Studies on Lattice Calibration With Frequency Analysis of Betatron Motion

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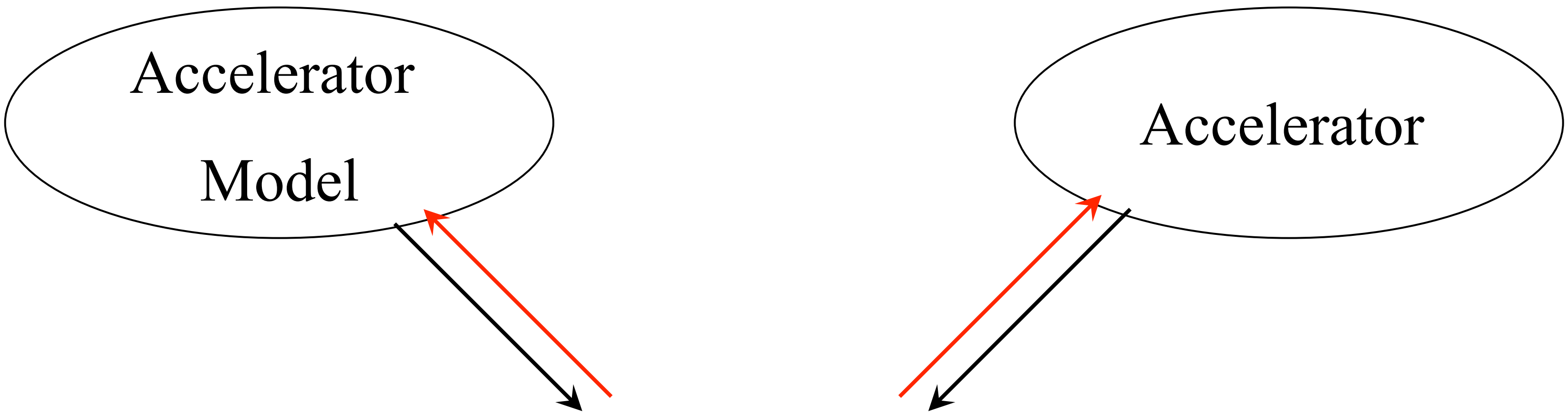
Most of the material in this lecture from
Ricardo Bartolini (Diamond),
Frank Schmidt, R. Tomas (CERN) , …

- Outline:
  - ❖Introduction
    - NAFF
    - perturbative theory of betatron motion
    - SVD fit of lattice parameters
  - ❖ DIAMOND Spectral Lines Analysis
    - Linear Model
    - Nonlinear Model

Real Lattice to Model Comparison



- Closed Orbit Response Matrix (LOCO–like)
- Frequency Map Analysis
- Frequency Analysis of Betatron Motion (resonant driving terms)

Frequency Analysis of Betatron Motion and Lattice Model Reconstruction (1)

Accelerator Model

- tracking data at all BPMs
- spectral lines from model (NAFF)
- build a vector of Fourier coefficients

Accelerator

- beam data at all BPMs
- spectral lines from BPMs signals (NAFF)
- build a vector of Fourier coefficients

$$\overline{A} = \left(a_1^{(1)} \quad \ldots \quad a_{NBPM}^{(1)} \quad \varphi_1^{(1)} \quad \ldots \quad \varphi_{NBPM}^{(1)} \quad a_1^{(2)} \quad \ldots \quad a_{NBPM}^{(2)} \quad \varphi_1^{(2)} \quad \ldots \quad \varphi_{NBPM}^{(2)} \quad \ldots \right)$$

Define the distance between the two vector of Fourier coefficients

$$\chi^2 = \sum_k \left(A_{Model}(j) - A_{Measured}(j) \right)^2$$

Least Square Fit (SVD) of accelerator parameters θ

to minimize the distance χ^2 of the two Fourier coefficients vectors

- Compute the “Sensitivity Matrix” M $$\Delta\bar{A} = M\bar{\theta}$$
- Use SVD to invert the matrix M $$M = U^T W V$$
- Get the fitted parameters $$\bar{\theta} = (V^T W^{-1} U)\,\Delta\bar{A}$$

MODEL → TRACKING → NAFF →

Define the vector of Fourier Coefficients – Define the parameters to be fitted

SVD → CALIBRATED MODEL

NAFF algorithm – J. Laskar (1988)
(Numerical Analysis of Fundamental Frequencies)

Given the quasi–periodic time series of the particle orbit $(x(n); p_x(n))$,

- Find the main lines in the signal spectrum

  $\Rightarrow \nu_1$ frequency, a_1 amplitude, ϕ_1 phase;

- build the harmonic time series

$$z_1(n) = a_1 e^{i\phi_1} e^{2\pi i \nu_1 n}$$

- subtract from the original signal
- analyze again the new signal $z(n) - z_1(n)$ obtained

The decomposition $z(n) = \sum_{k=1}^{n} a_k e^{i\phi_k} e^{2\pi i \nu_k n}$ **allows the**

Measurement of Resonant driving terms of non linear resonances

The quasi periodic decomposition of the orbit

$$x(n)-ip_x(n)=\sum_{k=1}^{n} c_k e^{2\pi i \nu_k n} \qquad c_k = a_k e^{i\phi_k}$$

can be compared to the perturbative expansion of the non linear betatron motion

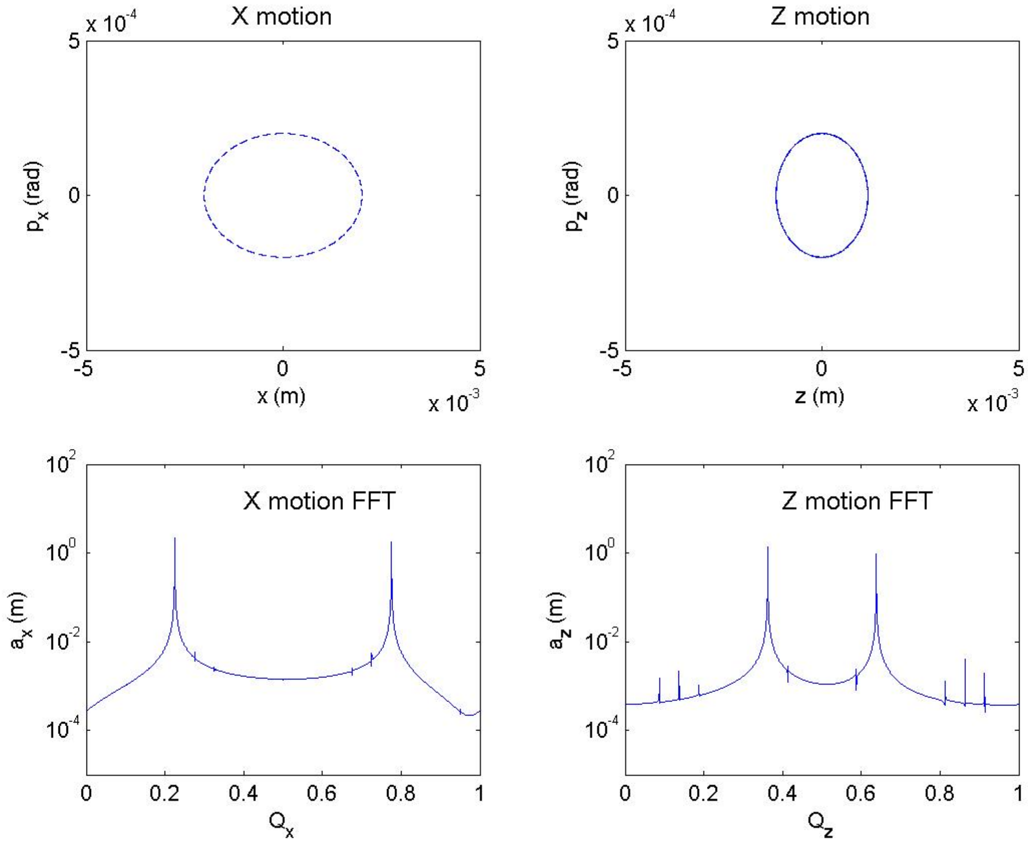
$$x(n)-ip_x(n)=\sqrt{2I_x}e^{i(2\pi Q_x n+\psi_0)}+$$

$$-2i\sum_{jklm} js_{jklm}(2I_x)^{\frac{j+k-1}{2}}(2I_y)^{\frac{l+m}{2}}e^{i\left[(1-j+k)(2\pi Q_x n+\psi_{x0})+(m-l)(2\pi Q_y n+\psi_{y0})\right]}$$

