Determination of the Linear Lattice through Analysis of turn-by-turn orbit data

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- Outline:
  - Motivation
  - Different Phase Advance Measurement Techniques:
    - Type of excitation
    - Method of data recording
    - Method of data Analysis
  - Examples of Applications

Motivation

Motivation for phase advance analysis is the same as for other methods we mentioned (direct measurement of beta functions, orbit response matrix analysis, MIA, PCA, ICA, resonance driving terms …)

Desire to understand and control the linear lattice

- **Beamsize and divergence**
- **Nonlinear dynamics is determined by the linear lattice functions and the sextupoles**

- **Phase advance measurement can be performed quickly**

Reminder: Beta Functions / Phase Advance

The solution can be parameterized by a pseudo-harmonic oscillation of the form

$$x_\beta(s) = \sqrt{\varepsilon}\sqrt{\beta(s)}\cos(\varphi(s)+\varphi_0)$$

$$x'_\beta(s) = -\sqrt{\varepsilon}\frac{\alpha}{\sqrt{\beta(s)}}\cos(\varphi(s)+\varphi_0) - \frac{\sqrt{\varepsilon}}{\sqrt{\beta(s)}}\sin(\varphi(s)+\varphi_0)$$

where $\beta(s)$ is the beta function,

$\alpha(s)$ is the alpha function,

$\varphi_{x,y}(s)$ is the betatron phase, and

ε is an action variable

$$\varphi = \int_0^s \frac{ds}{\beta}$$

Measurement Techniques

- Vary quadrupole strengths and look at tune-changes – (James's basic measurements talk)
- Fit orbit response matrix data: LOCO (James' talk)
- Ping the beam and analyze turn-by-turn data: phase advance, MIA, … (this talk, tomorrow, Thursday)
- Resonantly excite the beam and look at turn-by-turn data
  - Mostly going to show Cornell example

- Turn-by-turn data also contains coupling information – will mention this tomorrow

Direct beta function measurement

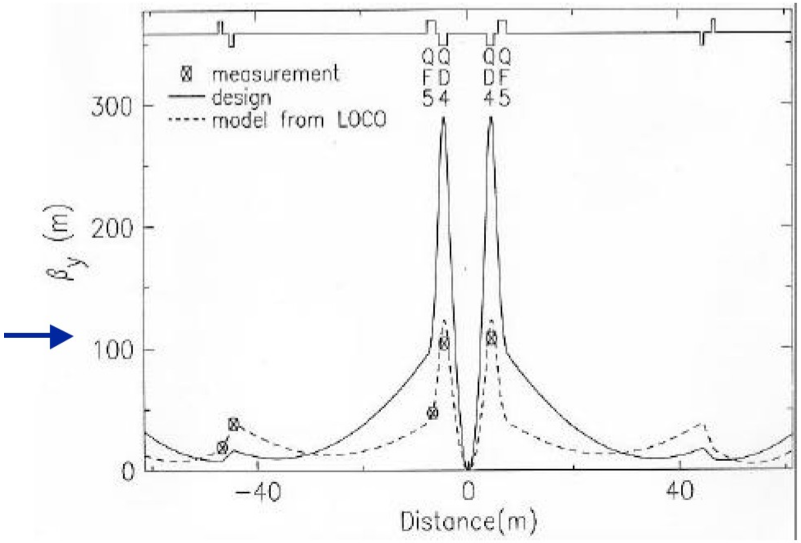
Vary quadrupole strengths and look at tune-changes

