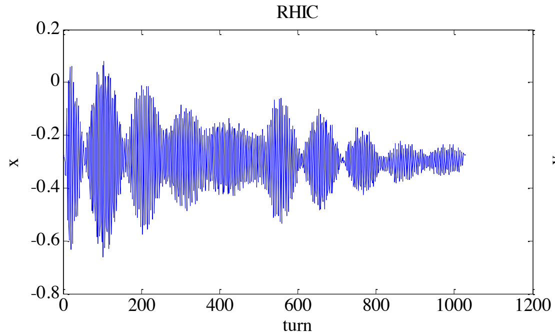
Model independent analysis and independent component analysis for BPM data analysis

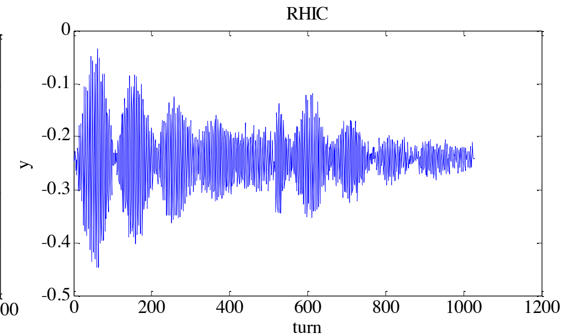
X. Huang

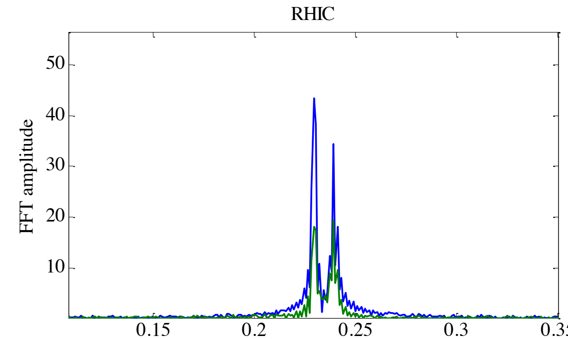
USPAS 2012 Summer – Beam based diagnostics

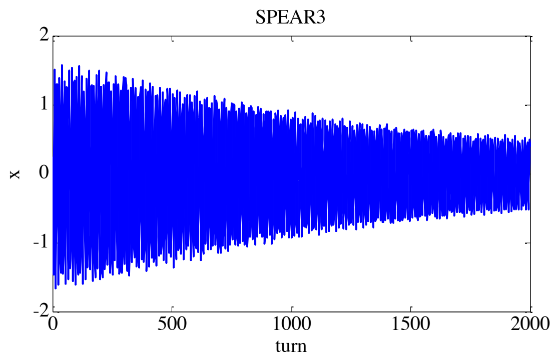
Examples of turn-by-turn BPM data

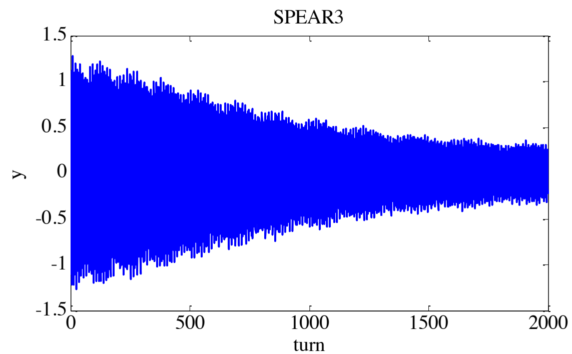
Many rings are equipped with multiple turn-by-turn BPMs. How to make efficient use of the vast amount of data?

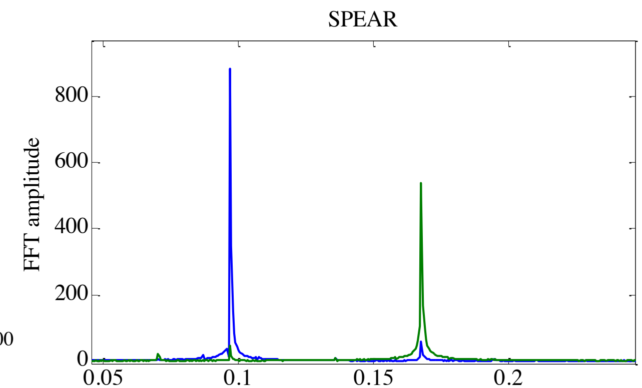


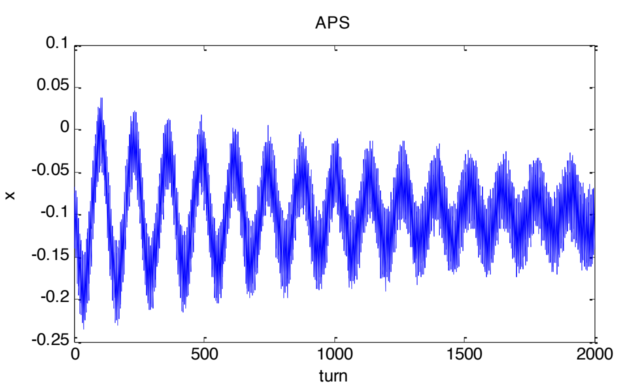


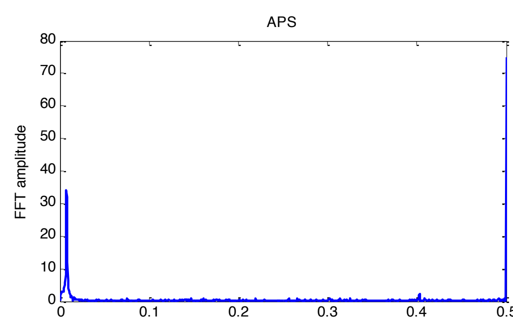












A model of BPM turn-by-turn data

- The turn-by-turn beam position signal is a combination of various source signals.

$$x_i(t) = \sum_j a_{ij} s_j(t) + n_j(t)$$

For the i'th BPM

or $$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$$

$\mathbf{A}$ is the mixing matrix

There are only a few meaningful source signals, such as betatron oscillation and synchrotron oscillation.