Each resonance driving term s_{jklm} contributes to the Fourier coefficient of a well defined spectral line

$$\nu(s_{jklm})=(1-j+k)Q_x+(m-l)Q_y$$

Spectral Lines for DIAMOND low emittance lattice (.2 mrad kick in both planes)

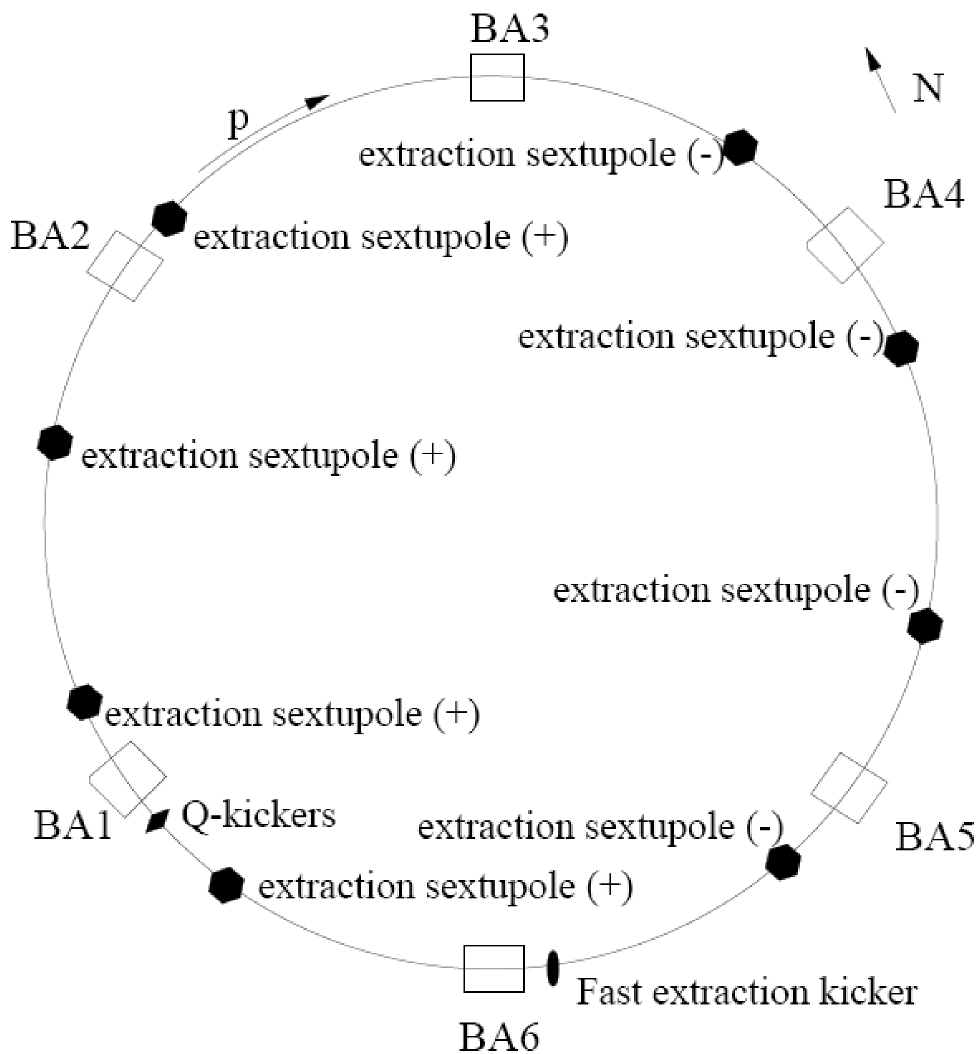


Spectral Lines detected with NAFF algorithm

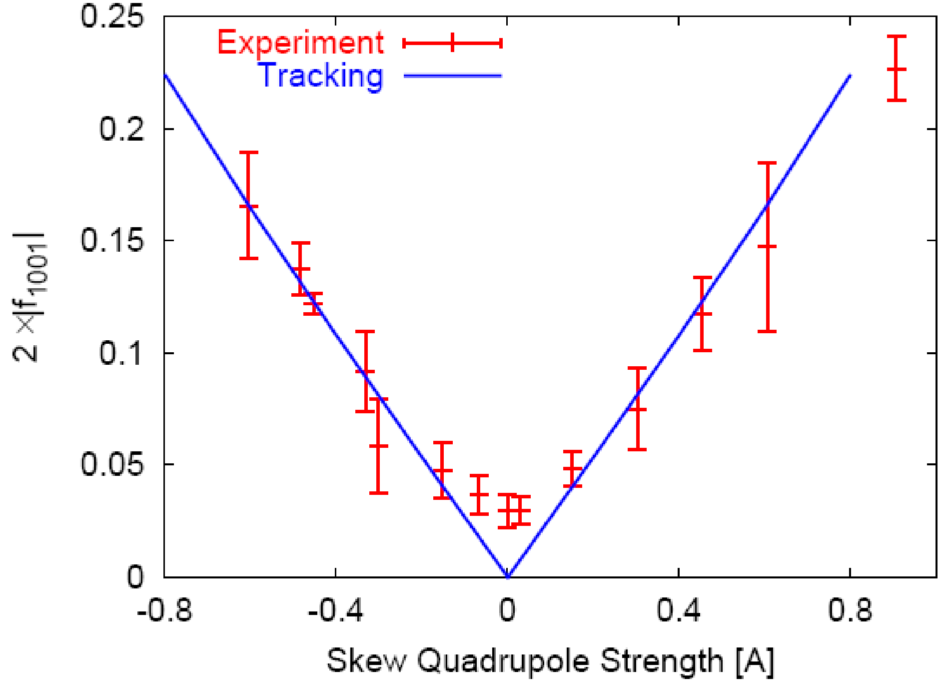
e.g. Horizontal:

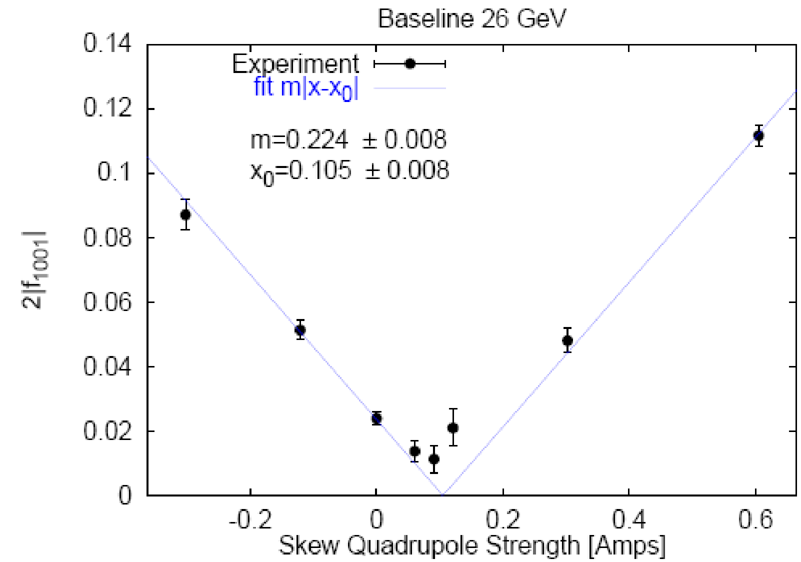
• (1, 0)	$1.10\ 10^{-3}$	horizontal tune
• (0, 2)	$1.04\ 10^{-6}$	$Q_x - 2\ Q_z$
• (–3, 0)	$2.21\ 10^{-7}$	$4\ Q_x$
• (–1, 2)	$1.31\ 10^{-7}$	$2\ Q_x + 2\ Q_z$
• (–2, 0)	$9.90\ 10^{-8}$	$3\ Q_x$
• (–1, 4)	$2.08\ 10^{-8}$	$2\ Q_x + 4\ Q_z$