β is computed via

$$\delta\nu_{x,y} = \frac{\beta_{h,v}}{4\pi}\Delta kl$$

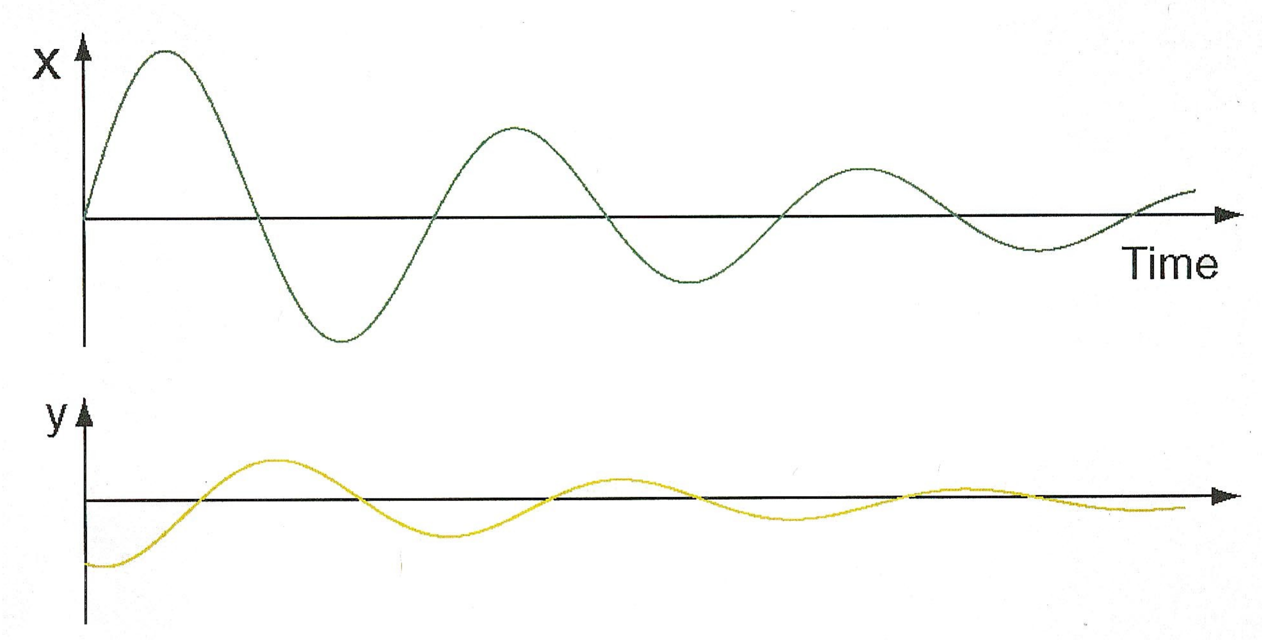
Disadvantages

- Hysteresis – accuracy
- Slow
- Limited information



Ping and analyze turn-by-turn data

Ping the beam and record turn-by-turn orbit data



Advantages

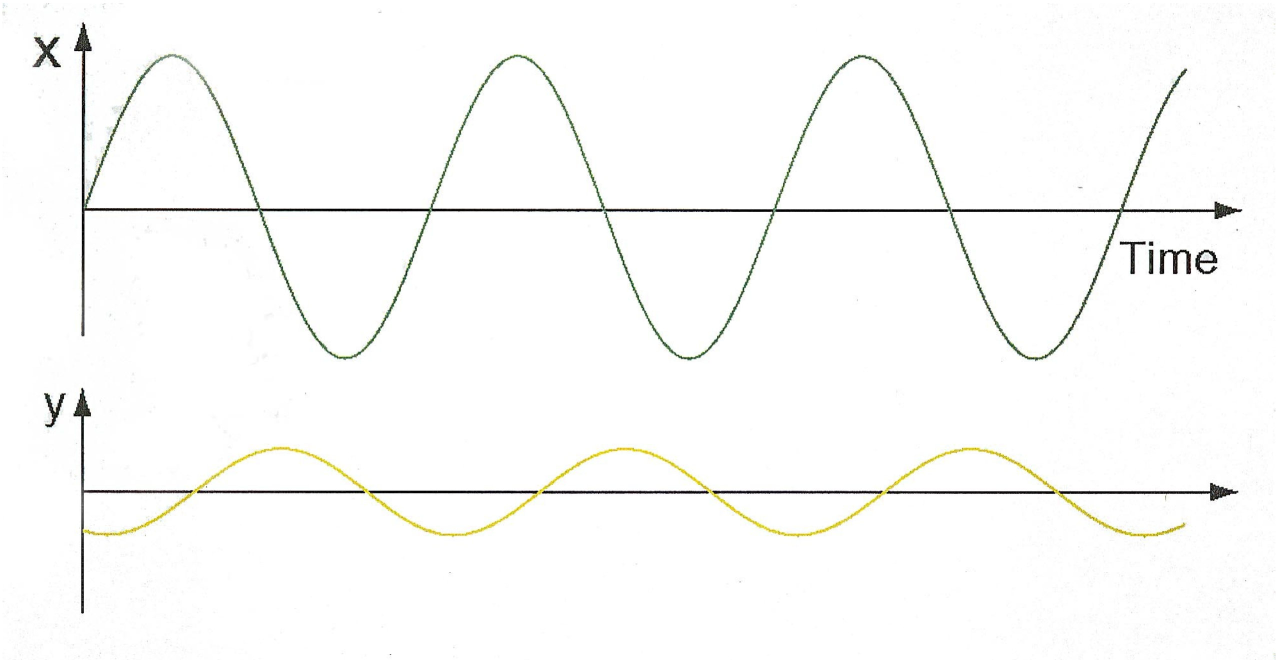
Fast

Disadvantages

Decoherence

Resonant excitation

Shake the beam at a betatron sideband and observe the beam motion (turn-by-turn) at the BPMs