Form a matrix of the BPM data

$$\mathbf{x} = \begin{pmatrix} x_1(1) & x_1(2) & \cdots & x_1(T) \\ x_2(1) & x_2(2) & \cdots & x_2(T) \\ \vdots & \vdots) & \ddots & \vdots \\ x_m(1) & x_m(2) & \cdots & x_m(T) \end{pmatrix}$$

m BPMs and T turns

X. Huang, PRSTAB, 8, 064001, 2005

Betatron modes via singular value decomposition

It has been proven* that when the BPM reading contains only one betatron mode, i.e.

$$x_m(t) = \sqrt{2J(t)\beta_m}\cos(\phi(t)+\psi_m)$$

Note the constant orbit offsets are always removed for each BPM. This is called “centering”.

then there are only two non-trivial SVD eigen-modes

$$\mathbf{x} = \mathbf{U}\mathbf{S}\mathbf{V}^{\mathrm{T}} = s_+\mathbf{u}_+\mathbf{v}_+^{\mathrm{T}} + s_-\mathbf{u}_-\mathbf{v}_-^{\mathrm{T}}$$

U,V are orthogonal matrices,
S is a block-diagonal matrix.

$$u_{+,m} = \frac{1}{s_+}\sqrt{<J>\beta_m}\cos(\phi_0+\psi_m),$$

$$u_{-,m} = \frac{1}{s_-}\sqrt{<J>\beta_m}\sin(\phi_0+\psi_m)$$

$$v_+(t) = \sqrt{\frac{2J(t)}{T<J>}}\cos(\phi(t)-\phi_0),$$

$$v_-(t) = -\sqrt{\frac{2J(t)}{T<J>}}\sin(\phi(t)-\phi_0)$$

u: spatial vector

v: temporal vector

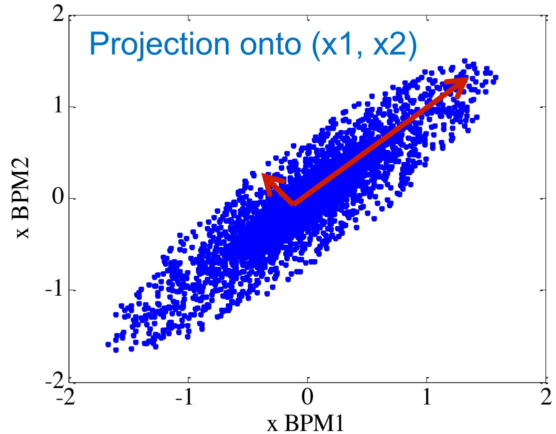
Beta function and betatron phase advance can be calculated from the spatial vector.

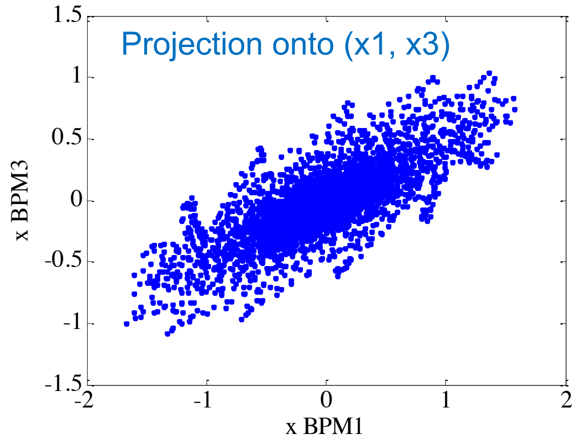
$$\psi_m = \tan^{-1}\left(\frac{s_-u_{-,m}}{s_+u_{+,m}}\right)$$

$$\beta_m = \frac{1}{<J>}\left[(s_+u_{+,m})^2 + (s_-u_{-,m})^2\right]$$

* Chun-xi Wang, et al. PR-STAB 6, 104001 (2003).

What does SVD do?





The BPM data can be viewed as T points in the m-dimensional space.

$$P(t) = (x_1(t), x_2(t), \cdots, x_m(t))$$

These points form an hyper-ellipsoid. What SVD does is to identify its principal-axes. This is called principal component analysis (PCA).
PCA: with a linear orthogonal transformation to obtain a set of linearly un-correlated components (variables) which holds (successively) the largest variances.

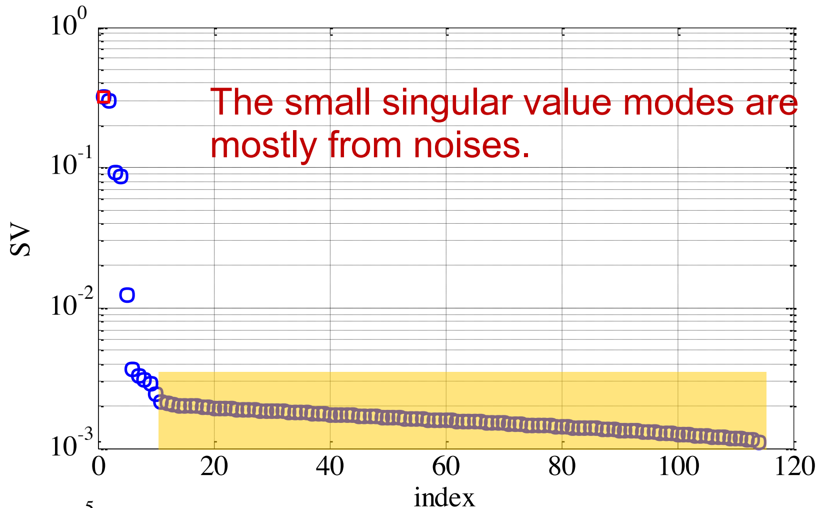
$$\mathbf{x}\mathbf{x}^T = \mathbf{U}\Sigma\mathbf{U}^T, \quad \Sigma = \mathbf{S}\mathbf{S}^T$$