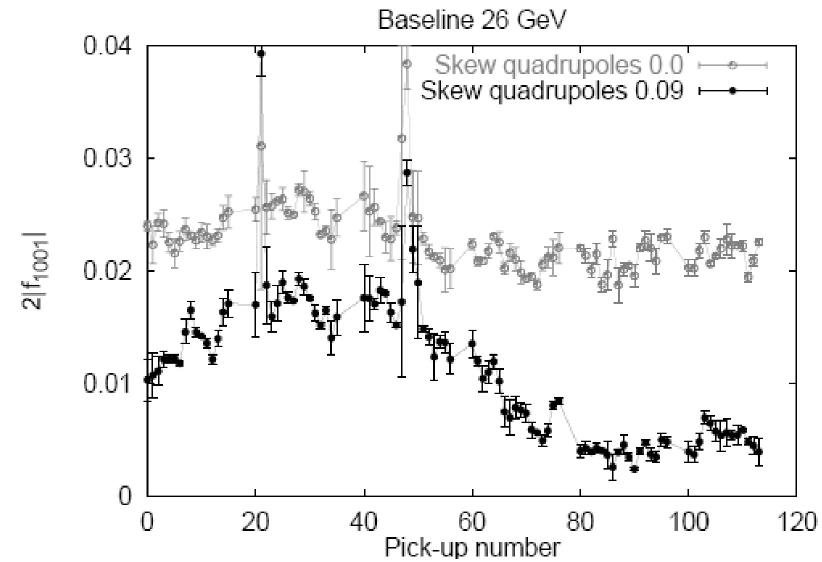
Early (2000-2002) Work: SPS experiments



Linear Coupling in SPS

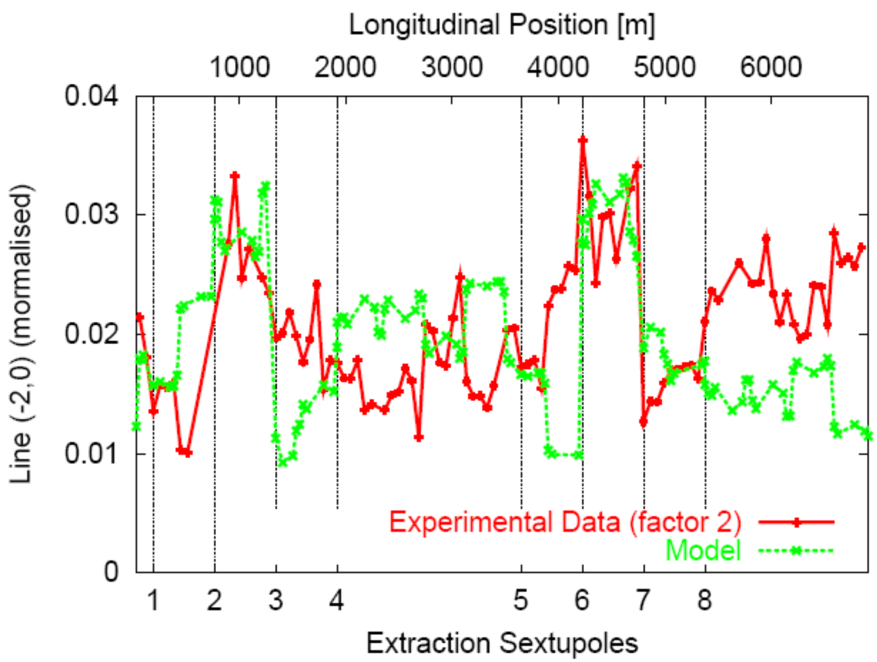






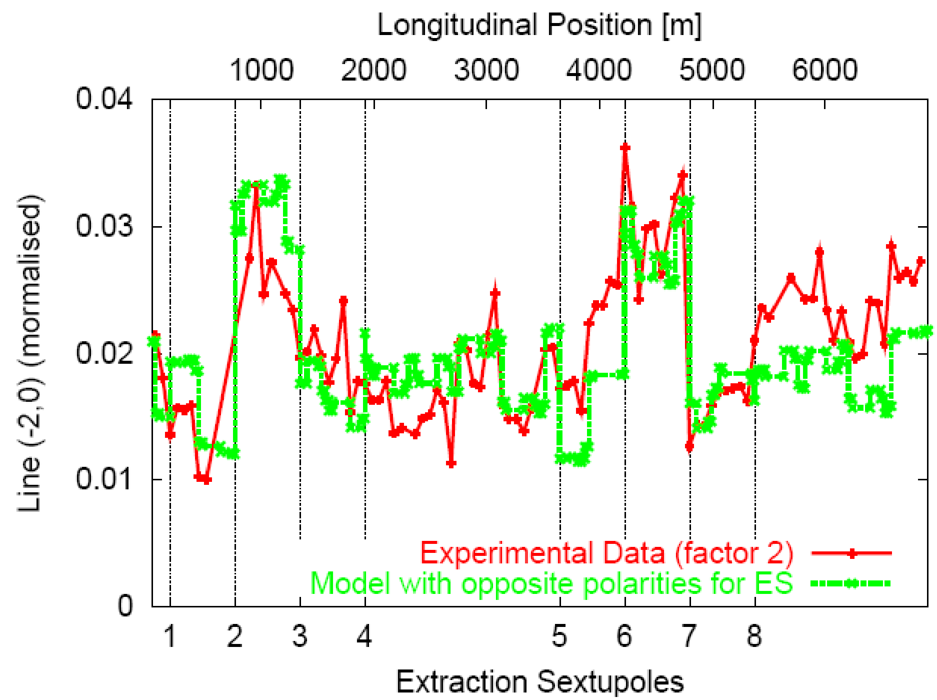
Sextupole Driving Terms with Extraction Sext.

The resonance (3,0) introduces the spectral line (-2,0).



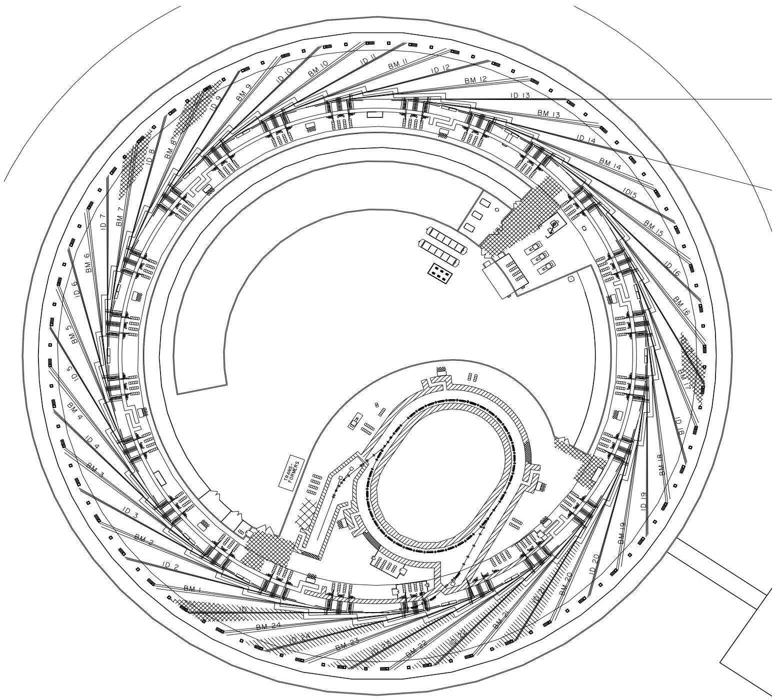
⇒ We have a problem!