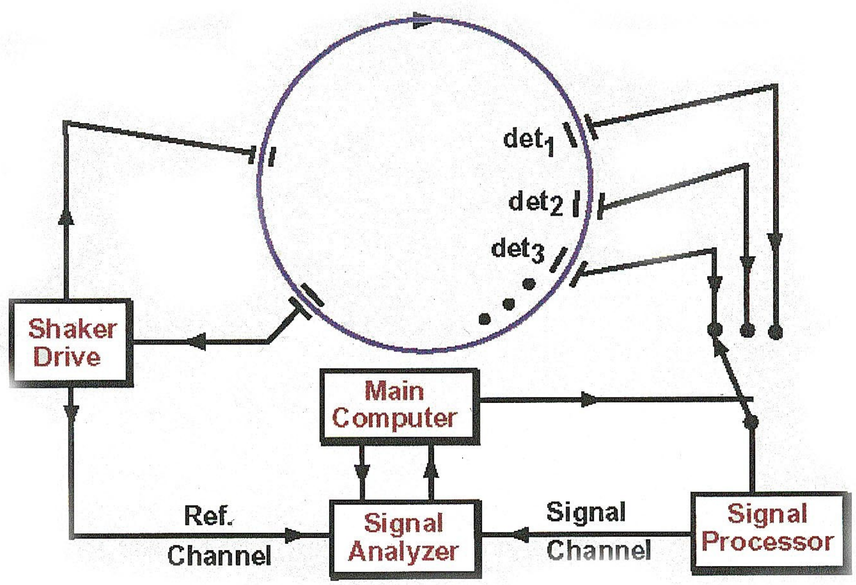
Advantages

- Fast
- Not limited by damping and decoherence

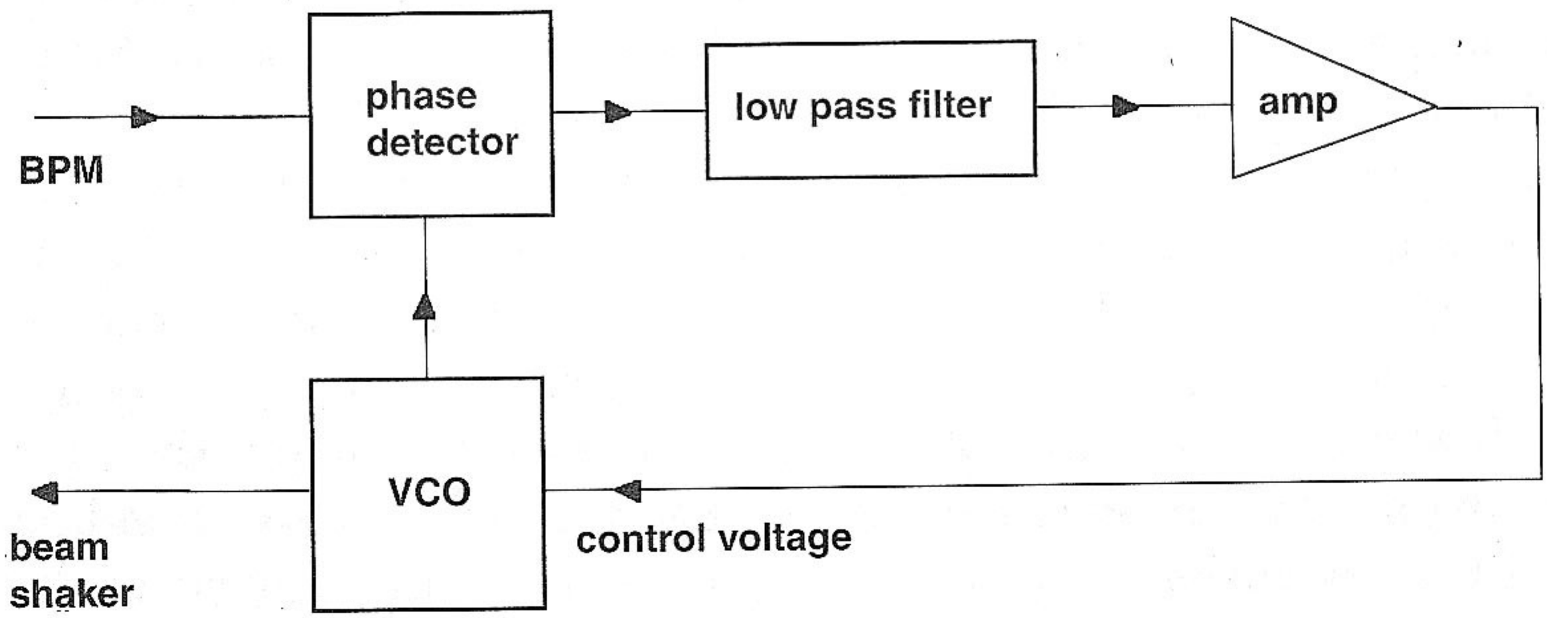
Resonant excitation

Cornell system:

- shaker is phased locked to beam
- shake beam horizontally and vertically
- analyze the signals from the BPMs sequentially



Phase locked loop



Phase detector compares the frequency of beam signal and local oscillator, computes the frequency difference and adjusts the oscillator

Determination of the Tunes

- Input signal is digitized
- Take N consecutive turns (say 1024, 16000, …)
- Compute frequency using fast Fourier transform and interpolation

Determination of the Tunes

Input: Turn-by-turn measured orbit data.

Analysis: Fourier transform of the turn-by-turn orbit data to compute the frequency, ν

$$x(n) = \sum_{j=1}^{N} \psi(\nu_i) \exp(2\pi i n \nu_i)$$

$$\psi(\nu_i) = \frac{1}{N} \sum_{n=1}^{N} x(n) \exp(-2\pi i n \nu_i)$$

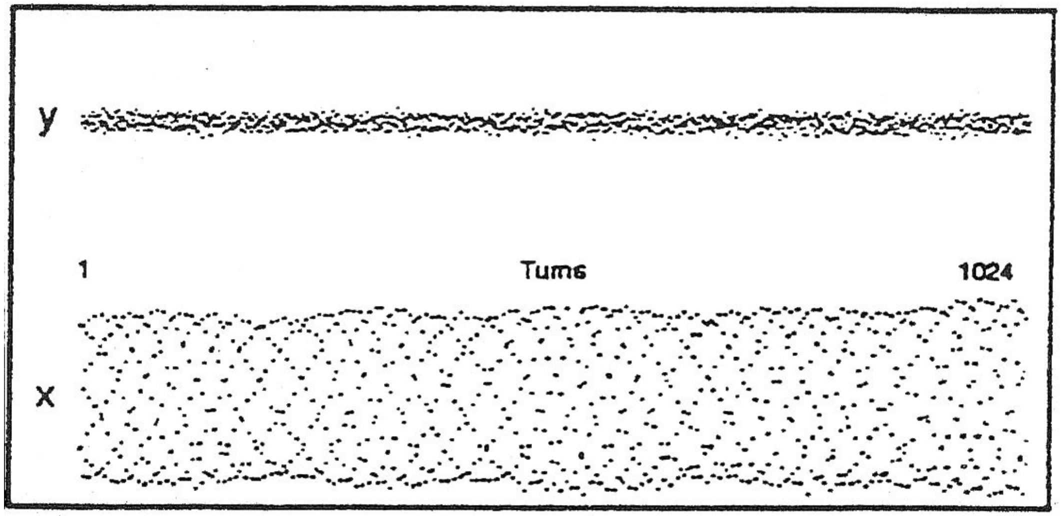


Figure 1: Single BPM recording the excited horizontal beam motion (scale: 8 mm peak to peak, time=88.9 μsec/turn)

Fast Fourier transform

The frequency corresponding to the largest value of ψ is taken as the approximate tune → $|\delta\nu|<1/2N$

The resolution can be improved by an interpolated FFT. If one assumes that the shape of the Fourier spectrum is known and corresponds to that of a pure sinusoidal oscillation with tune, ν_{int}

$$\nu_{int} = \frac{1}{N}\left[k - 1 + \frac{A(k)}{A(k-1) + A(k)}\right], k - 1 \le N\nu \le k$$

with a sin window

$$y_k = x_k \sin\left(\frac{\pi k}{N}\right), k = 0,1,2,\ldots,N-1$$

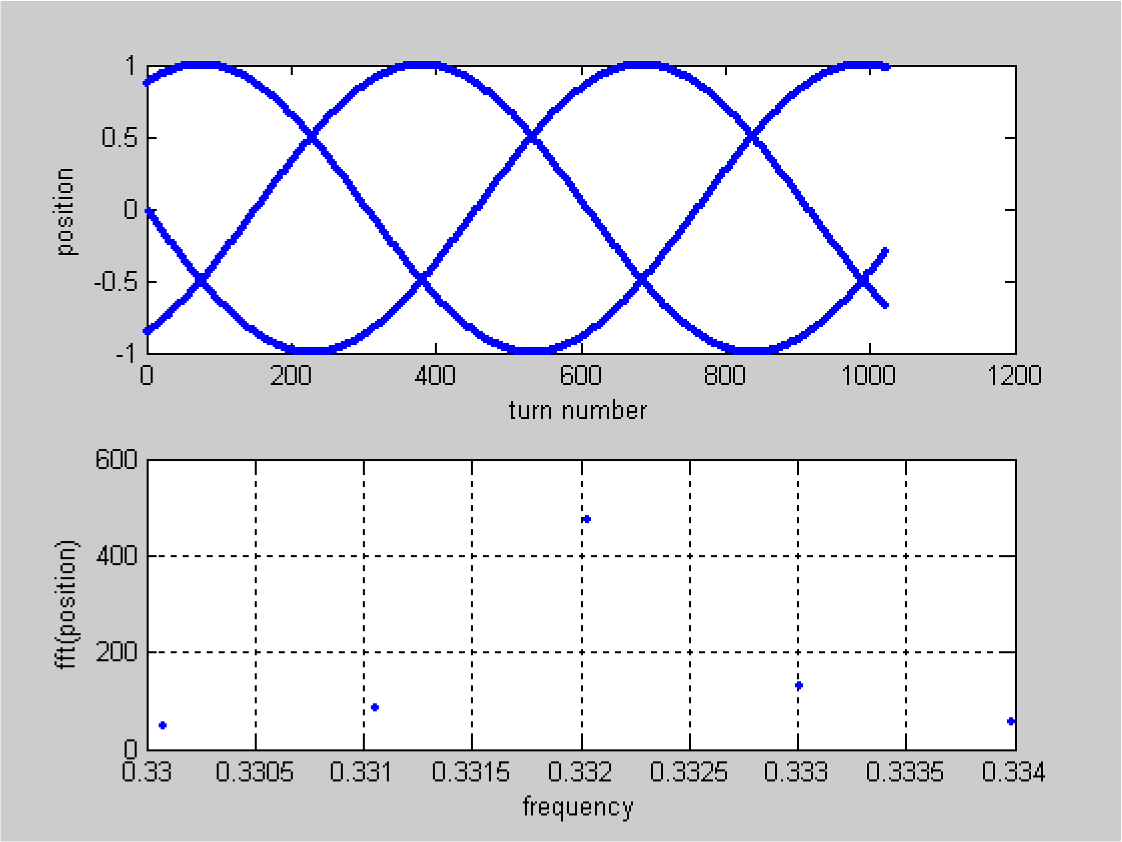
$$\nu_{int} = \frac{1}{N}\left[k - 1 + \frac{2A(k)}{A(k-1) + A(k)} - \frac{1}{2}\right]$$

