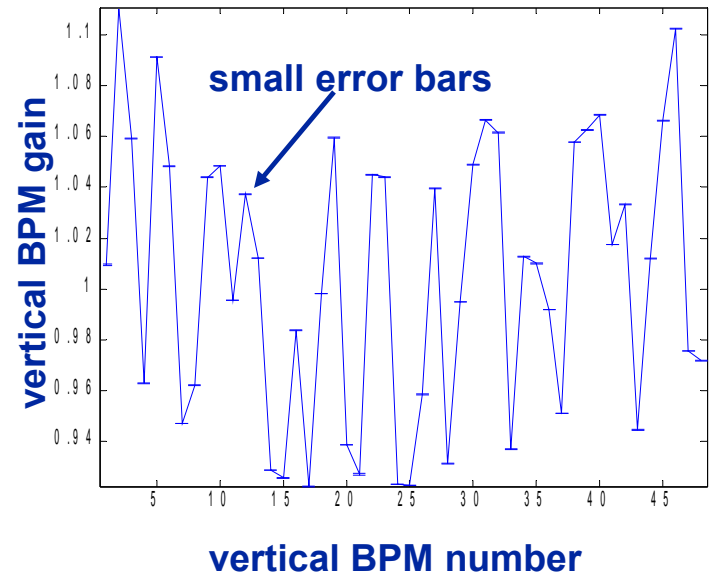
Analyzing multiple data sets
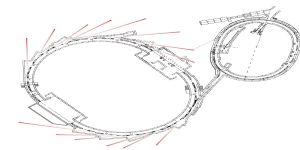
Analyzing multiple data sets provides a second method for investigating the variation in fit parameters from measurement noise. The results shown here are for the NSLS X-Ray ring, and are in agreement with the error bars calculated from analytical propagation of errors.

ERROR BARS ON THE FIT PARAMETERS DUE TO RANDOM ERROR IN THE MEASURED ORBIT.

The variations given in this table are the rms error bars on the fit parameters due to random orbit measurement errors. We measured the response matrix ten times, and fit a model to each response matrix. Then, for each of the parameters we took the average over the ten data sets and calculated the rms variation from the average.

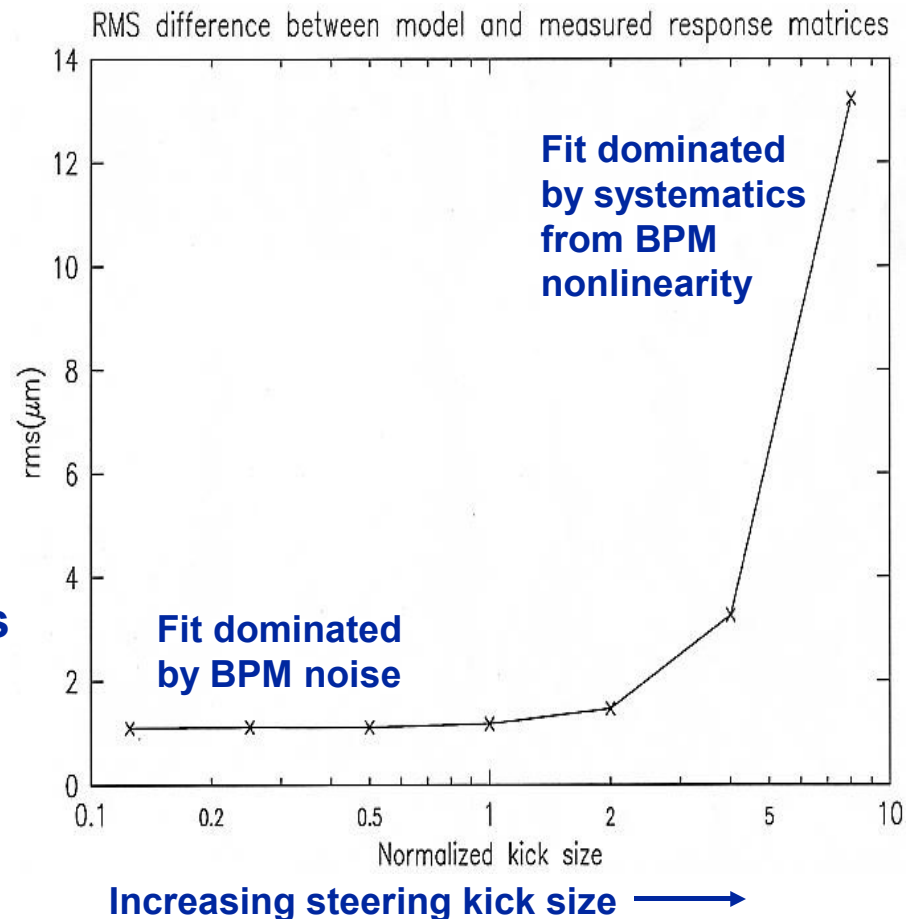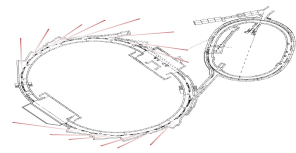| Parameter	rms variation
quadrupole gradients	.04 %
quadrupole rolls	.4 mrad
BPM gain	.05 %
BPM rolls	.5 mrad
BPM C-parameter	.0004
steering magnet calibration	.05 %
steering magnet rolls	.8 mrad
steering magnet longitudinal center	2 mm
steering magnet fractional energy shift	3.4E-7
β functions	.08%

Systematic error
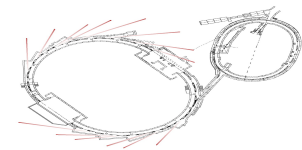
The error in fit parameters from systematic differences between the model and real rings is difficult to quantify.