Change polarities of the extraction sextupoles?



Hardware checks confirmed that these sextupoles had opposite polarities.

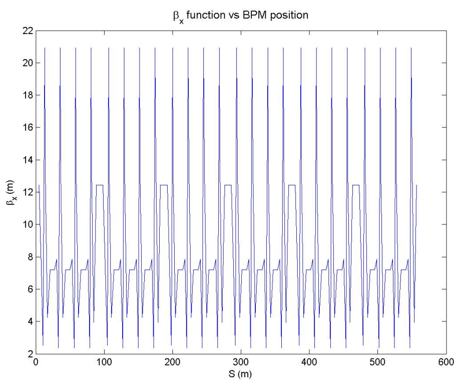
Diamond Light Source

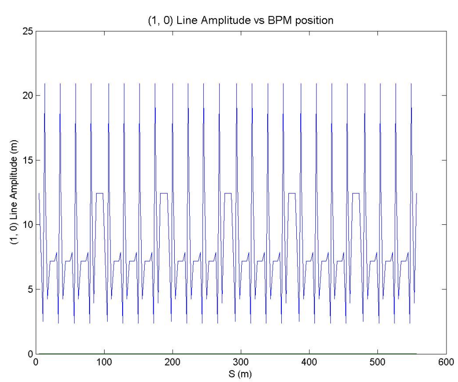


- Diamond is one of the newest light sources in Great Britain: 3 GeV, 2.75 nm horizontal emittance, extensive beam diagnostics equipment (all BPMs have high resolution turn by turn ability)

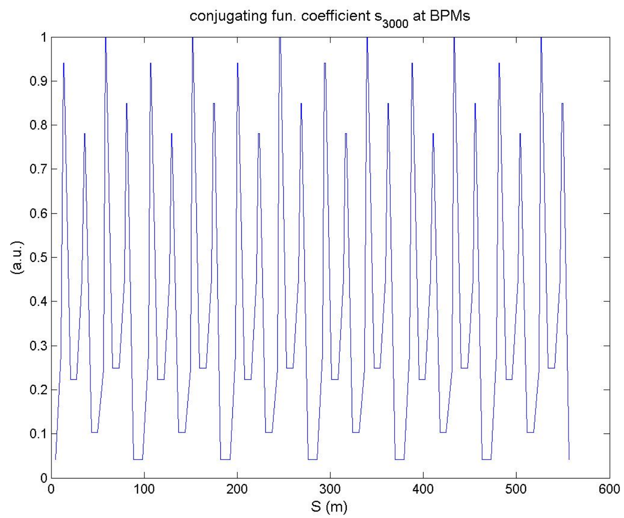
Amplitude of Spectral Lines for low emittance DIAMOND lattice computed at all the BPMs

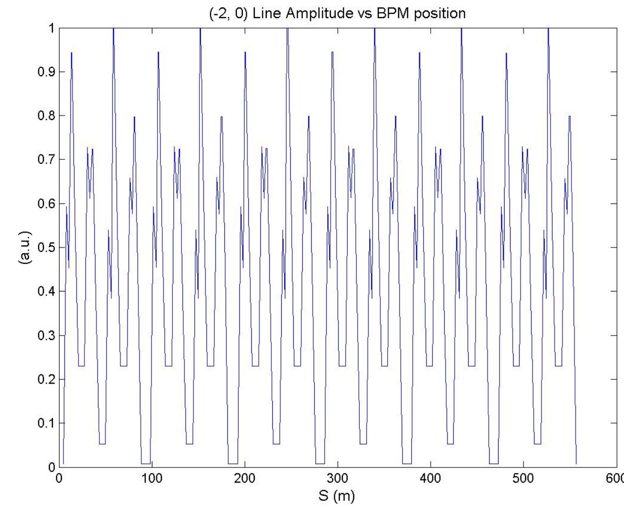
Main spectral line (Tune Q_x)





(-2, 0) spectral line: resonance driving term h_{3000} ($3Q_x = p$) at all BPMs





- The amplitude of the tune spectral line replicates the β functions
- The amplitude of the (– 2, 0) show that third order resonance is well compensated within one superperiod. Some residual is left every two cells (5π/2 phase advance)

DIAMOND Spectral Lines Analysis

- Horizontal Misalignment of sextupoles (β – beating)
- Vertical Misalignment of sextupoles (linear coupling)
- Gradient errors in sextupoles (non linear resonances)

Horizontal misalignment of a set of 24 sextupoles with 100 μm rms (β - beating correction)

The generated normal quadrupole components introduce a β - beating.

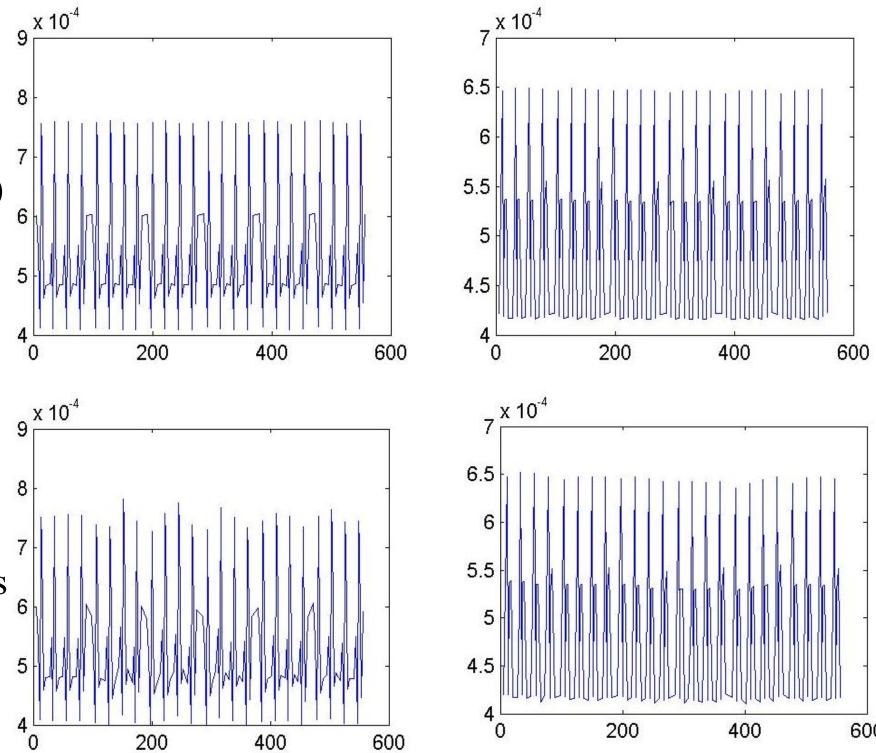
- we build the vector of Fourier coefficients of the horizontal and vertical tune line
- we use the horizontal misalignments as fit parameters

H tune line (no misalignments)

V tune line (no misalignments)