(Asseo CERN PS Note 87-1 (1987))

Improving the resolution

Example: tune = 0.33224

x(i) = sin(2π(0.33224)i)



Straight fft

ν = 0.3320

With interpolation

ν = 0.332239998

Determination of the phases

One method (Castro et. al. PAC 1993)

Define two functions C and S using the turn-by-turn data x and analyzed frequency ν.

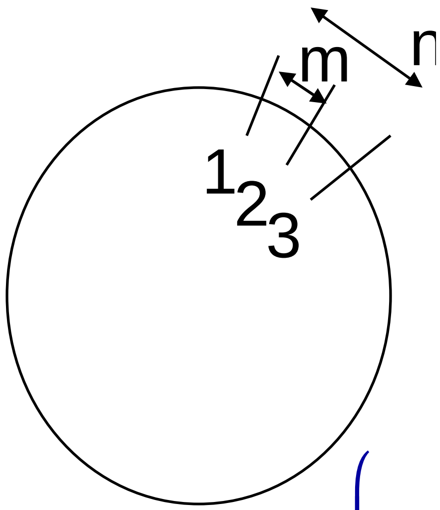
$$C = \sum_{i=1}^{N} x_i \cos(2\pi i\nu) \quad \text{and} \quad S = \sum_{i=1}^{N} x_i \sin(2\pi i\nu)$$

Then the amplitude, *A*, and phase μ are

$$A = \frac{2\sqrt{C^2 + S^2}}{N} \quad \text{and} \quad \mu = -\cot\left(\frac{S}{C}\right)$$

Amplitude is not as reliable as the phase

Determination of the β-functions – Method 1



(Castro et. al. PAC 1993)

$$\begin{pmatrix} x \\ x' \end{pmatrix}_2 = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_1 \quad , \quad \begin{pmatrix} x \\ x' \end{pmatrix}_3 = \begin{pmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}_1$$

$$R_{fi} = \begin{pmatrix} \sqrt{\frac{\beta_f}{\beta_i}}\left(\cos\varphi_{fi} + \alpha_i \sin\varphi_{fi}\right) & \sqrt{\beta_f \beta_i}\sin\varphi_{fi} \\ -\frac{1+\alpha_i\alpha_f}{\sqrt{\beta_f\beta_i}}\sin\varphi_{fi} + \frac{\alpha_i - \alpha_f}{\sqrt{\beta_f\beta_i}}\cos\varphi_{fi} & \sqrt{\frac{\beta_i}{\beta_f}}\left(\cos\varphi_{fi} - \alpha_f \sin\varphi_{fi}\right) \end{pmatrix}$$

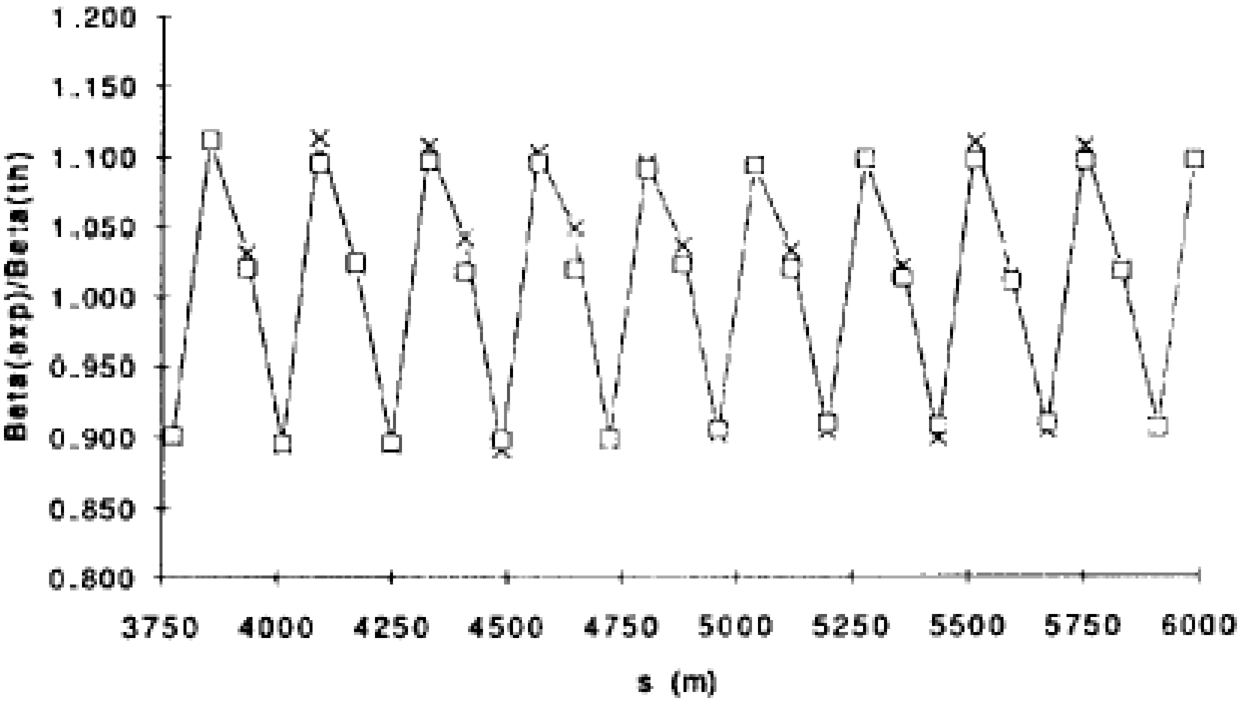
Using the ideal values for the machine and the measured phases