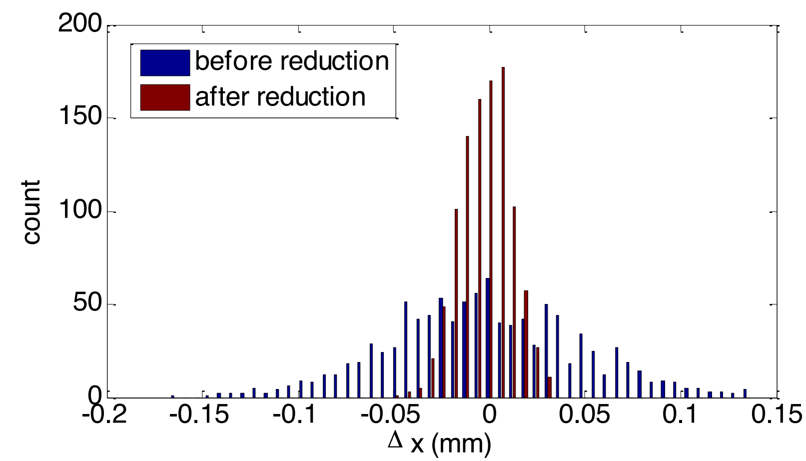
The U matrix diagonalize the covariance matrix.

The results in the previous slide states: with only one betatron mode in the BPM data, the hyper-ellipsoid degenerates to an ellipse (2D).

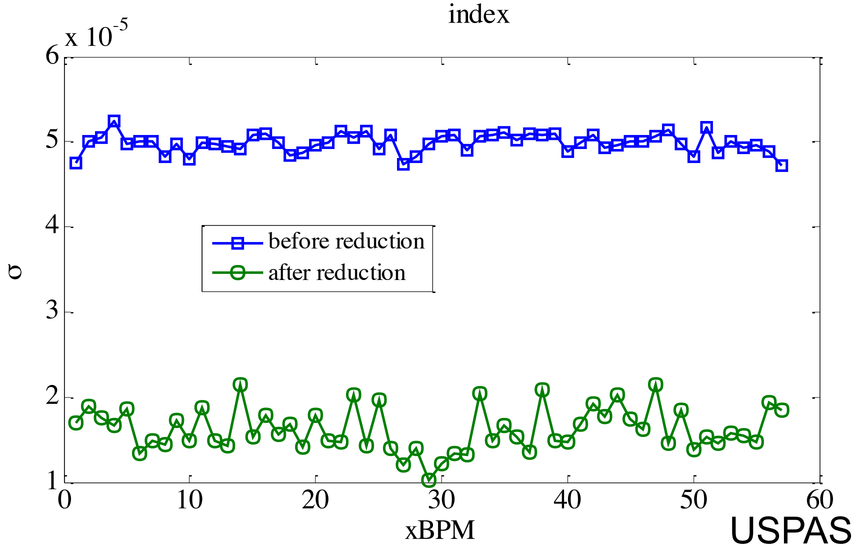
Noise reduction with SVD

As the random noises are distributed in all eigen-modes while the signals are concentrated in the leading eigen-modes, noise can be reduced by re-constructing the data after removing the noise-only (with small singular values) modes.





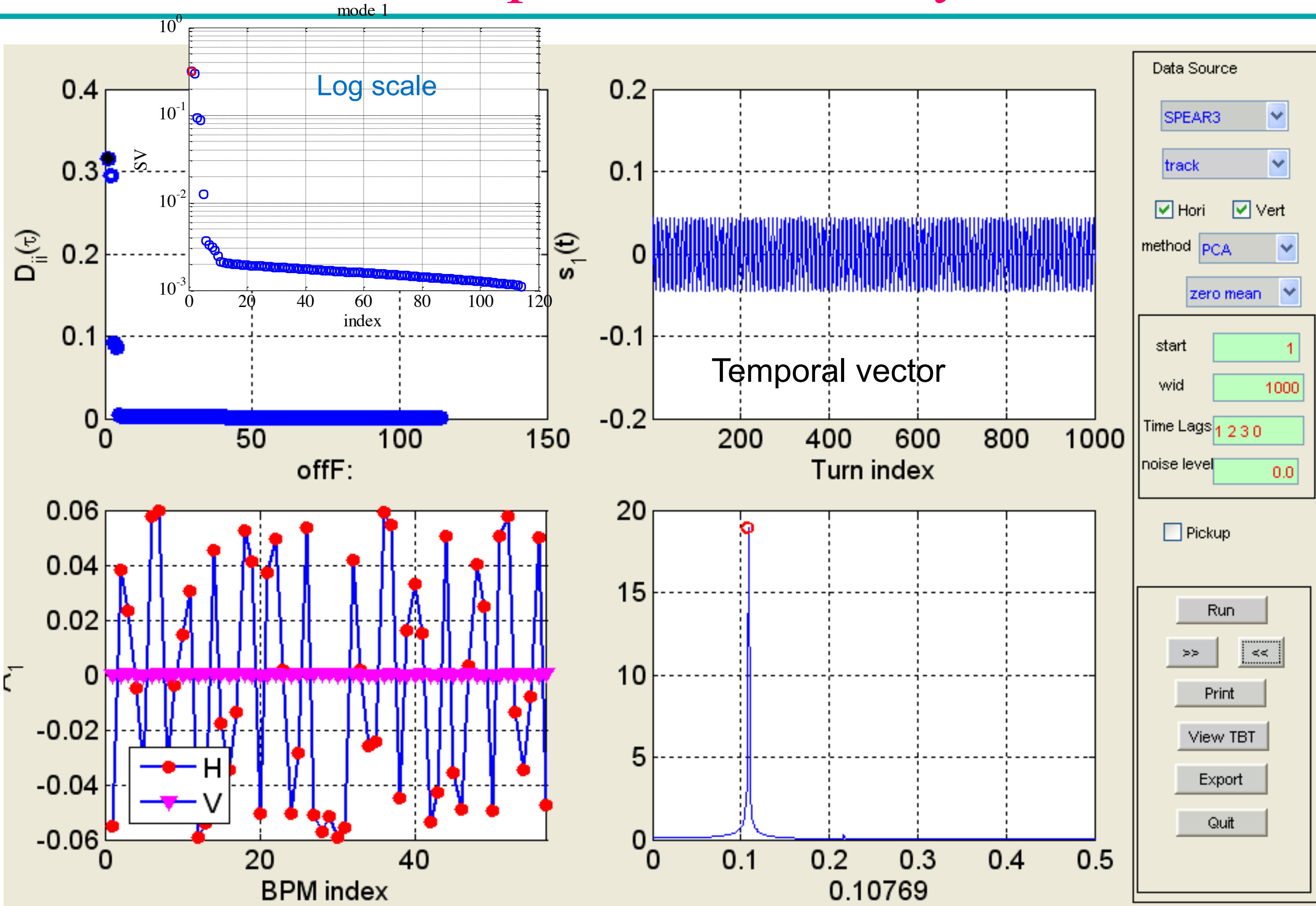
Keep 10 out of 114 modes.