H tune line with misalignments

V tune line with misalignments

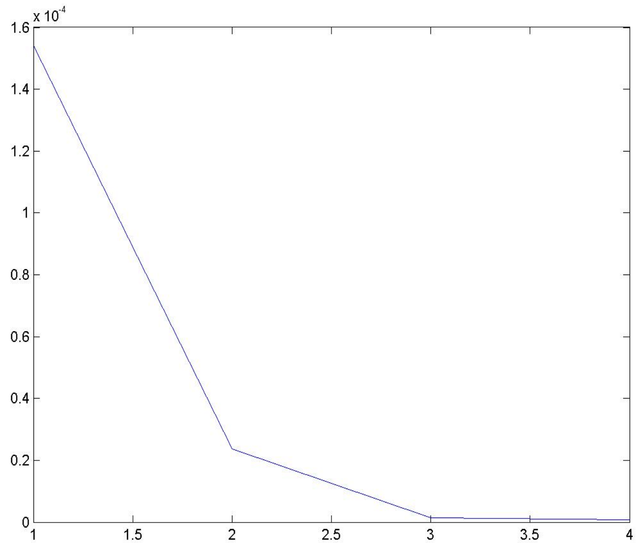


SVD on sextupoles horizontal misalignments

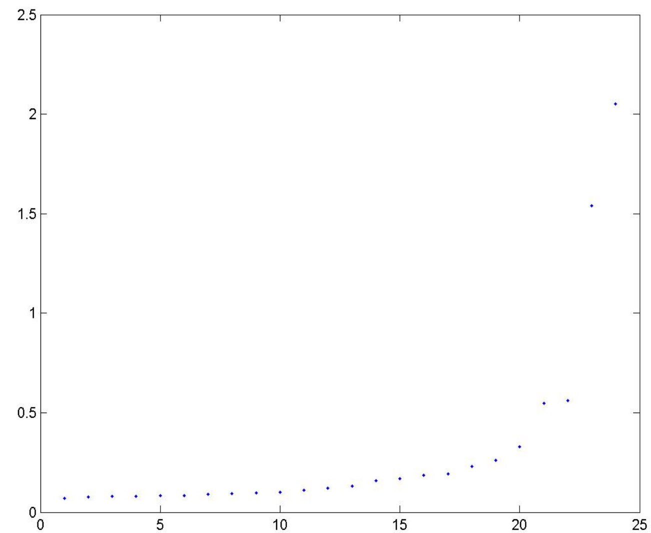
We build the vector $\overline{A} = \left(a_1^{H(1,0)} \quad \ldots \quad a_{NBPM}^{H(1,0)} \quad a_1^{V(0,1)} \quad \ldots \quad a_{NBPM}^{V(0,1)}\right)$

containing the amplitude of the tune lines in the two planes at all BPMs

We minimize the sum $\chi^2 = \sum_j \left(A_{Model}(j) - A_{Measured}(j)\right)^2$



χ^2 as a function of the iteration number

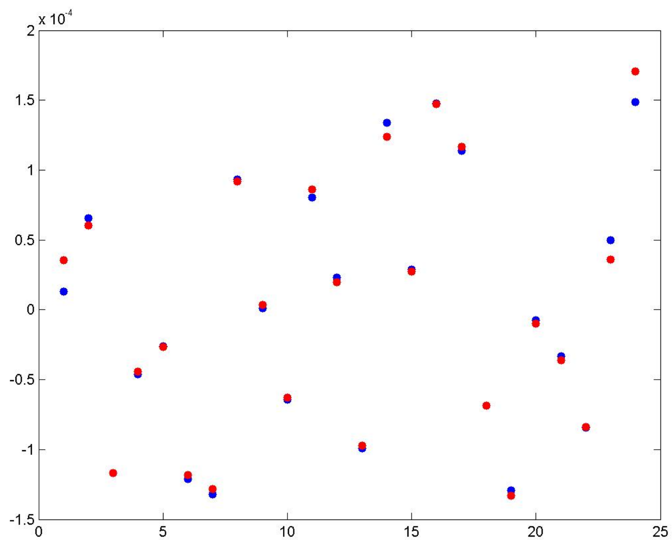


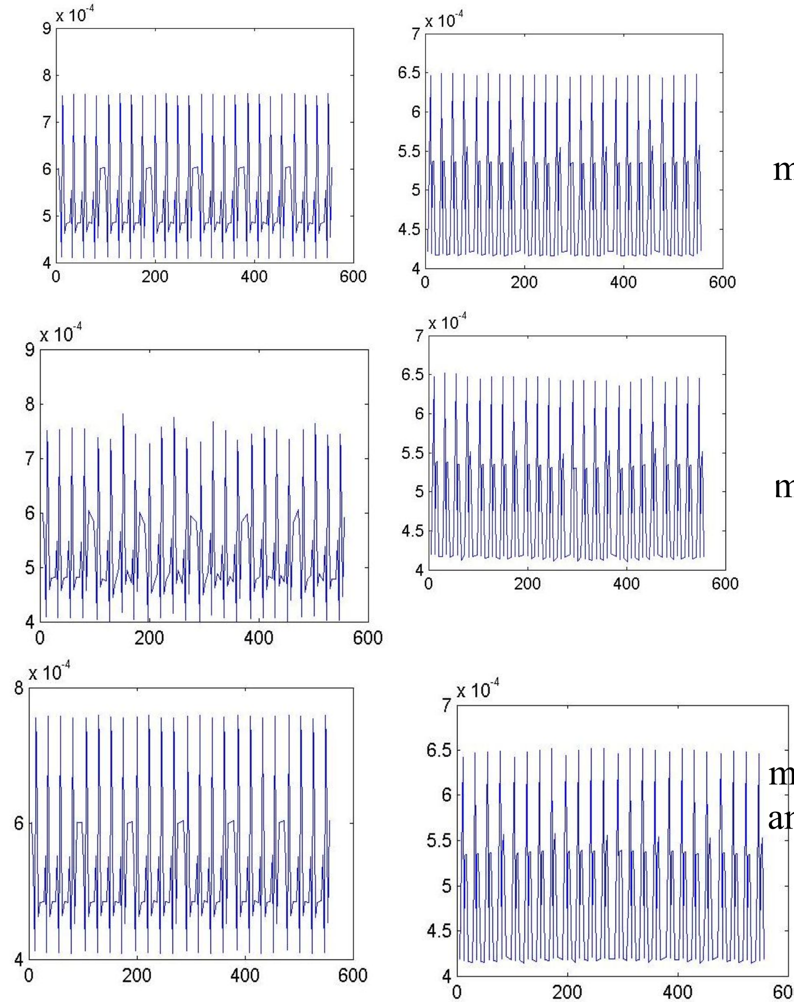
Example of SVD principal values

Fitted values for the 24 horizontal sextupole misalignments obtained from the SVD