$$\beta_1^* = \beta_1 \frac{\left(\cot\psi_{12}^* - \cot\psi_{13}^*\right)}{\left(\cot\psi_{12} - \cot\psi_{13}\right)} \quad \text{and} \quad \alpha_1^* = \alpha_1 \frac{\left(\cot\psi_{12}^* - \cot\psi_{13}^*\right) + \cot\psi_{12}^*\cot\psi_{13} - \cot\psi_{12}\cot\psi_{13}^*}{\left(\cot\psi_{12} - \cot\psi_{13}\right)}$$

Quantities with * are measured, those without are ideal

Beta beating at LEP

(Castro et. al. PAC 1993)



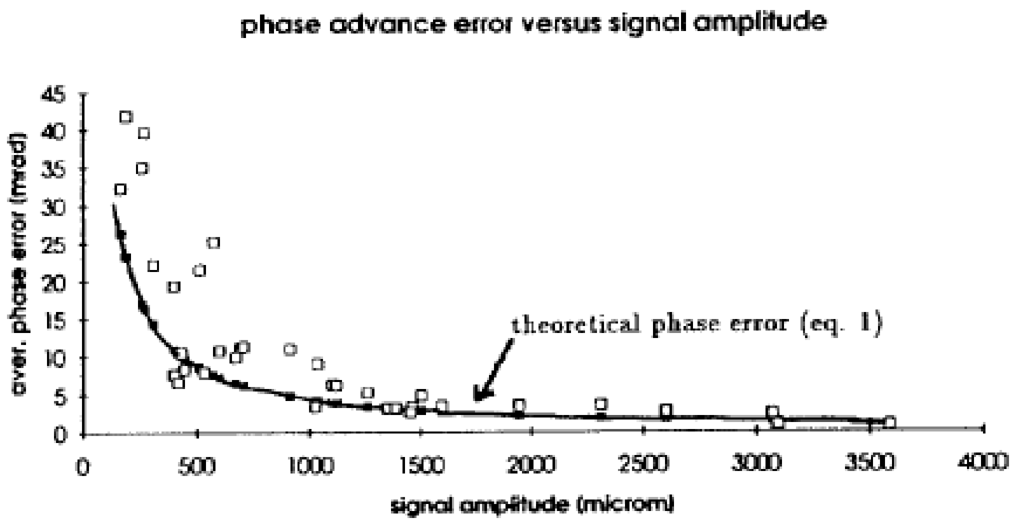
Error in the determination

Uncertainty in the phase

First there is noise of the BPMs, σ_x

$$\sigma_\mu = \frac{1}{A}\sqrt{\frac{2}{N}}\sigma_x$$

Th **en**



Phase Determination 2 (Cornell)

- Use digitized position signal, then

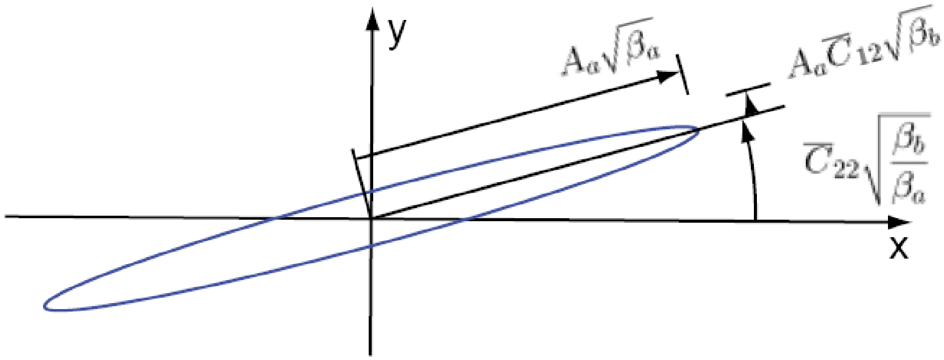
  The phase of the reference signal at turn n is used to construct sine cosine references

$$R_{\sin}(n) = \sin \phi_{ref}(n)$$
$$R_{\cos}(n) = \cos \phi_{ref}(n)$$

- Convolute the beam signal with these (digital lock-in) and integrate

  Results are used to solve for the lattice functions

$$\begin{aligned} x &= A_a \sqrt{\beta_a} \cos(n\omega_a + \phi_a), \\ y &= -A_a \sqrt{\beta_b}(\overline{C}_{22} \cos(n\omega_a + \phi_a) + \overline{C}_{12} \sin(n\omega_a + \phi_a)). \end{aligned}$$



In practice assume $\beta = \beta(\text{design})$ and solve for ϕ and $\overline{C}_{ij}$.

Determination of the β-functions – Method 2

Sagan et. al. PRST 2000

General relationship between beta and phase

$$\frac{1}{\beta} = \frac{d\phi}{ds}$$

The relative error in the beta function can be calculated from

$$\frac{\delta\beta}{\beta_{\mathrm{design}}} = \frac{d(\delta\phi)}{d\phi_{\mathrm{design}}}$$