The noise level (sigma) is reduced (keeping p out of 2m modes) to

$$\sigma_n = \sigma\sqrt{\frac{p}{2m}}$$

Example of SVD analysis



Spatial vector

This data set is from tracking the SPEAR3 lattice with added random noise (sigma=0.05 mm). You will play with this program (and the data sets) in the computer-lab class.

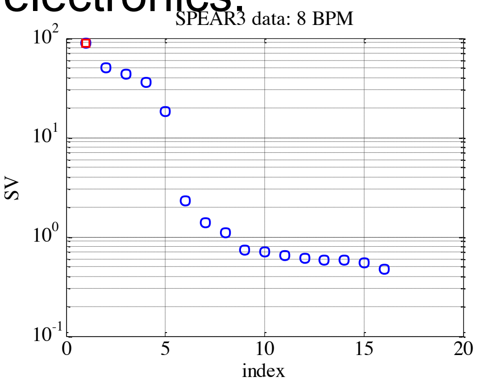
Limitation of the PCA method

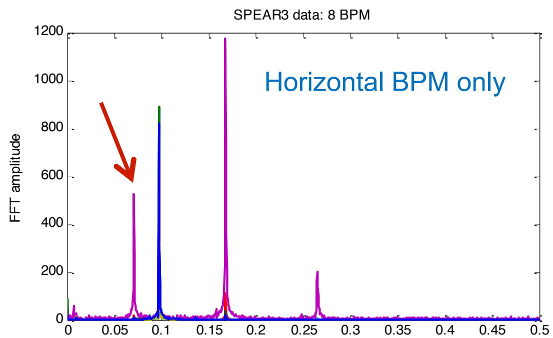
The eigen-modes are determined by the orthogonality and variances (strengths) of the components. If two signals have nearly the same strengths, they will be mixed in the eigen-modes (degeneracy in eigen-analysis). In reality this is common:

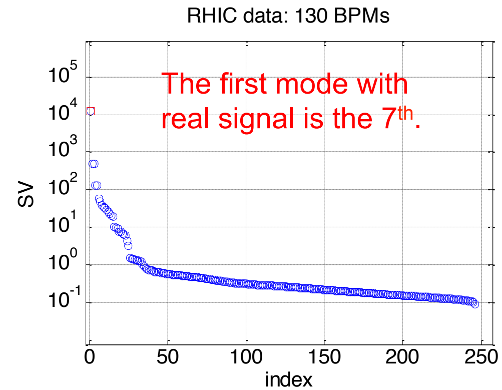
(1) Horizontal and vertical betatron modes can be mixed.

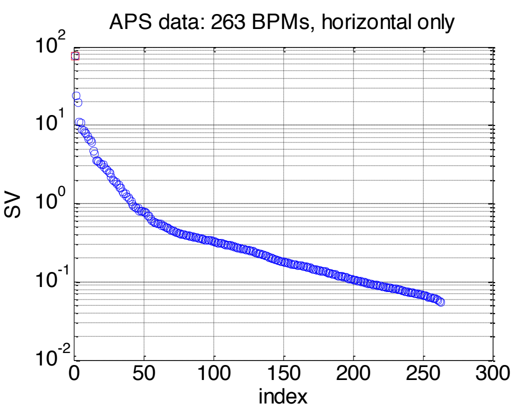
(2) Betatron modes can be mixed with the synchrotron mode.

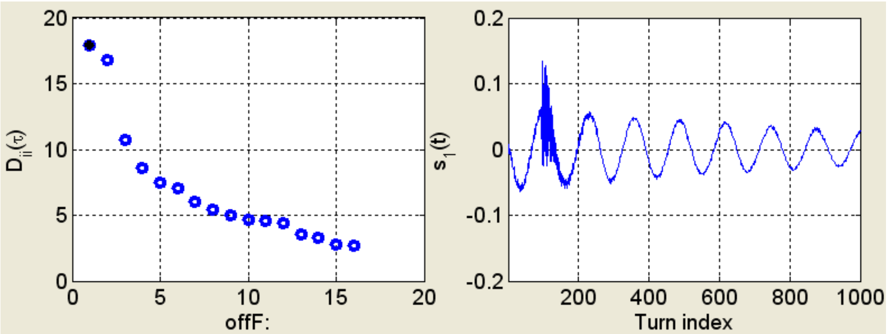
(3) Actual BPM data are often plagued by signal contamination or failing electronics.











The independent component analysis (ICA)

- The source signals are assumed statistically independent.

$$p(x_1, x_2) = p(x_1)p(x_2)$$

This is a strong condition that the PCA analysis does not make full use of.

$$E\{h_1(x_1)h_2(x_2)\} = E\{h_1(x_1)\}E\{h_2(x_2)\}$$

For any function h_1,h_2.