Typical sources of systematic error are:

• Magnet model limitations – unknown multipoles; end field effects.

• Errors in the longitudinal positions of BPMs and steering magnets.

• Nonlinearities in BPMs.

   • electronic and mechanical

   • avoid by keeping kick size small.

RMS difference between model and measured response matrices

Fit dominated by systematics from BPM nonlinearity

Fit dominated by BPM noise

rms(μm)

Normalized kick size

Increasing steering kick size ⟶

Systematic error, II

With no systematic errors, the fit should converge to

$$\chi^2/\text{D.O.F.} = 1 \pm \sqrt{2/\text{D.O.F.}}$$

$\text{D.O.F.} = N - M = $ degrees of freedom

N **= # of data points**

M **= # of fit parameters**

**This plot shows results with simulated data with** $\chi^2/\text{D.O.F.} = 1.01.$ **With real data the best fit I've had is** $\chi^2/\text{D.O.F.} \approx 1.2$ **fitting NSLS XRay ring data to 1.2 μm for 1.0 μm noise levels. Usually** $\chi^2/\text{D.O.F.}$ **is considerably larger.**

Error vector histogram

Number of points (8640 total)

$$(M^{\text{meas}} - M^{\text{model}})/\sigma_{\text{BPM}}$$

The conclusion: In a system as complicated as an accelerator it is impossible to eliminate systematic errors. The error bars calculated by LOCO are only a lower bound. The real errors include systematics and are unknown. The results are still not useless, but they must be compared to independent measurements for confirmation.
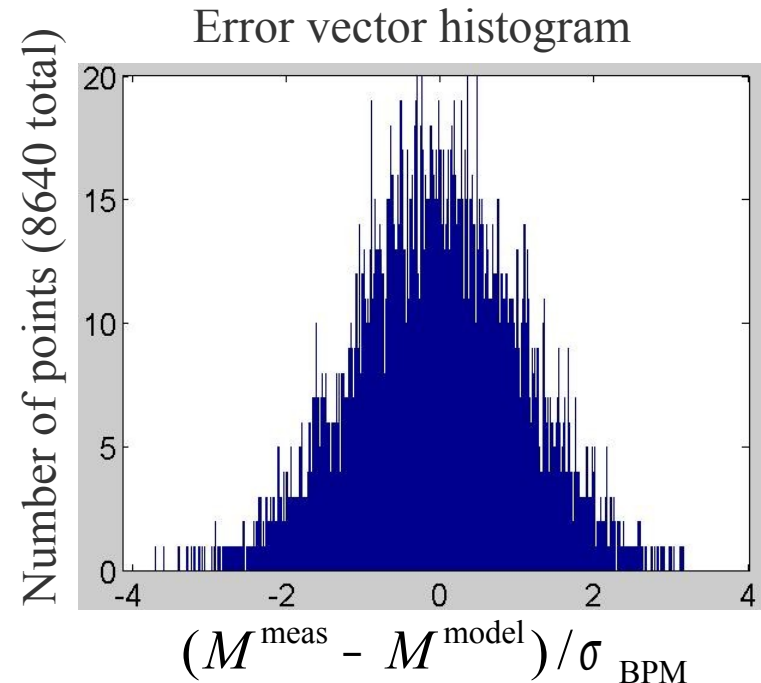
LOCO fit for NSLS X-Ray Ring
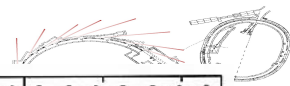
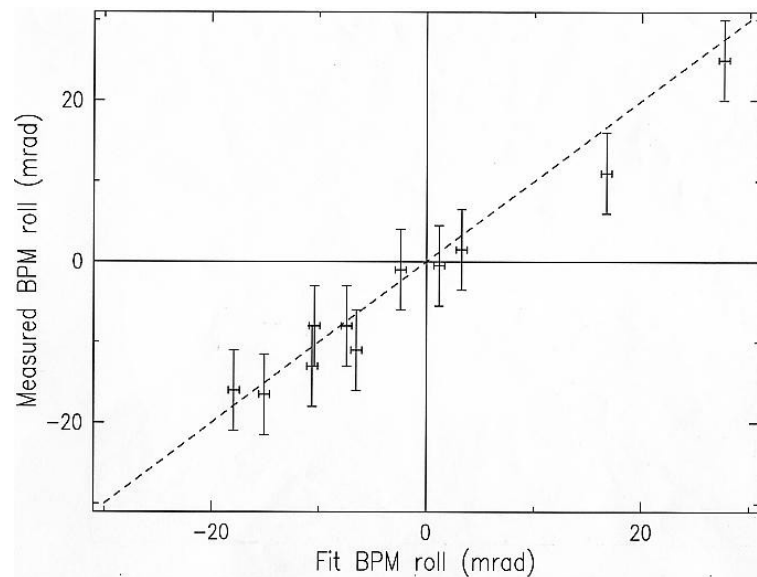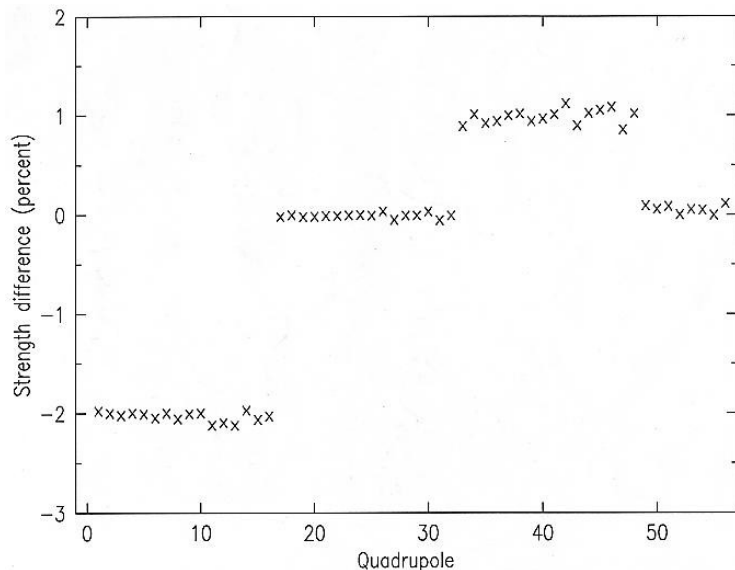**Before fit, measured and model response matrices agree to within ~20%.**