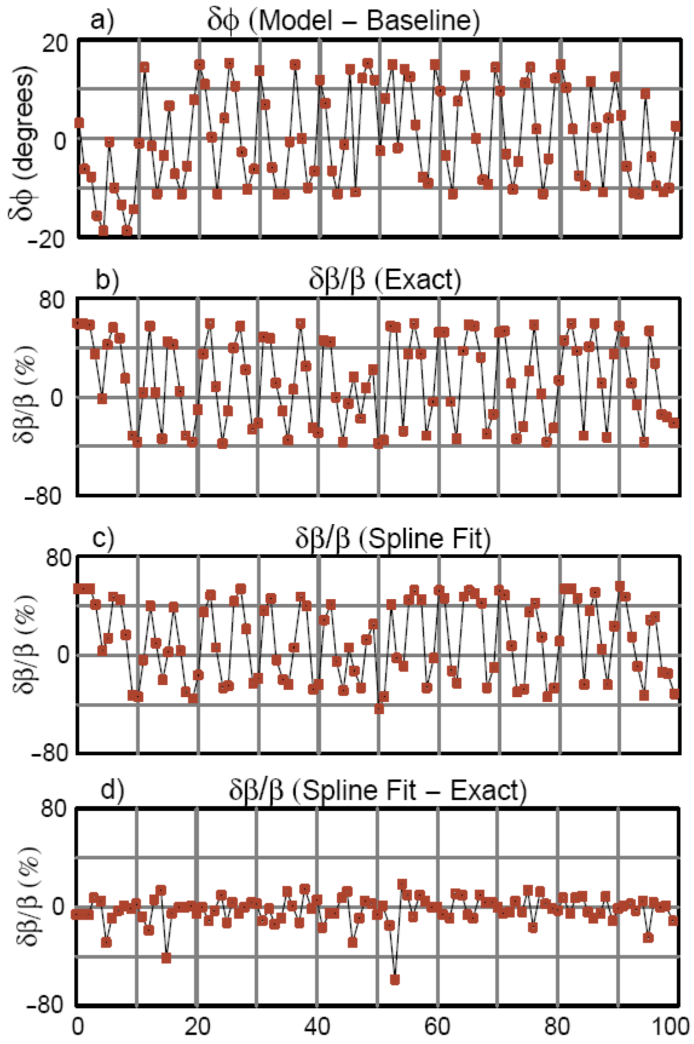
$$\delta\beta \equiv \beta_{\mathrm{meas}} - \beta_{\mathrm{design}}$$

$$\delta\phi \equiv \phi_{\mathrm{meas}} - \phi_{\mathrm{design}}$$

Determination of the β-functions – Method 2

Sagan et. al. PRST 2000

To do the differentiation, one can use a spline fit to interpolate between measured datapoints

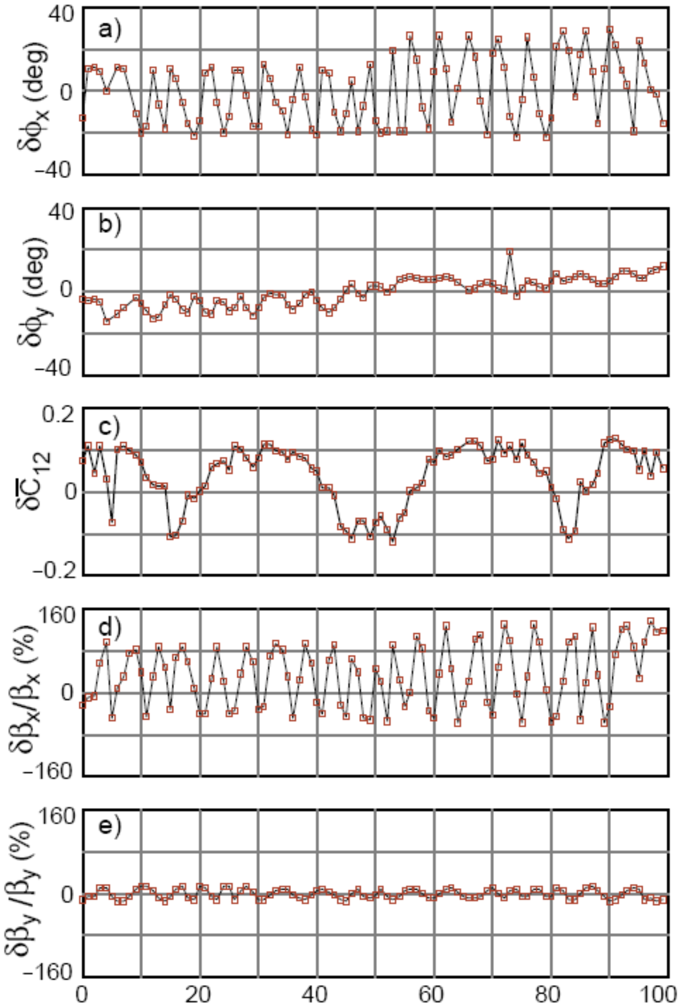


Correction

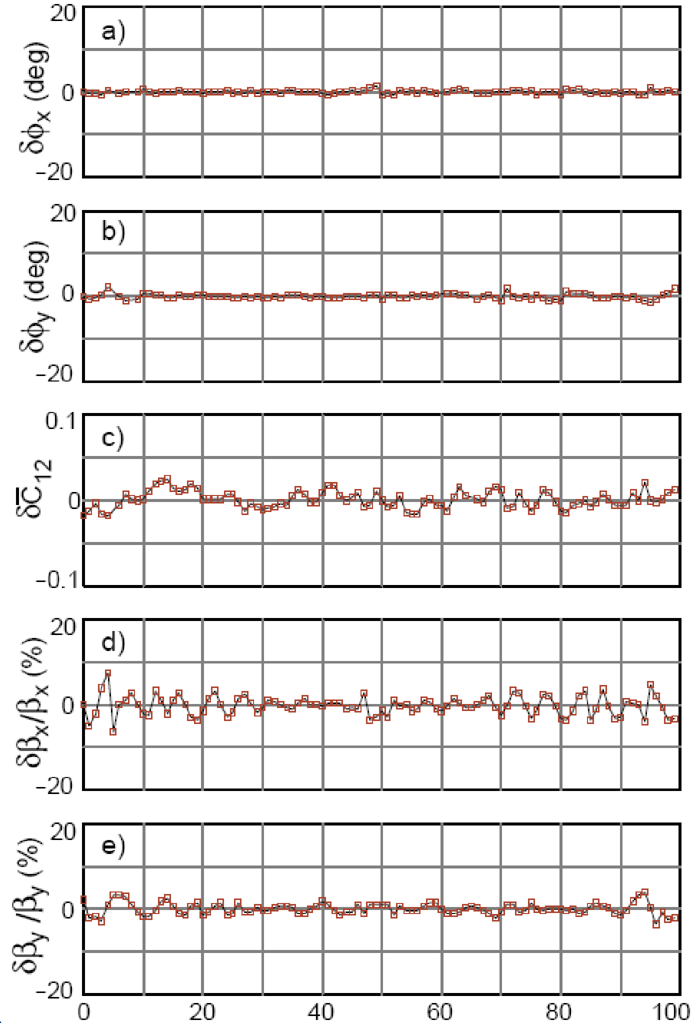
- In order to correct the lattice error one can now compare the measured phase advance (or the derived beta function beating) with a lattice simulation code
- Use any multiparameter fit to vary quadrupole strengths (or potentially skew quads, …) to make model agree with measurement
- One important optimization is to select the best set of fitparameters (plus need enough adjustable quadrupoles, or orbit offsets in sextupoles, …) to implement correction
- Corrections are made using:
  - Quadrupole strengths (in CESR all quadrupoles have independent power supplies)
  - Interaction Region Quadrupole rotation angles