PCA only requires the components to be linearly uncorrelated, i.e., the covariance between two variables is zero.

$$E\{x_1 x_2\} - E\{x_1\}E\{x_2\} = 0$$

For two Gaussian variables, uncorrelatedness is equivalent to independence. Many ICA algorithms exploit the non-gaussianity of the signals, such as fastICA.

It is possible to use non-gaussianity based methods for BPM data analysis. But we will focus on an algorithm that relies on the time-spectrum of the source signals.

The Principle

- The source signals are assumed to be narrow-band with non-overlapping spectra, so their un-equal time covariance matrices are diagonal.

$$\langle \mathbf{s}(t)\mathbf{s}(t+\tau)^T \rangle = \mathrm{diag}[\rho_1(\tau), \rho_2(\tau), \ldots, \rho_n(\tau)]$$

Since $\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t)$

$$\mathbf{C}_x(0) \equiv \langle \mathbf{x}(t)\mathbf{x}(t)^T \rangle = \mathbf{A}\mathbf{C}_s(0)\mathbf{A}^T + \sigma^2\mathbf{I}$$

$$\mathbf{C}_x(\tau) \equiv \langle \mathbf{x}(t)\mathbf{x}(t+\tau)^T \rangle = \mathbf{A}\mathbf{C}_s(\tau)\mathbf{A}^T, \tau \neq 0$$

The mixing matrix A diagonalizes the un-equal time sample covariance matrices simultaneously.

The Algorithm* - 1

- Diagonalize the equal-time covariance matrix (data whitening)

D_1,D_2 are diagonal

Set to remove noise

$$\mathbf{C}_x(0) = [\mathbf{U}_1, \mathbf{U}_2] \begin{bmatrix} \mathbf{D}_1 & \\ & \mathbf{D}_2 \end{bmatrix} [\mathbf{U}_1, \mathbf{U}_2]^T \quad \text{with} \quad 0 \le \max(\mathbf{D}_2) < \lambda_c \le \min(\mathbf{D}_1)$$

Construct an intermediate "whitened" data matrix

$$\mathbf{z} = \mathbf{D}_1^{-\frac{1}{2}} \mathbf{U}_1^T \mathbf{x} = \mathbf{V}\mathbf{x} \qquad \text{which satisfies} \qquad < \mathbf{z}\mathbf{z}^T >= \mathbf{I}$$

This pre-processing step is just PCA. Matrix z contains the temporal vectors.

* The second order blind identification (SOBI) algorithm of A. Belouchrani, et al. in IEEE Trans. Signal Processing, 48, 900, (2003).

The Algorithm - 2

- Jointly diagonalize* the un-equal time covariance matrices of matrix z of selected time-lag constants.

$$\mathbf{C}_z(\tau) = \mathbf{W}\mathbf{C}_s(\tau)\mathbf{W}^T \quad \text{for} \quad \tau = \{\tau_i \mid i = 1,2,\cdots,k\}$$

Then

$$\mathbf{s} = \mathbf{W}^T\mathbf{V}\mathbf{x} \quad \text{and} \quad \mathbf{A} = (\mathbf{U}_1\mathbf{D}_1^{\frac{1}{2}})\mathbf{W}$$

The columns of A (spatial vectors) and corresponding rows (temporal vectors) of s are the resulting modes.

*Algorithm for joint diagonalization can be found in J.F. Cardoso and A. Souloumiac, SIAM J. Matrix Anal. Appl. 17, 161 (1996)

Linear Lattice Functions Measurements

- There are two betatron modes because each BPM sees different phase.

The betatron component $x = A_{b1}s_1 + A_{b2}s_2$

Beta function and phase advance

$$\beta = a(A_{b1}^2 + A_{b2}^2) \qquad \psi = \tan^{-1}\left(\frac{A_{b1}}{A_{b2}}\right)$$

- There is one synchrotron mode.