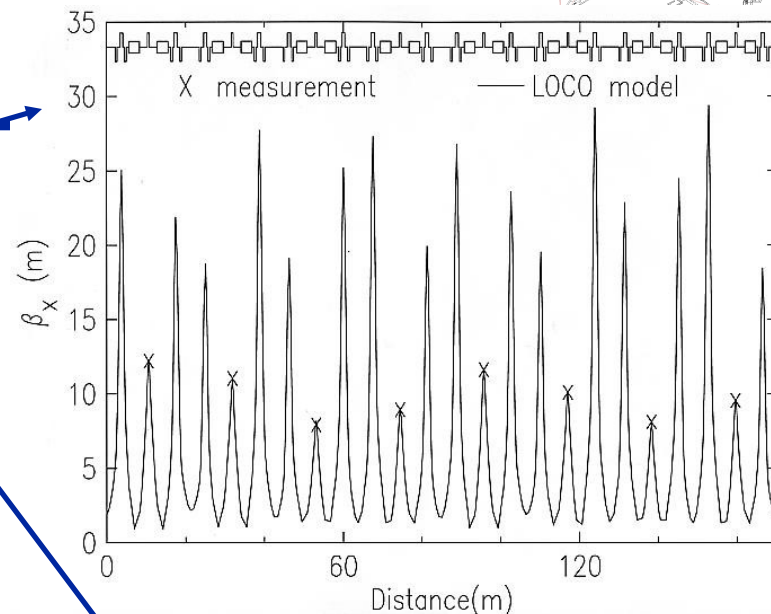$(M_{meas}-M_{model})_{rms}$ = 1.17 μm
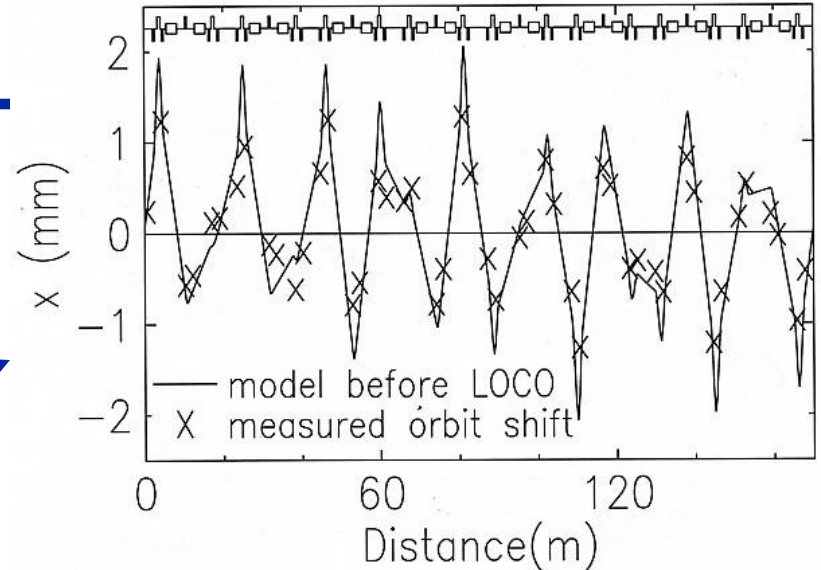
After fit, response matrices agree to 10^{-3}.