Correction of the beta beating – Method 2

Before



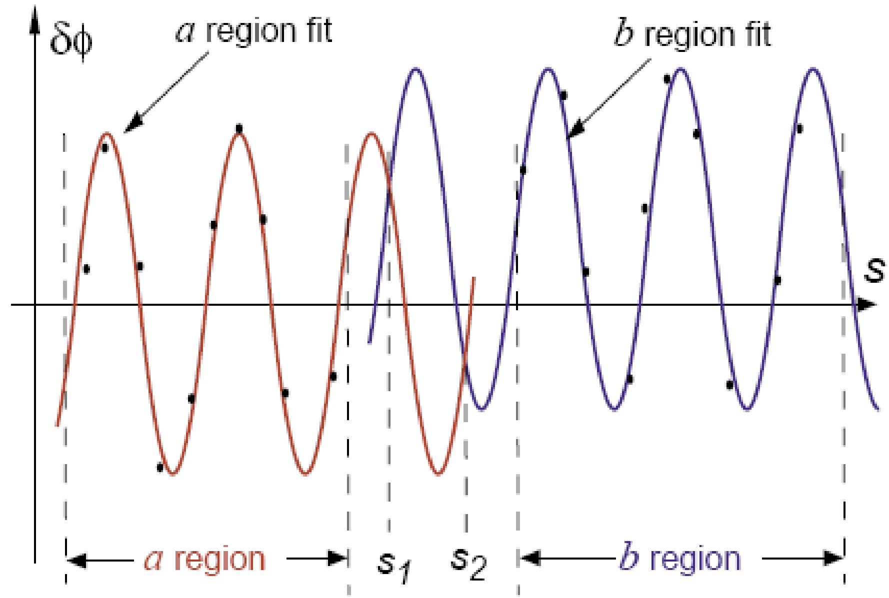
After



Note the change in scale

Location of Quadrupole Errors

Assume that one is suspicious about a certain area. Take two areas around the region and fit to free waves. See where the amplitude begins to change.



$$\delta\phi(s) = \begin{cases} \xi_a \sin 2\phi(s) + \eta_a \cos 2\phi(s) + C_a & s \in A \\ \xi_b \sin 2\phi(s) + \eta_b \cos 2\phi(s) + C_b & s \in B \end{cases}$$