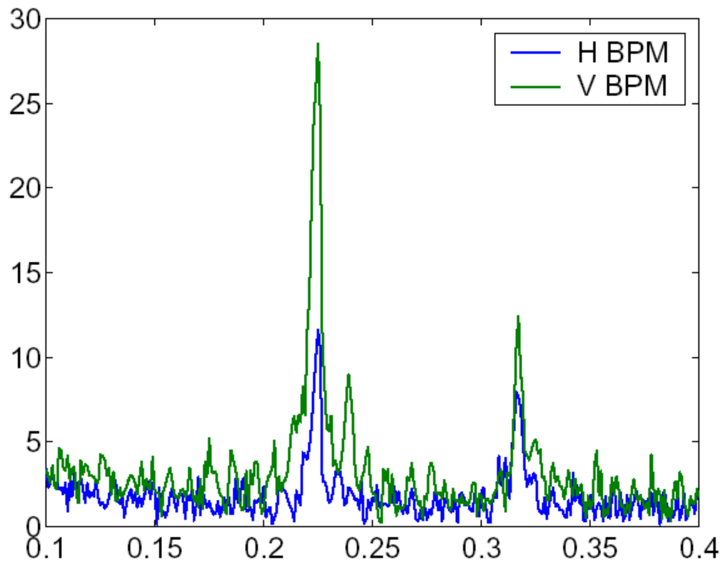
The synchrotron component $x = A_l s_l$

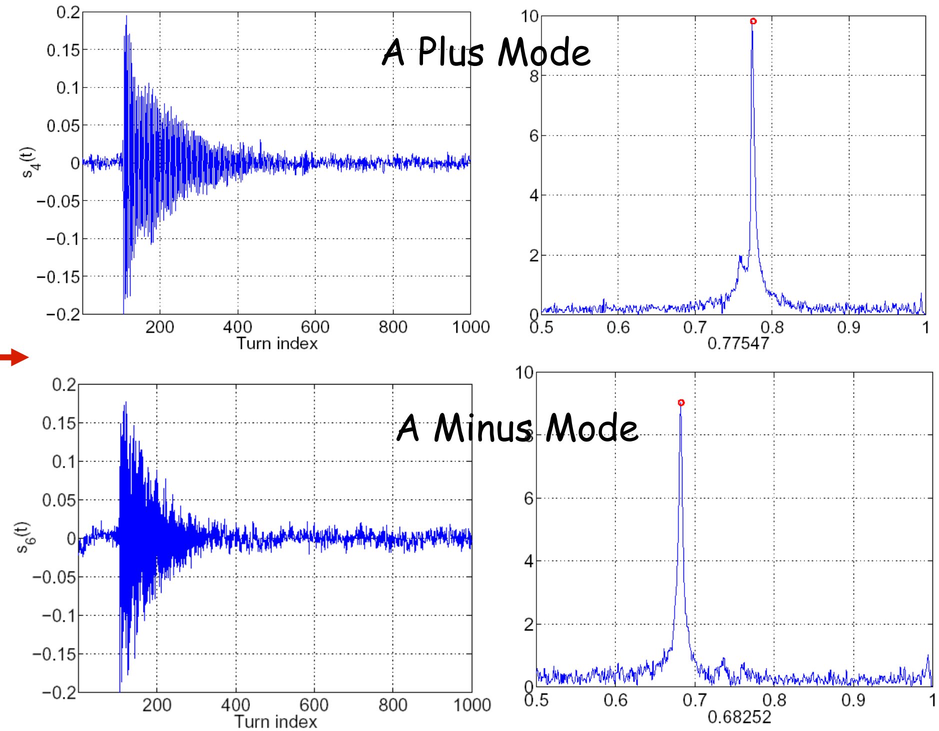
Dispersion function and momentum deviation

$$D_x = bA_l \qquad \delta = \frac{s_l}{b}$$

Example: de-coupling

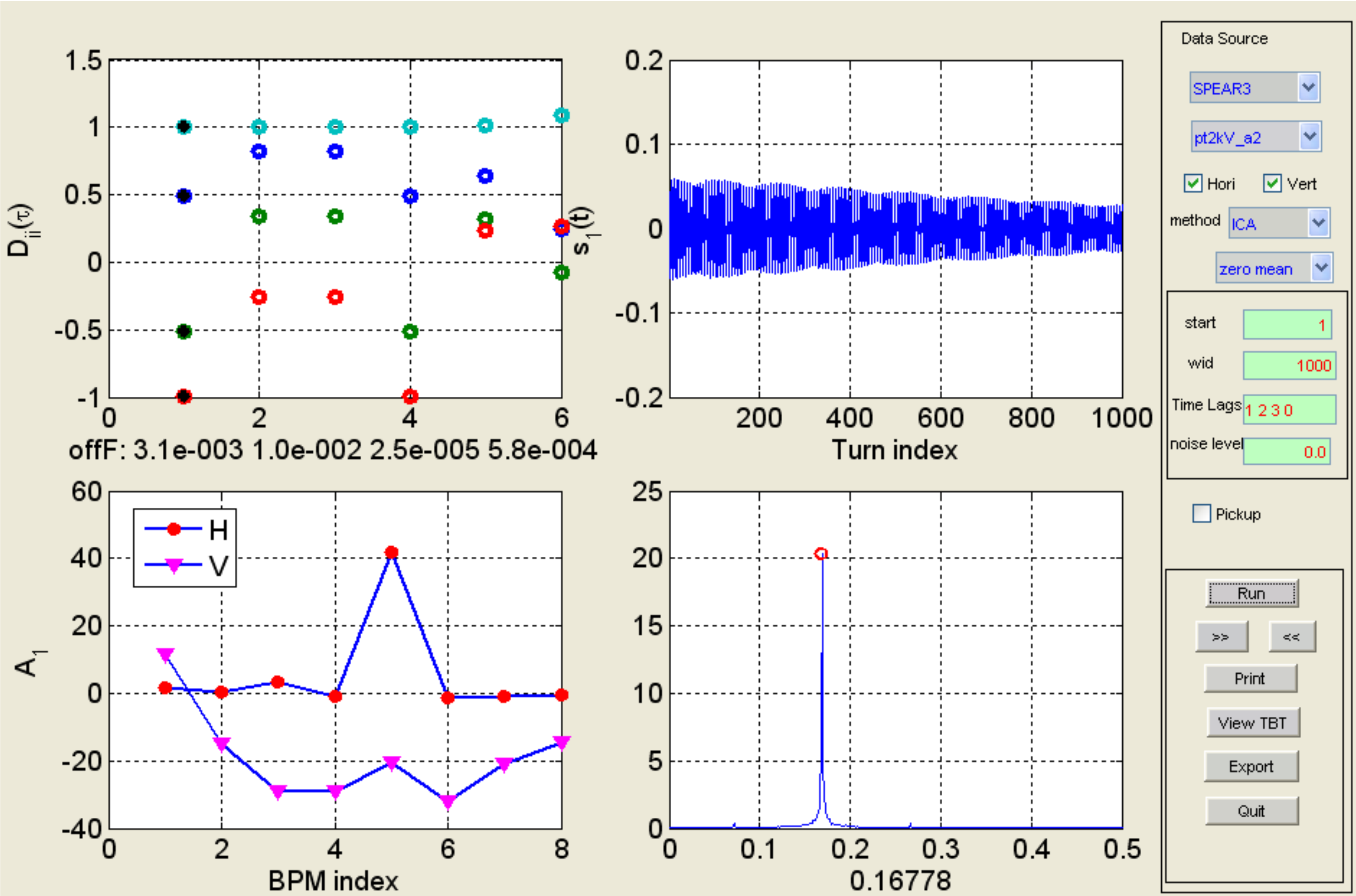
The ICA method can de-couple the normal modes in presence of linear coupling.