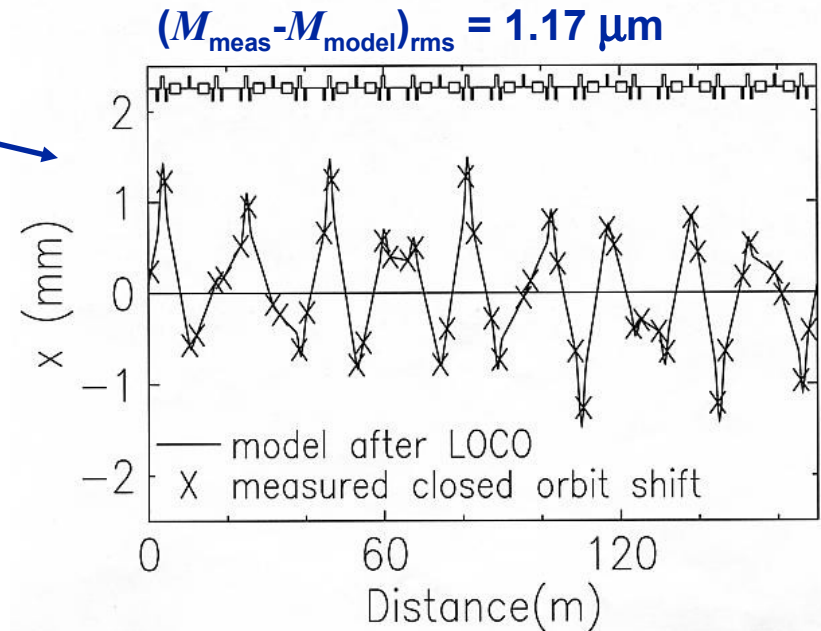
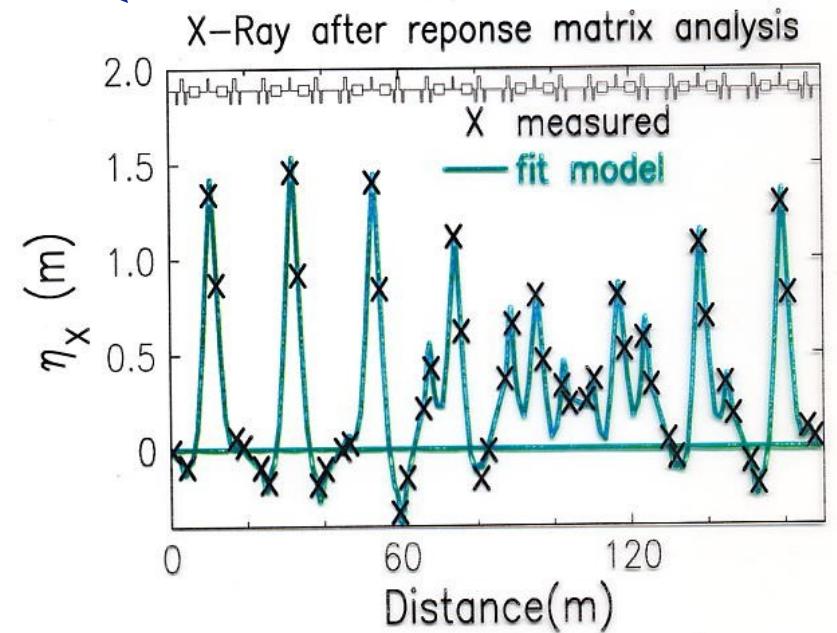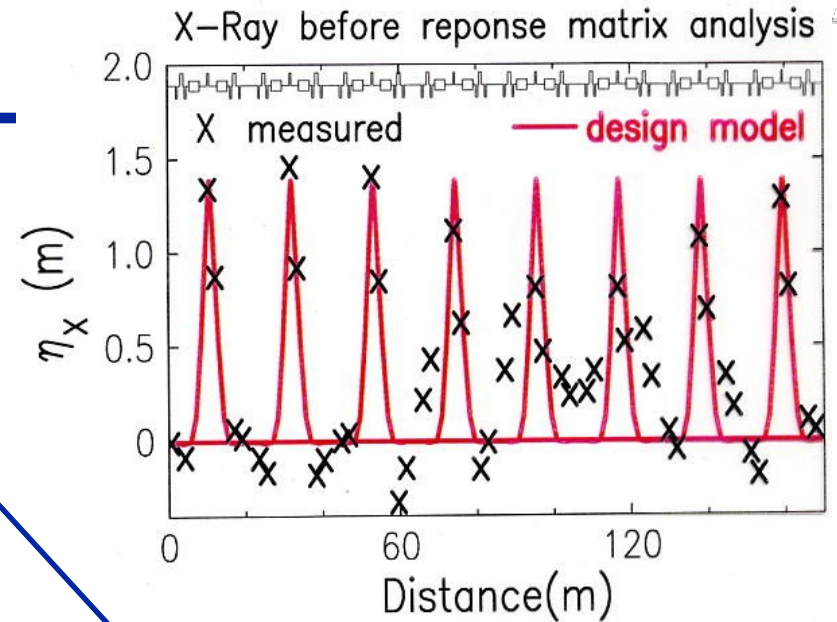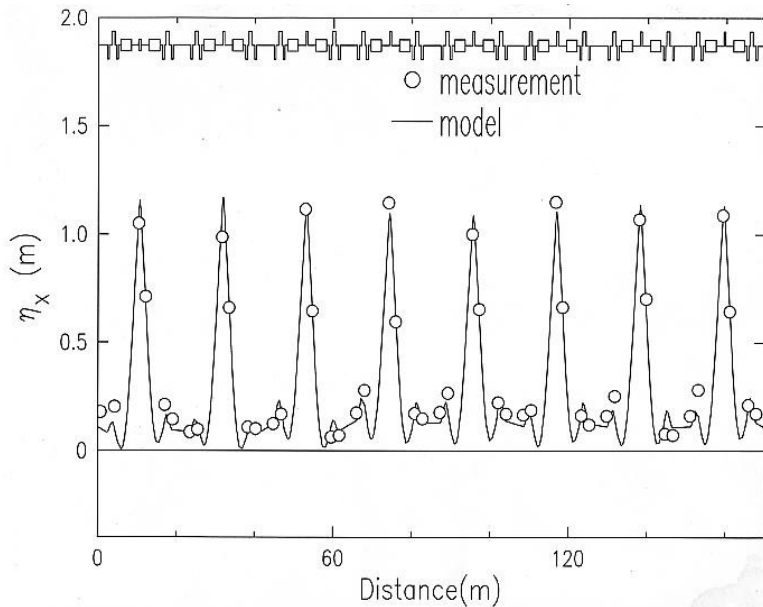# Confirming LOCO fit for X-Ray Ring