Blu dots = assigned misalignments

Red dots = reconstructed misalignments





no misalignments

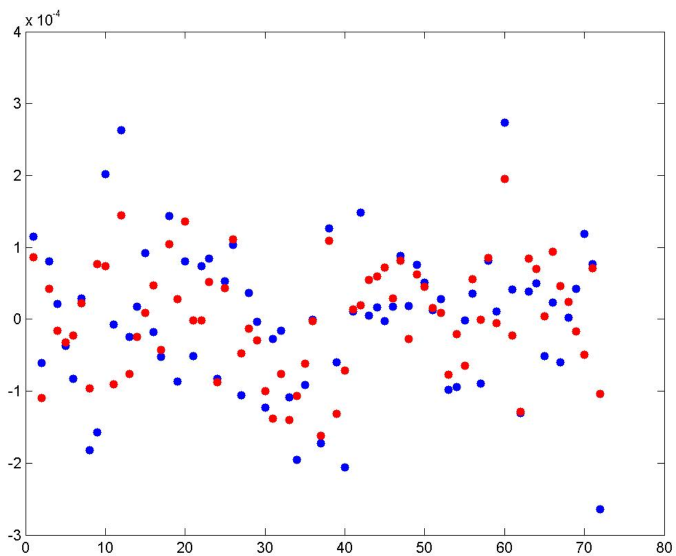
with misalignments

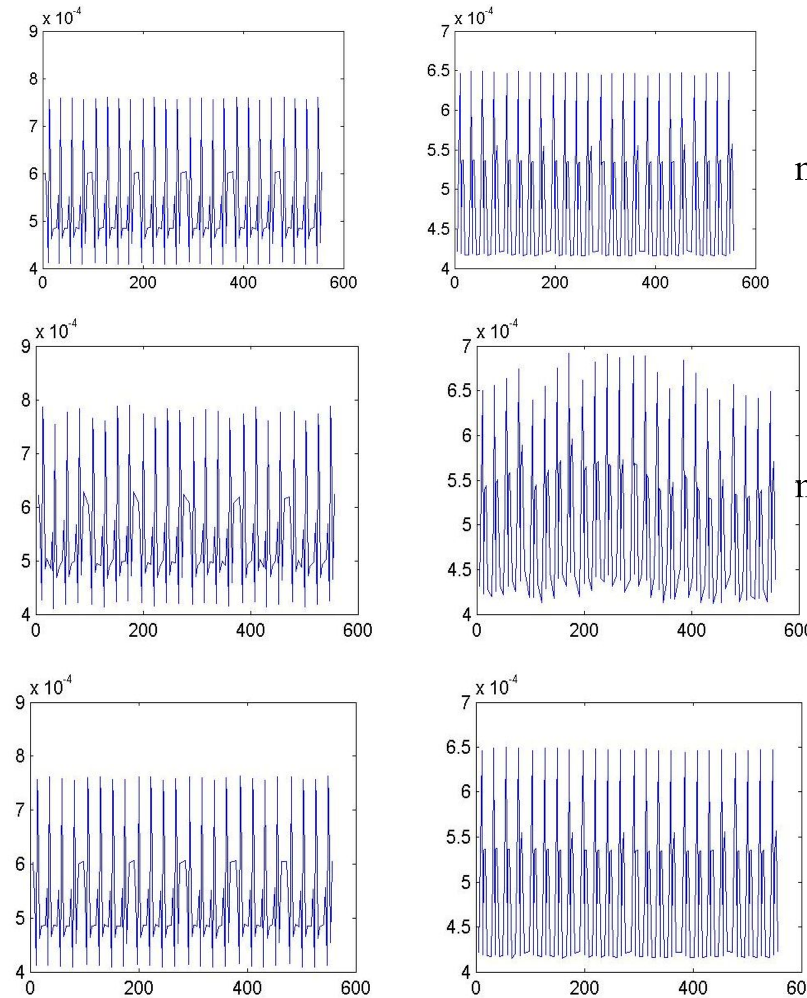
with misalignments and corrections

Fitted values for the 72 horizontal sextupole misalignments obtained from the SVD

Blu dots = assigned misalignments

Red dots = reconstructed misalignments





no misalignments

with misalignments

with salignments d corrections

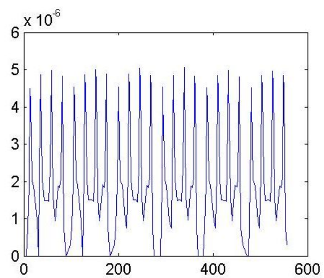
Sextupoles gradient errors applied to 24 sextupoles (dK_2/K_2 = 5%)

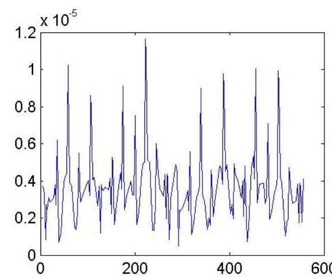
The sextupole gradient errors spoil the compensation of the third order resonances, e.g $3Q_x = p$ and $Q_x - 2Q_z = p$

- we build the vector of Fourier coefficients of the H(-2,0) and H(0,2) line
- we use the errors gradients as fit parameters

(0,2) line amplitude in H plane

(no gradient errors)



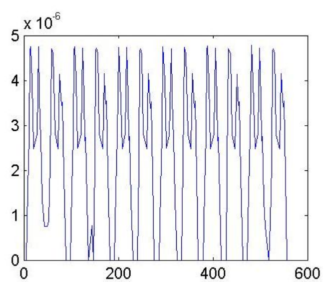


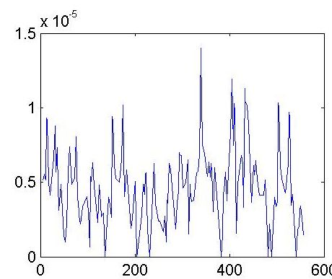
(0,2) line amplitude in H plane

with gradient errors

(-2,0) line amplitude in H plane

(no gradient errors)





(-2,0) line amplitude in H plane

with gradient errors

SVD on sextupole gradient errors

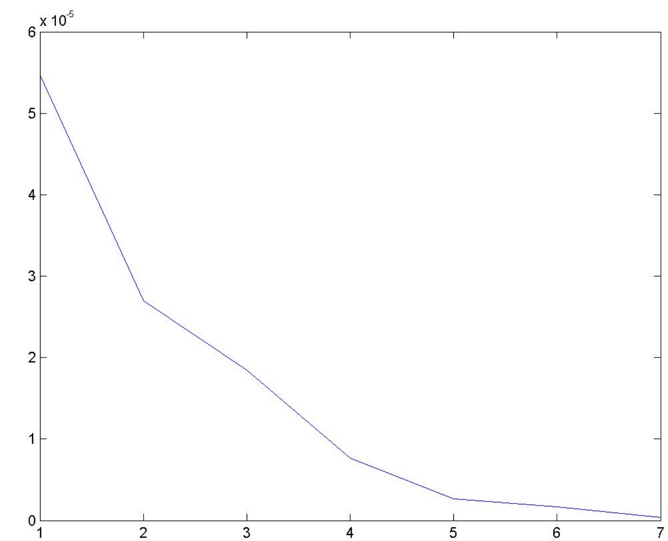
We build the vector $\bar{A} = \left(a_1^{H(-2,0)} \quad \ldots \quad a_{NBPM}^{H(-2,0)} \quad a_1^{H(0,2)} \quad \ldots \quad a_{NBPM}^{H(0,2)} \right)$

containing the amplitudes at all BPMs

- the (–2, 0) line in the H plane related to h_{3000}
- the (0, 2) line in the H plane related to h_{1002}