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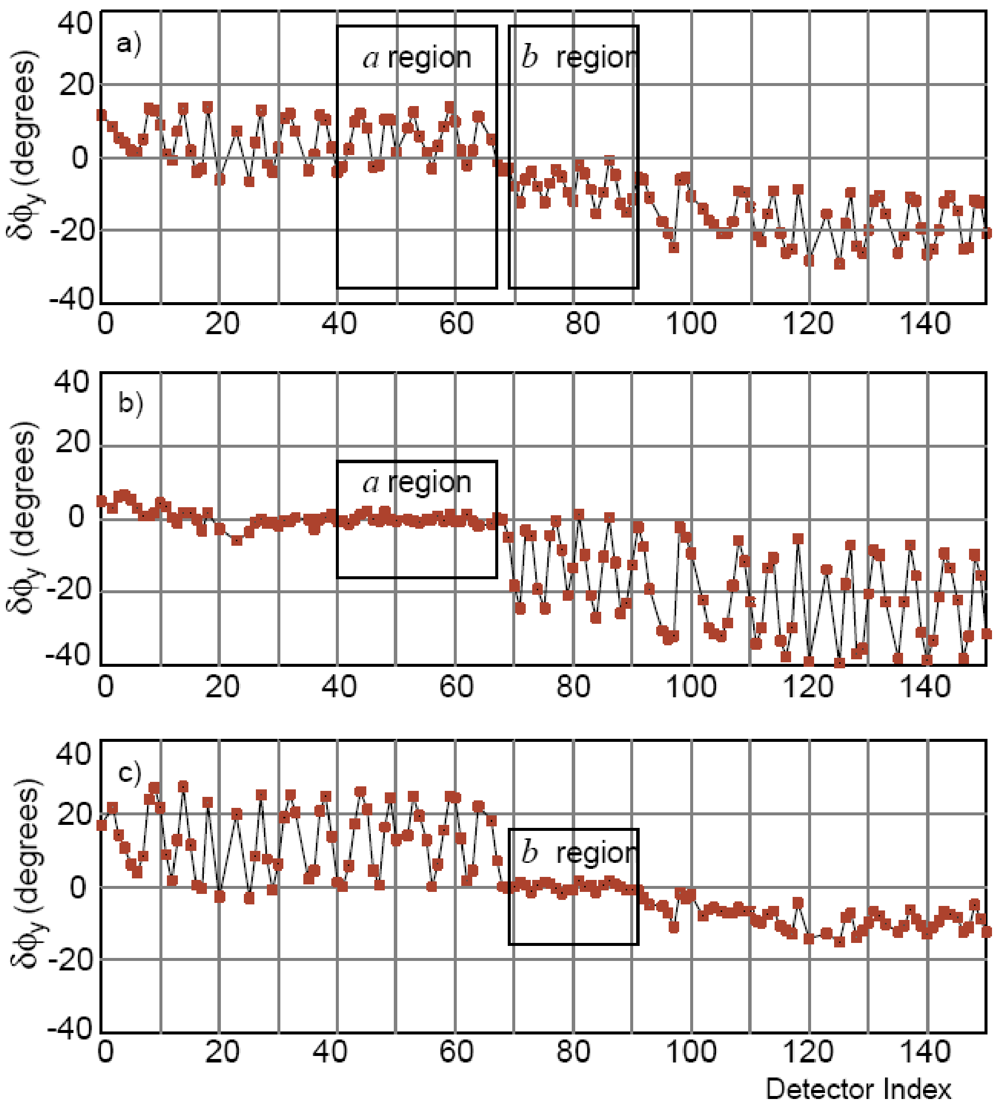
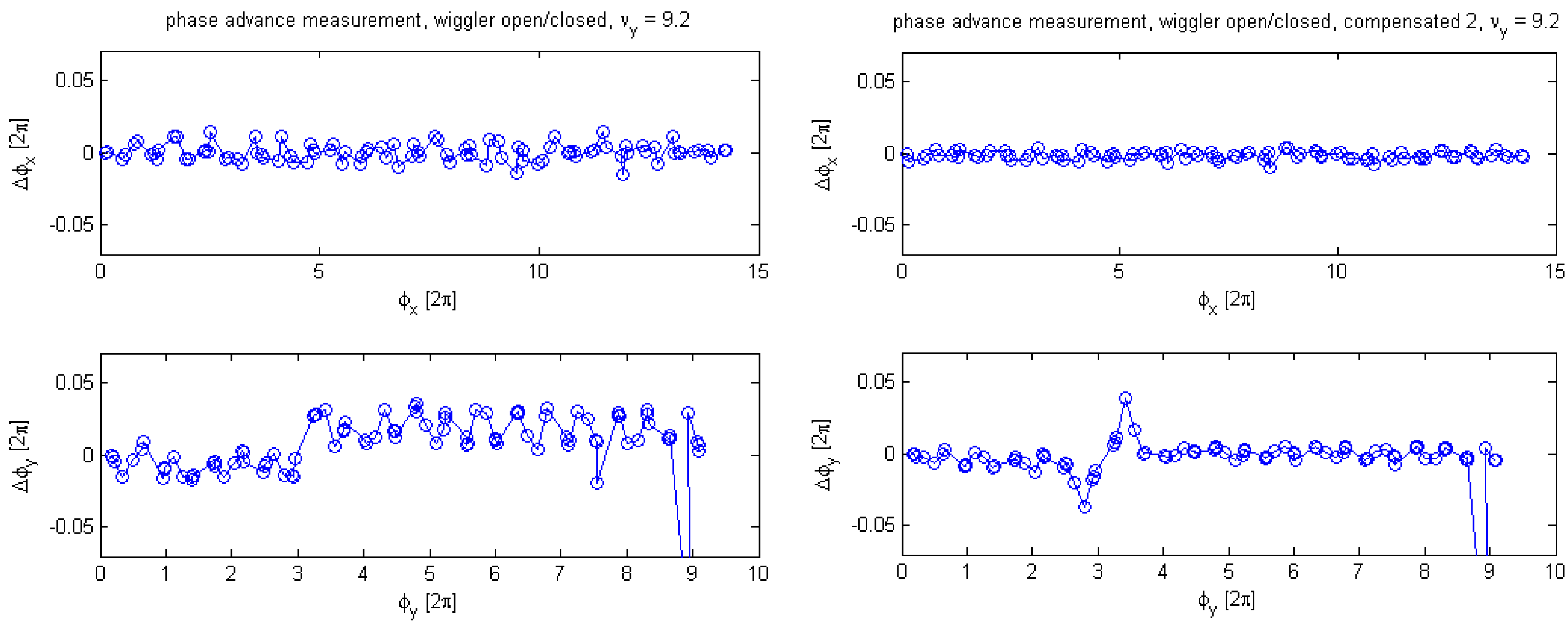
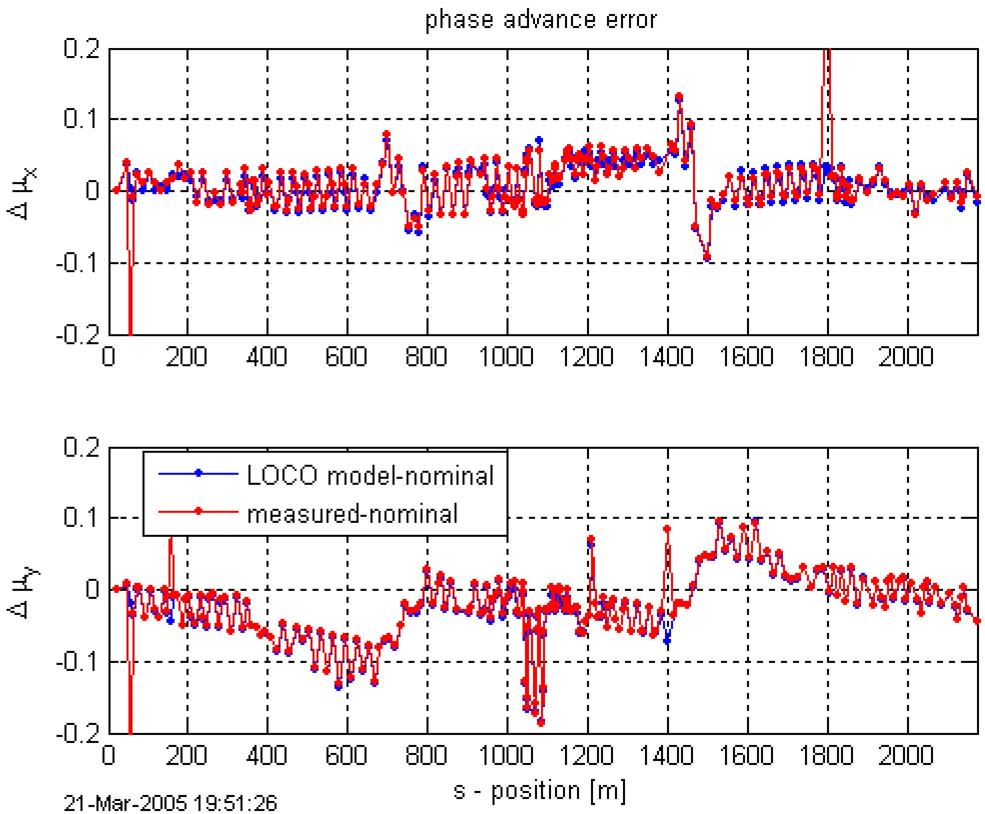


Example of another Method: Single turn kick



- Phase advance measurements allow to very quickly (few seconds) verify whether a (precomputed) compensation of a local lattice distortion works
- Example shown above is the systematic focusing change due to a wiggler

Phase advance as independent test



- ❖ Can compare a phase advance measurement with the predictions of an accelerator model, which has been calibrated using orbit response matrix analysis (this morning) or MIA (tomorrow)
- ❖ Particularly helpful in complicated/large accelerators (b-factories)

Summary

• Using resonant excitation and analysis of turn-by-turn data, lattice function measurements can be done quickly and accurately

Further reading:

• P. Castro et al., PAC93 conference proceedings 2103 (1993)
•Asseo, CERN PS Note 87-1 (1987)
•D. Sagan et al
    •PRST 2 074001 (1999)
    •PRST 3 092801 (2000)
    •PRST 3 102801 (2000)