LOCO predicts measured β's, BPM roll.

LOCO confirms known quadrupole changes, when response matrices are measured before and after changing optics.
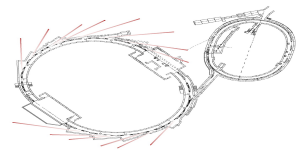
Correcting X-Ray Ring η_x

LOCO predicts measured η_x,

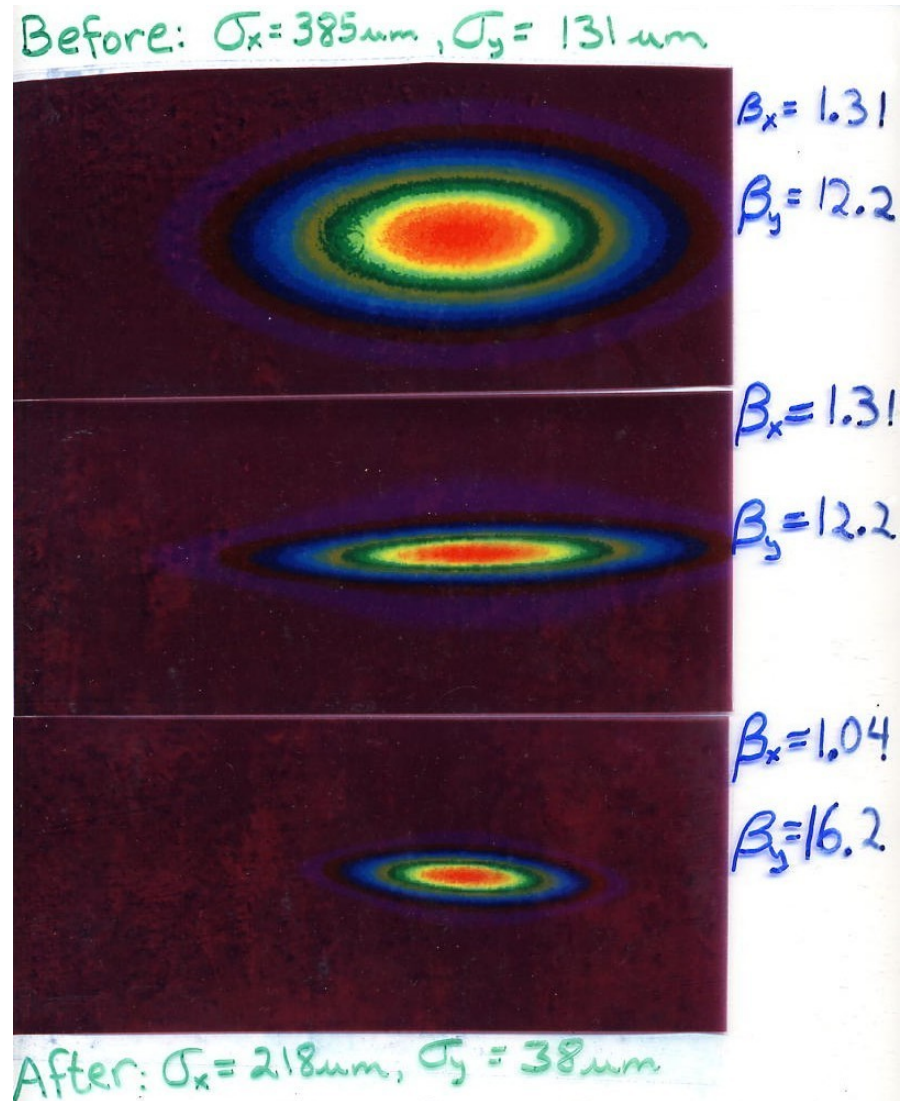**and is used to find gradient changes that best restore design periodicity.**

X-Ray before reponse matrix analysis

X measured — design model

measurement — model

X-Ray after reponse matrix analysis

X measured — fit model

NSLS X-Ray Ring Beamsize

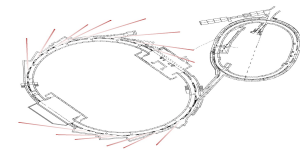The improved optics control in led to reduction in the measured electron beam size.

The fit optics gave a good prediction of the measured emittances. The vertical emittance is with coupling correction off.

	model	measurement
horizontal emittance	93.3 nm*rad	94.2 nm*rad
vertical emittance	6.6 nm*rad	8.6 nm*rad