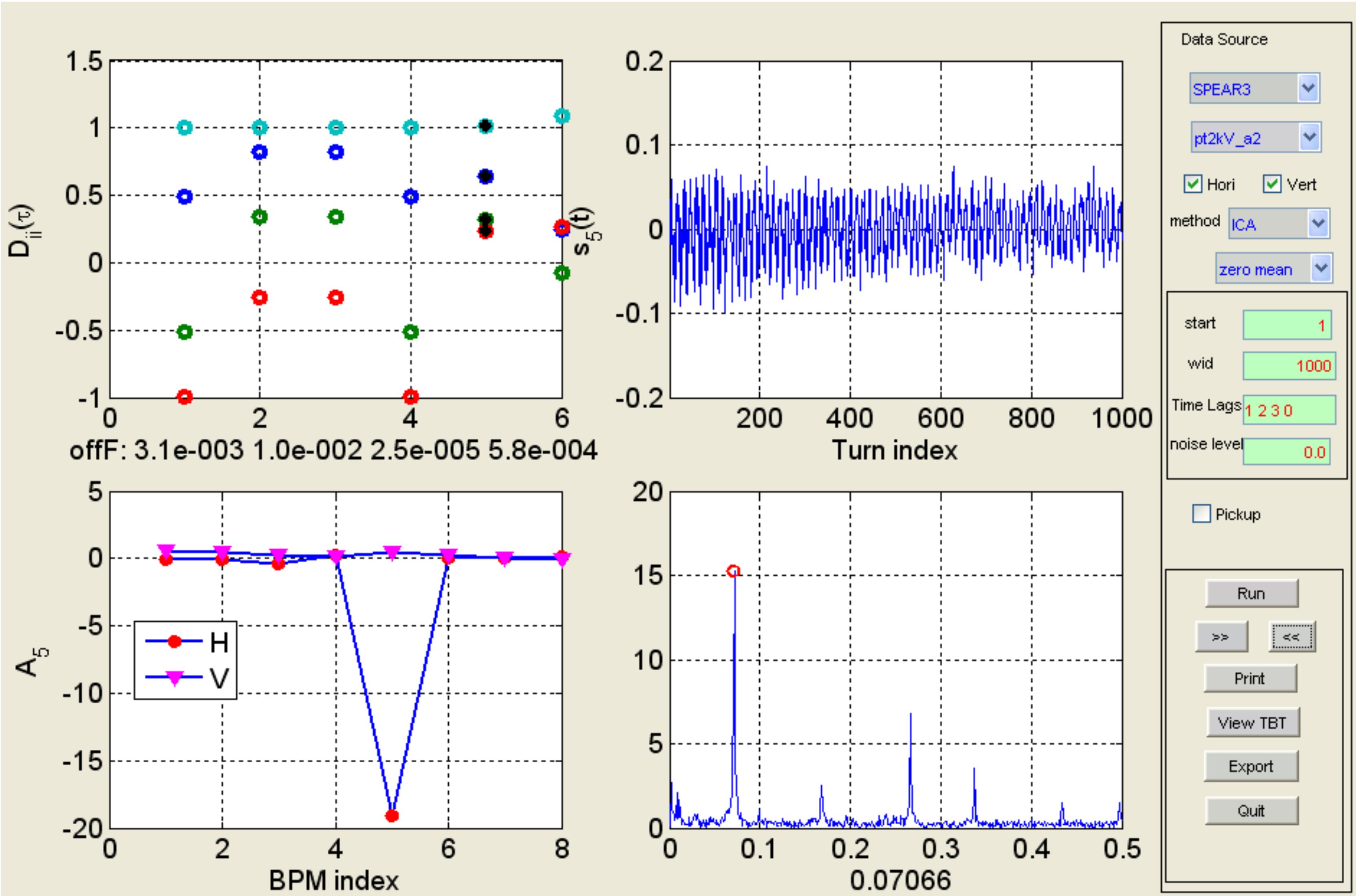


FFT spectra of raw horizontal and vertical BPM signals at section L1. Both BPMs see a mixture of the "plus" mode and "minus" mode.