We minimize the sum

$$\chi^2 = \sum_j \left(A_{Model}(j) - A_{Measured}(j) \right)^2$$

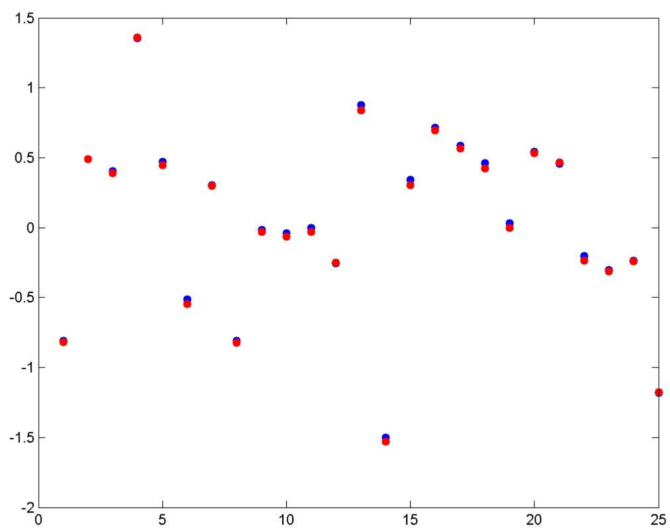


χ^2 as a function of the iteration number

Fitted values for the 24 sextupoles gradients errors obtained from SVD

Blu dots = assigned misalignments

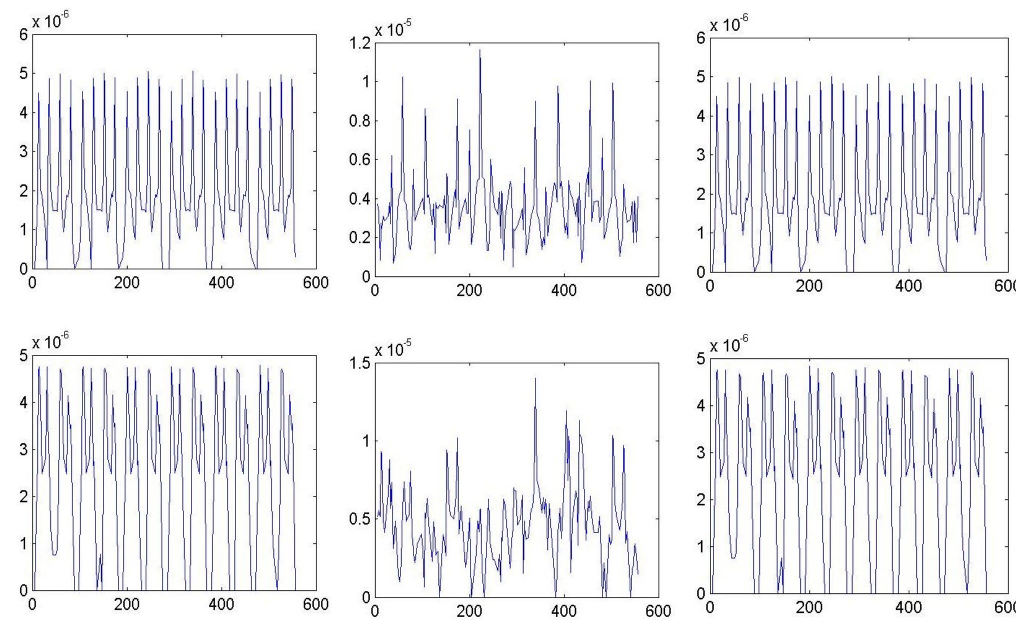
Red dots = reconstructed misalignments



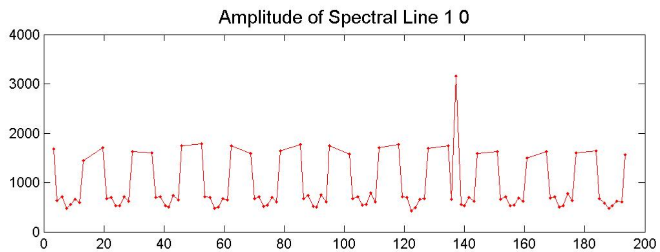
no gradient errors

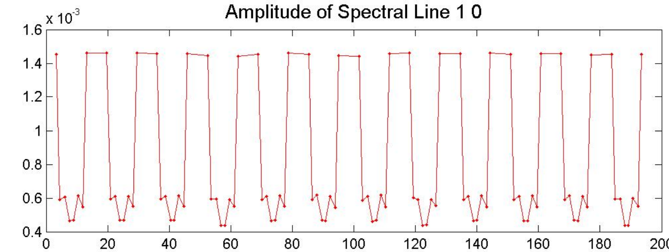
with gradient errors

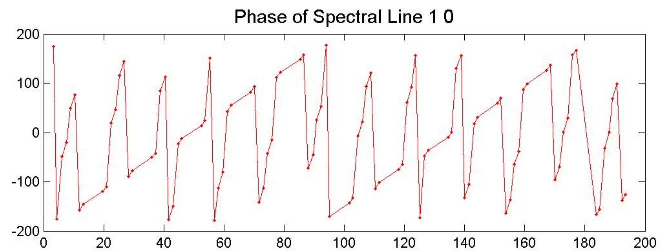
with gradient errors and corrections

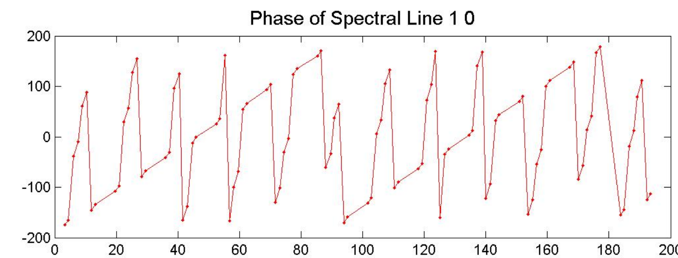


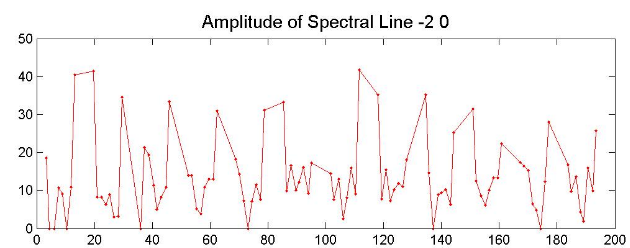
Measurements: ALS example (very early results)

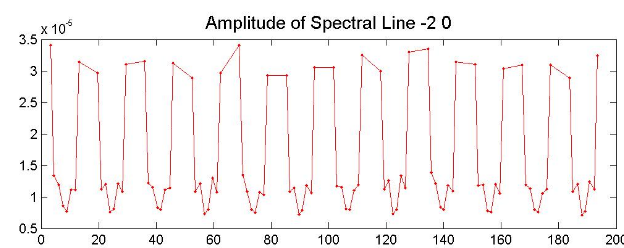


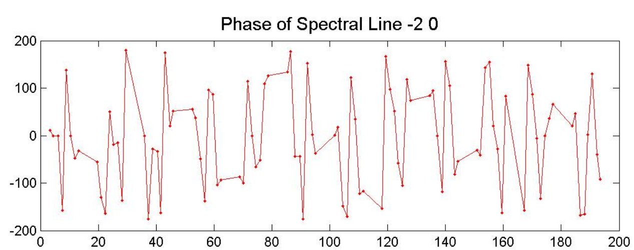


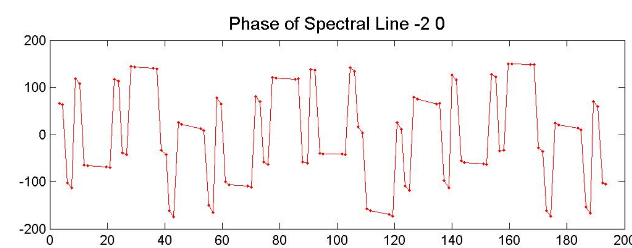












All BPMs have turn-by-turn capabilities

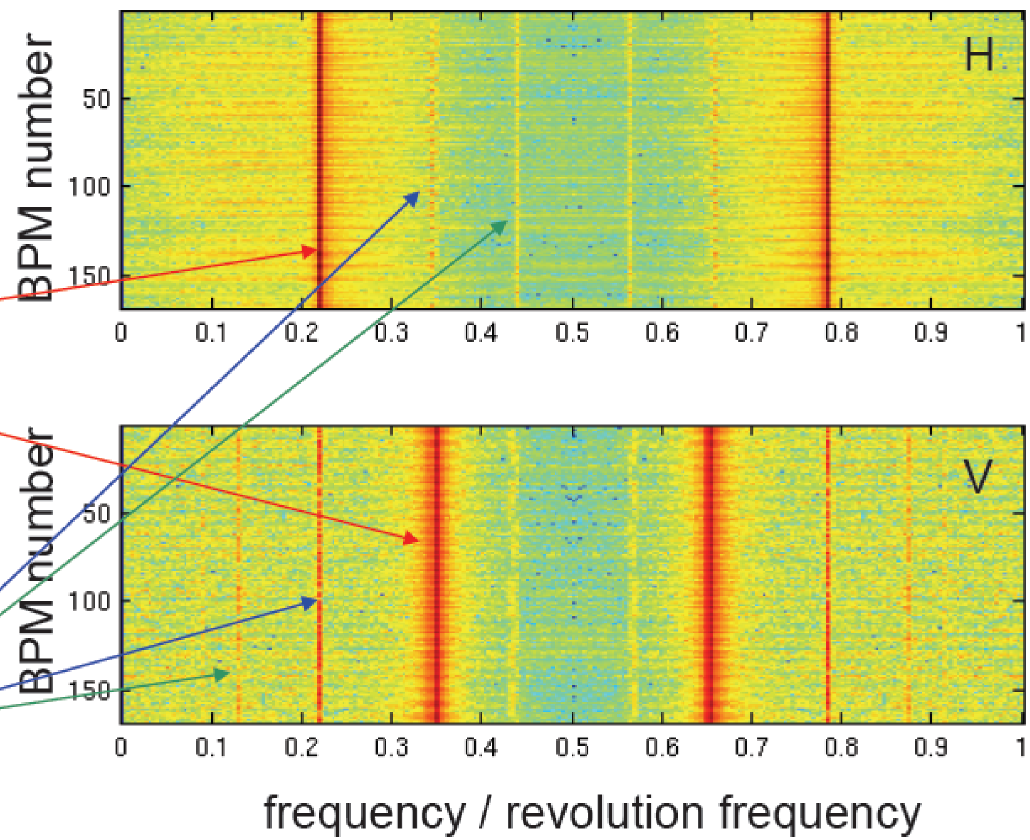
- excite the beam diagonally
- measure tbt data at all BPMs
- colour plots of the FFT