Before: $\sigma_x = 385 \mu m$, $\sigma_y = 131 \mu m$

$\beta_x = 1.31$
$\beta_y = 12.2$

$\beta_x = 1.31$
$\beta_y = 12.2$

$\beta_x = 1.04$
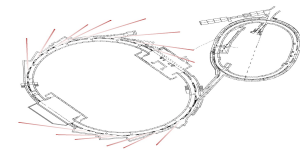$\beta_y = 16.2$

After: $\sigma_x = 218 \mu m$, $\sigma_y = 38 \mu m$

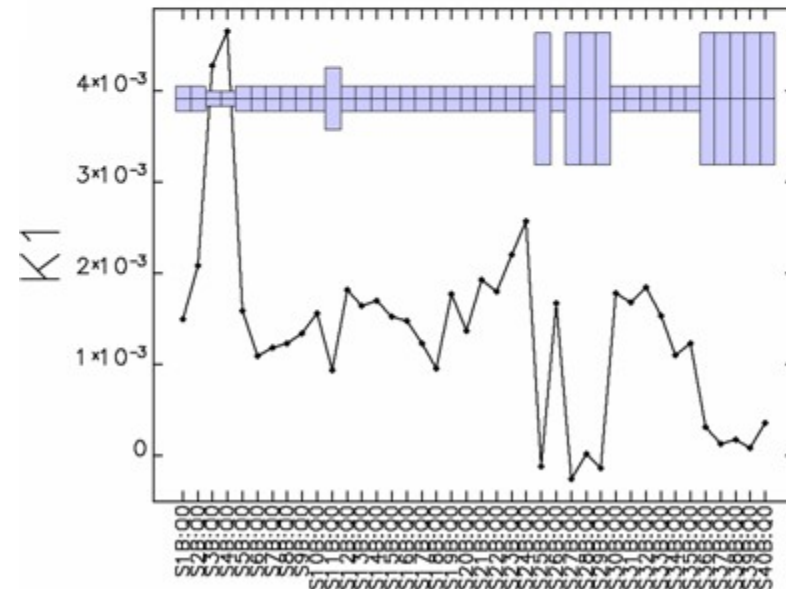**Nonlinear ξ:**

(ν_x, ν_y) vs. f_{rf} agrees with model.
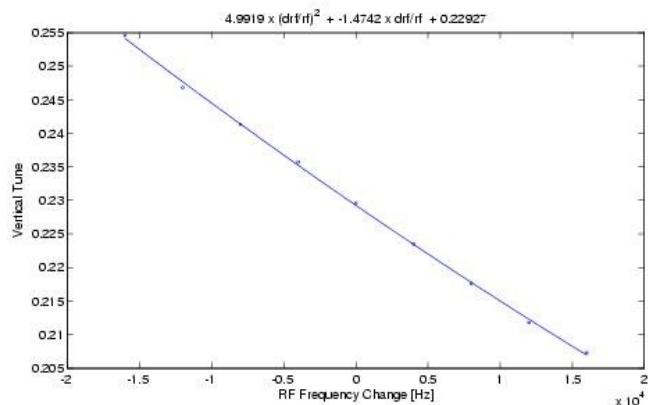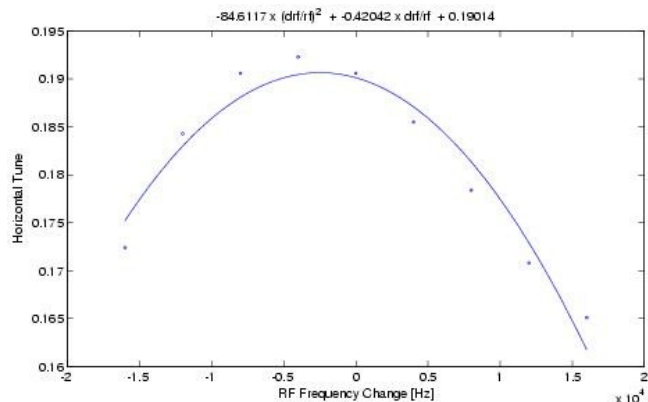
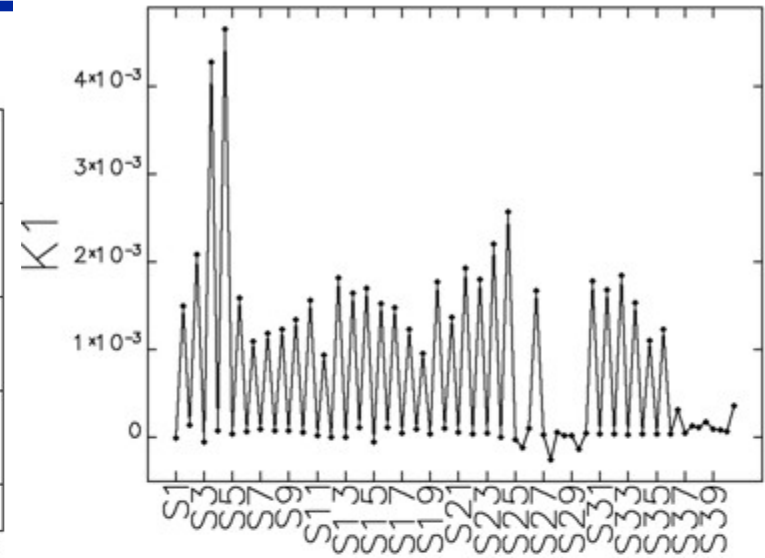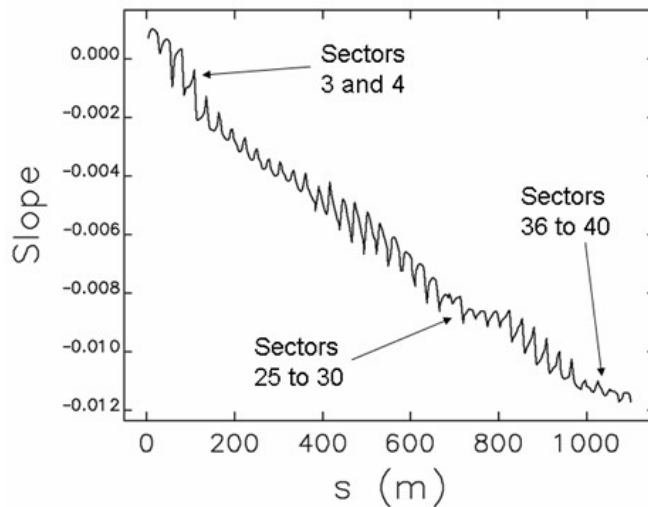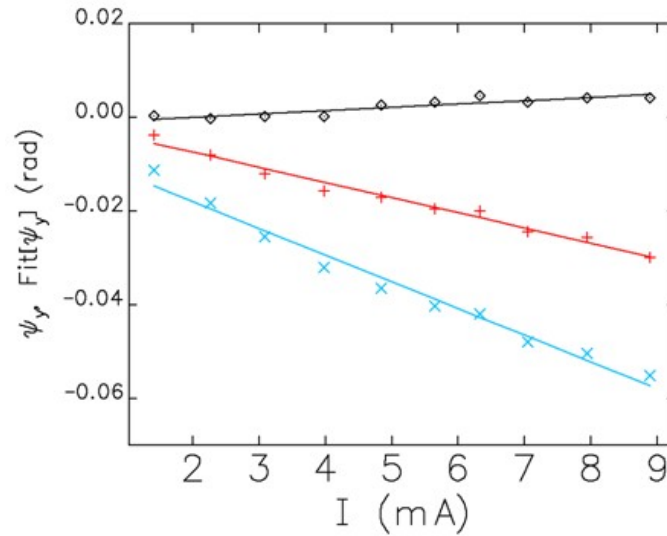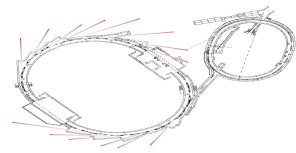**Local chromaticity calibrated with LOCO shows no sextupole errors:**
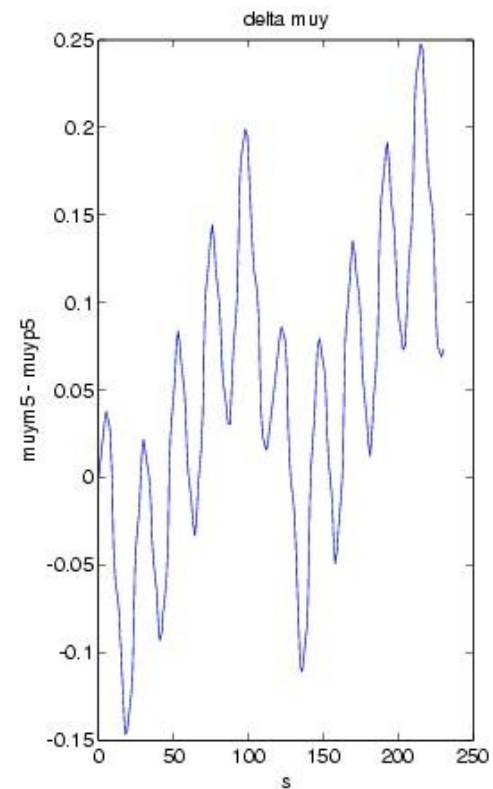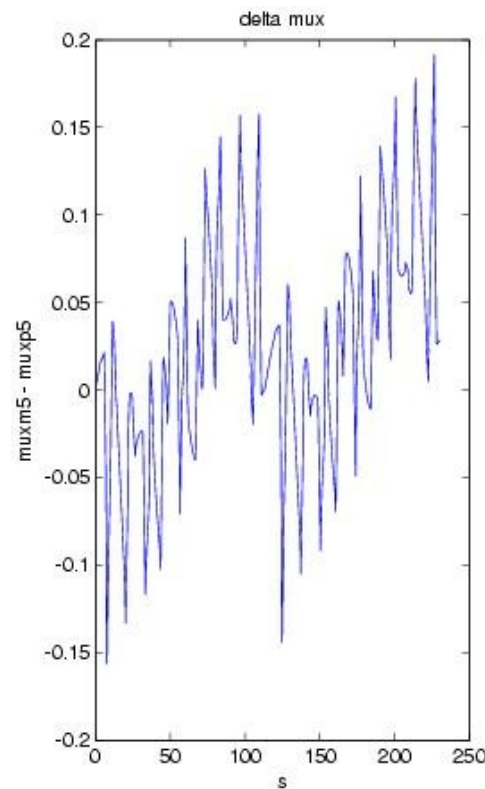
Transverse impedance at APS (V. Sajaev)