Q_x = 0.22 H tune in H

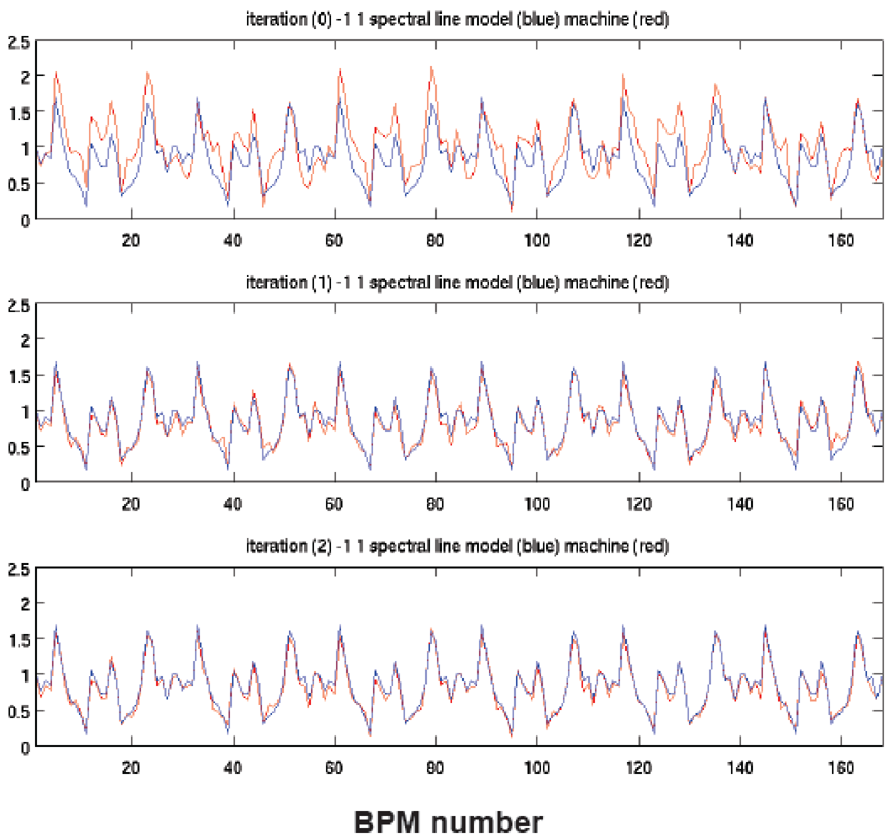
Q_y = 0.36 V tune in V

All the other important lines are linear combination of the tunes Q_x and Q_y

$m\, Q_x + n\, Q_y$



First attempt at correction …



Blue model; red measured

A first attempt to fit the spectral line (-1,1), determined by the resonance (-1,2), improved the agreement of the spectral line with the model

However the lifetime was worse by 15%

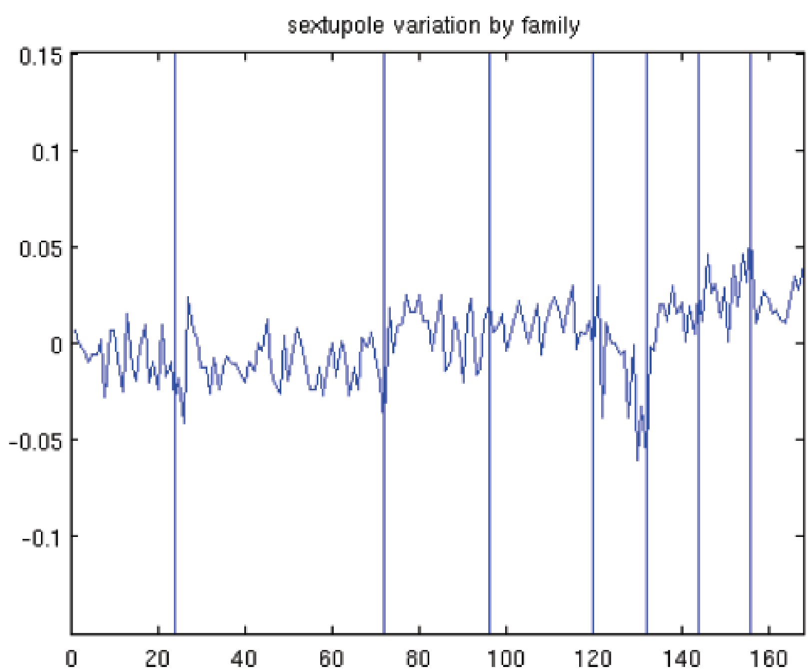
The fit produced non realistic large deviation in the sextupoles (>10%);

The other spectral lines were spoiled

Potential problems could be that:

- problem is undercontrained (too many sextupole knobs, too few BPMs)
- Linear (coupled) lattice errors contributed to variation in driving terms – however correction only involved sextupoles

Simultaneous fit of (-2,0) in H and (1,-1) in V



Now the sextupole variation is limited to < 5%

Both resonances are controlled

and the **lifetime improved by 10%**

Limits of resonance driving term measurements

BPMs precision in turn by turn mode (+ gain, coupling and non-linearities)

10 μm with ~10 mA

very high precision required on turn-by-turn data (not clear yet is few tens of μm is sufficient); Algorithm for the precise determination of the betatron tune lose effectiveness quickly with noisy data. R. Bartolini et al. Part. Acc. 55, 247, (1995)

BPM gain and coupling can be corrected by LOCO, but nonlinearities remain (especially for diagonal kicks)

Decoherence of excited betatron oscillation reduce the number of turns available Studies on oscillations of beam distribution shows that lines excited by resonance of order m+1 decohere m times faster than the tune lines. This decoherence factor m has to be applied to the data R. Tomas, PhD Thesis, (2003)

Summary

- **Resonance driving term analysis provides quantitative information about nolinearities in the machine**

- **It allows to measure the local distribution of the dominant nonlinearities**

- **However, it does not give a direct information about how harmful the nonlinearities are**

- **Theoretically it can provide a method similar to orbit response matrix analysis (or phase advance, …) to measure not just the gradient and skew gradient distribution, but also the setxupole, (octupole), … How well this will work experimentally is still not clear:**

    - **Can we use the spectral lines to recover the LINEAR and NON LINEAR machine model with a Least Square method?**

- SPS with a few very large nonlinearities worked well. Diamond simulations look encouraging, measurements ran into some problems but gave some improvements as well. ALS measurements were BPM resolution limited (BPM upgrade started).

Further Reading

- Proceedings of the 2008 workshop on nonlinear dynamics, held at ESRF: http://www.esrf.eu/Accelerators/Conferences/non-linear-beam-dynamics-workshop
- R. Bartolini, et al. 'Measurement of Resonance Driving Terms by Turn-by-Turn Data', Proceedings of PAC 1999, 1557, New York (1999)
- W. Fischer, et al. 'Measurement of Sextupolar Resonance Driving Terms in RHIC', Proceedings of PAC 2003, Portland
- R. Bartolini and F. Schmidt, LHC Project note 132, Part. Accelerators. 59, pp. 93-106, (1998).
- F. Schmidt, R. Tom´as, A. Faus–Golfe, CERN-SL-2001-039-AP Geneva, CERN and IPAC 2001.
- M. Hayes, F. Schmidt, R. Tom´as, EPAC 2002 and CERNSL-2002-039-AP Geneva, 25 Jun 2002.
- F. Schmidt, CERN SL/94-56 (AP) Update March 2000.
- R. Bartolini and F. Schmidt, CERN SL–Note–98–017 (AP).