$$Z_{eff}^i = \frac{1}{2} \frac{E\sigma_s}{R} \frac{\beta_q}{\beta_i} \frac{d(K_1 L)}{dI}$$

○ **Constrained fitting to limit quadrupole gradient changes in fit.**

↳ **Sometimes data is insufficient to fully constrain fit gradients**

↳ **Reducing # of singular values gave unreliable results**

↳ **Instead, add constraints to restrict magnitude of gradient changes:**

$$A \begin{bmatrix} \Delta K_1 \\ \Delta K_2 \\ \vdots \end{bmatrix} = R^{\mathrm{meas}} - R^{\mathrm{mod}} \qquad \longrightarrow \qquad \begin{bmatrix} A \\ ----- \\ w_1\,0\,0\cdots \\ 0\,w_2\,0\cdots \\ \vdots \end{bmatrix} \begin{bmatrix} \Delta K_1 \\ \Delta K_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} R^{\mathrm{meas}} - R^{\mathrm{mod}} \\ ----- \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

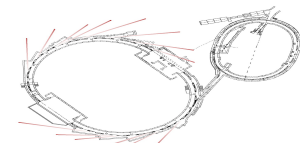○ **Levenberg-Marquardt fitting algorithm**

↳ **More robust convergence for LOCO fitting**

$$A\Delta K = R^{\mathrm{meas}} - R^{\mathrm{mod}} \qquad \longrightarrow \qquad (A'A + \lambda\,\mathrm{diag}(\mathrm{diag}(A'A)))\Delta K = A'(R^{\mathrm{meas}} - R^{\mathrm{mod}})$$
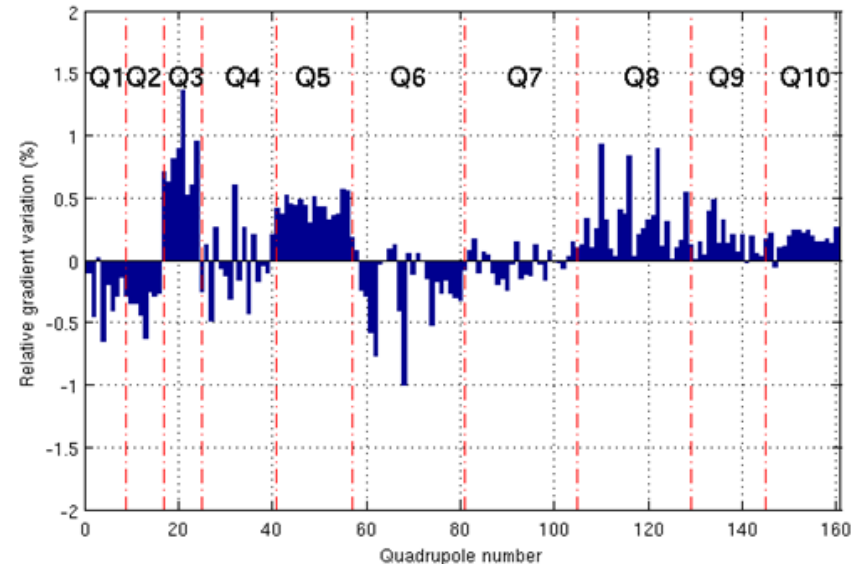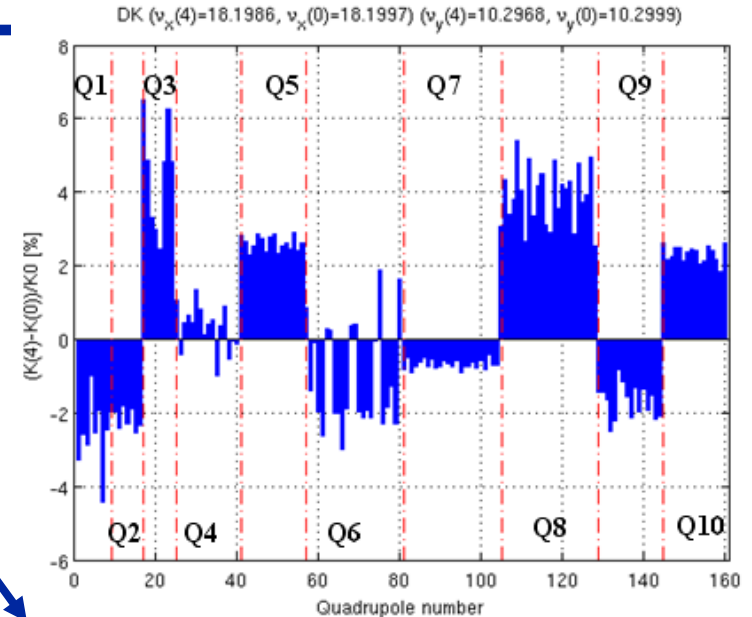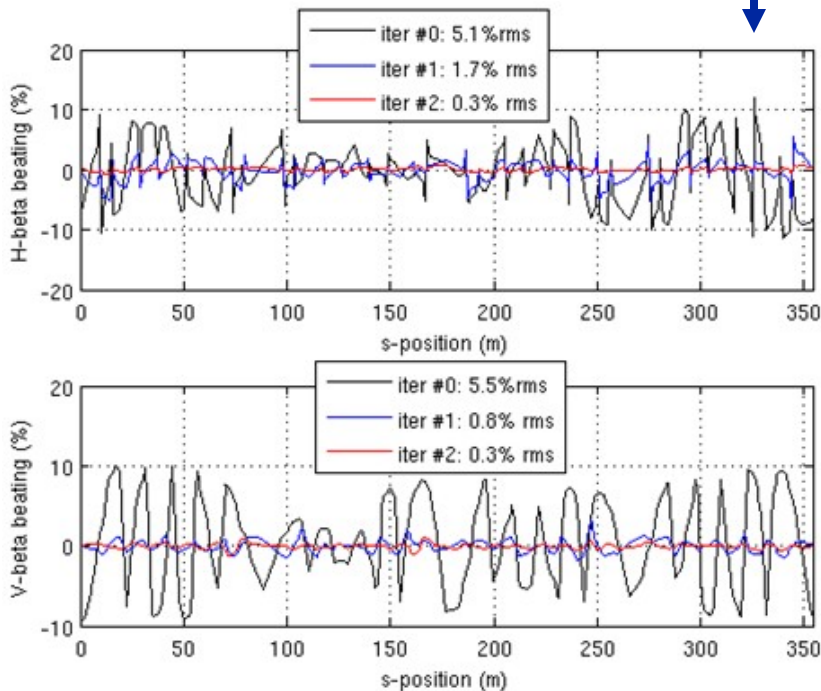
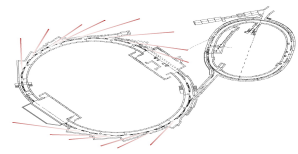**Gauss-Newton**   **Gauss-Newton/steepest-descent, depending on λ**

Recent results from SOLEIL

- ○ **Fit gradients w/out constraints (>6% corrections)**

- ○ **Fit gradients with contraints (<1.5% corrections)**

- ○ **Successfully corrected optics errors**

DK ($\nu_x(4)$=18.1986, $\nu_x(0)$=18.1997) ($\nu_y(4)$=10.2968, $\nu_y(0)$=10.2999)

Further reading

See December, 2007 ICFA Beam Dynamics Newsletter.

Numerical Recipes, Cambridge University Press, is an excellent reference for SVD, χ^2 model fitting, and error bars, as well many other numerical techniques for analyzing data.

J. Safranek, "Experimental determination of storage ring optics using orbit response measurements", Nucl. Inst. and Meth. A388, (1997), pg. 27.

D. Robin, J. Safranek, W. Decking, "Realizing the benefits of restored periodicity in the advanced light source", Phys. Rev. Special Topics-AB, v. 2 (1999).

Search http://accelconf.web.cern.ch/AccelConf/ "Text of paper" for LOCO.

The LOCO code is available at http://als.lbl.gov/als_physics/portmann/MiddleLayer/applications

LOCO uses Andrei Terebilo's AT accelerator modeling code to calculate response matrices. AT is available at http://ssrl.slac.stanford.edu/at/

The idea for LOCO came from previous work:

W.J. Corbett, M.J. Lee, and V. Ziemann, "A fast model calibration procedure for storage rings," SLAC-PUB-6111, May, 1993.