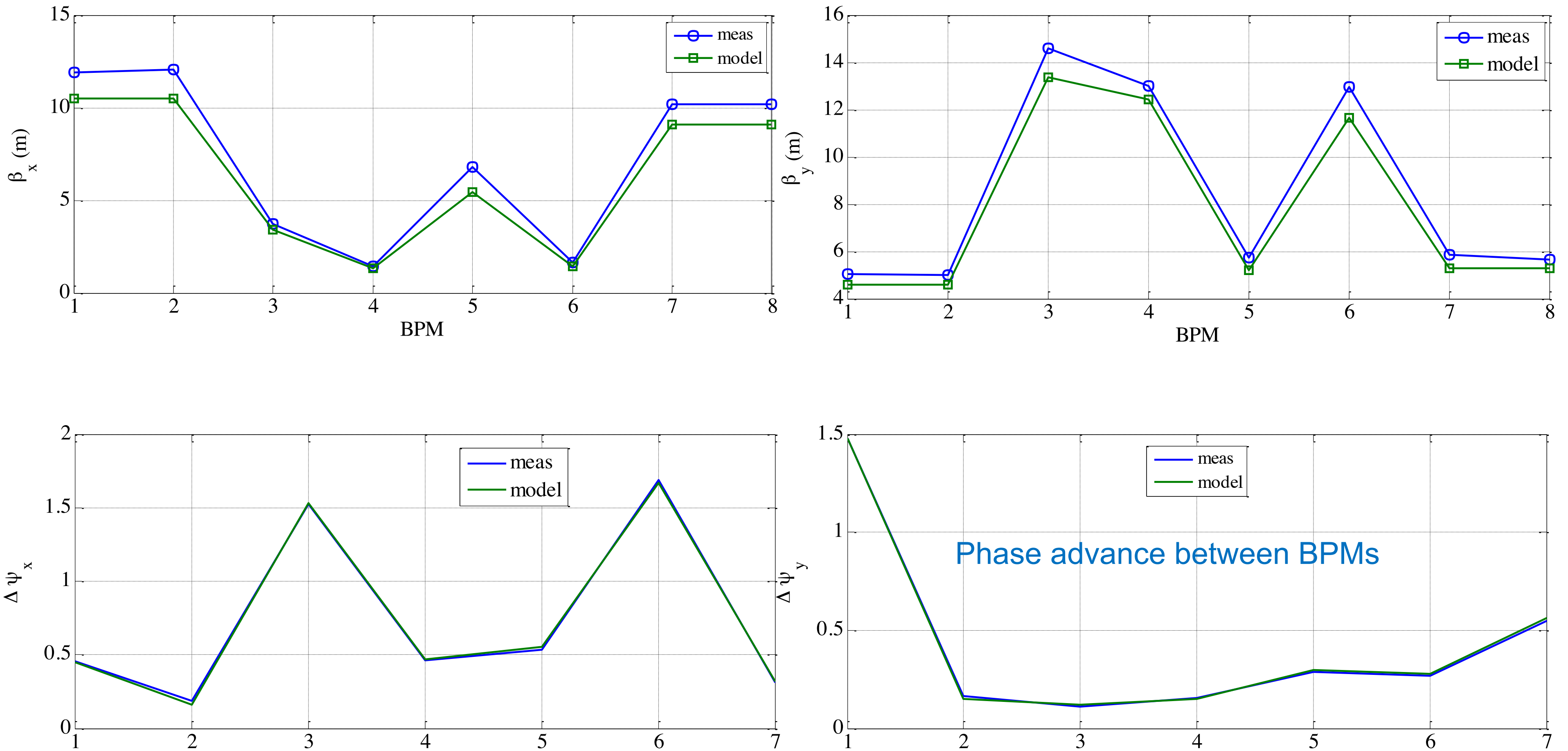
Example: SPEAR3 data



SPEAR3: the contaminated BPM

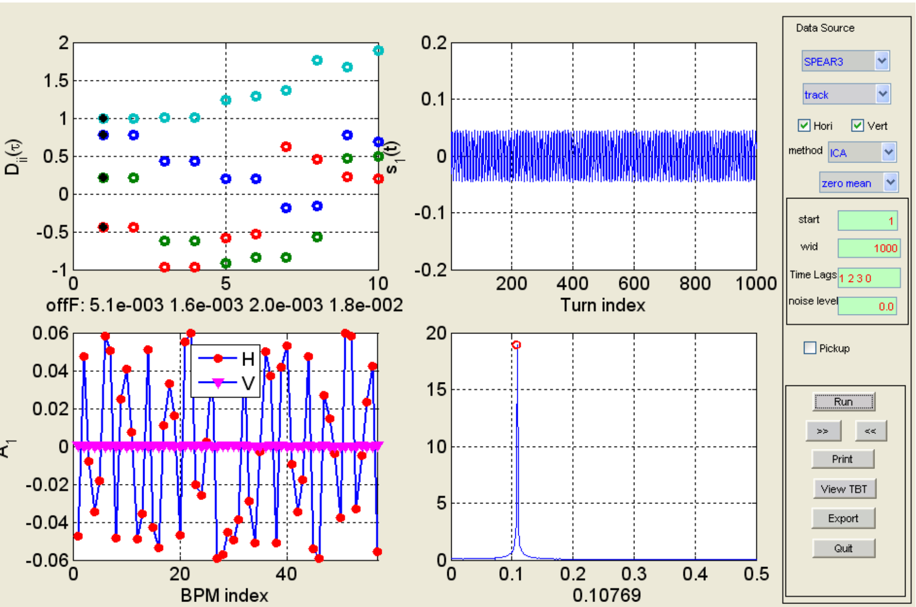


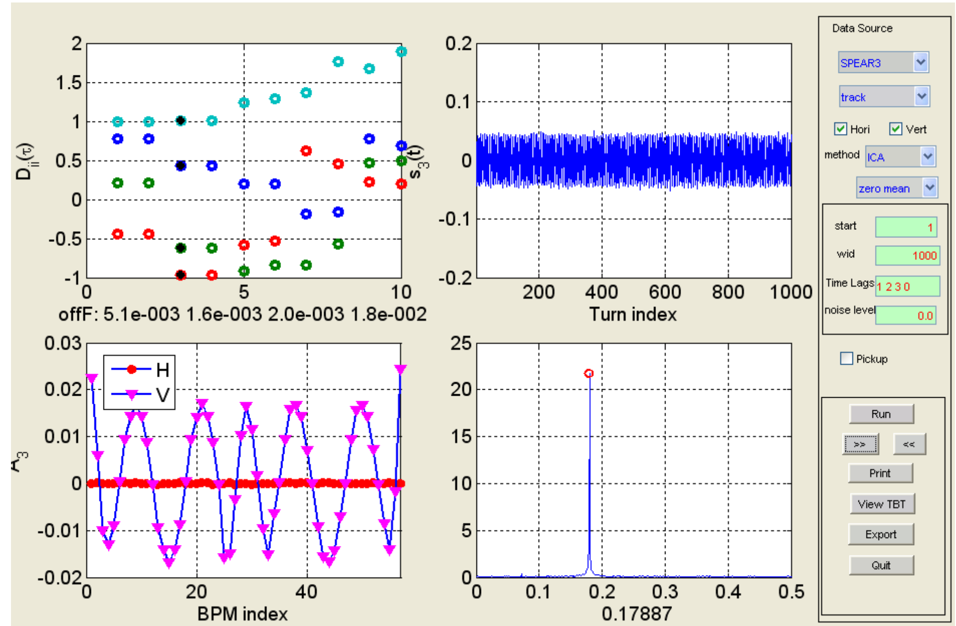
SPEAR3: the measured phase advance



There are a BPM gain errors. But the phase advances are in excellent agreement with the model.

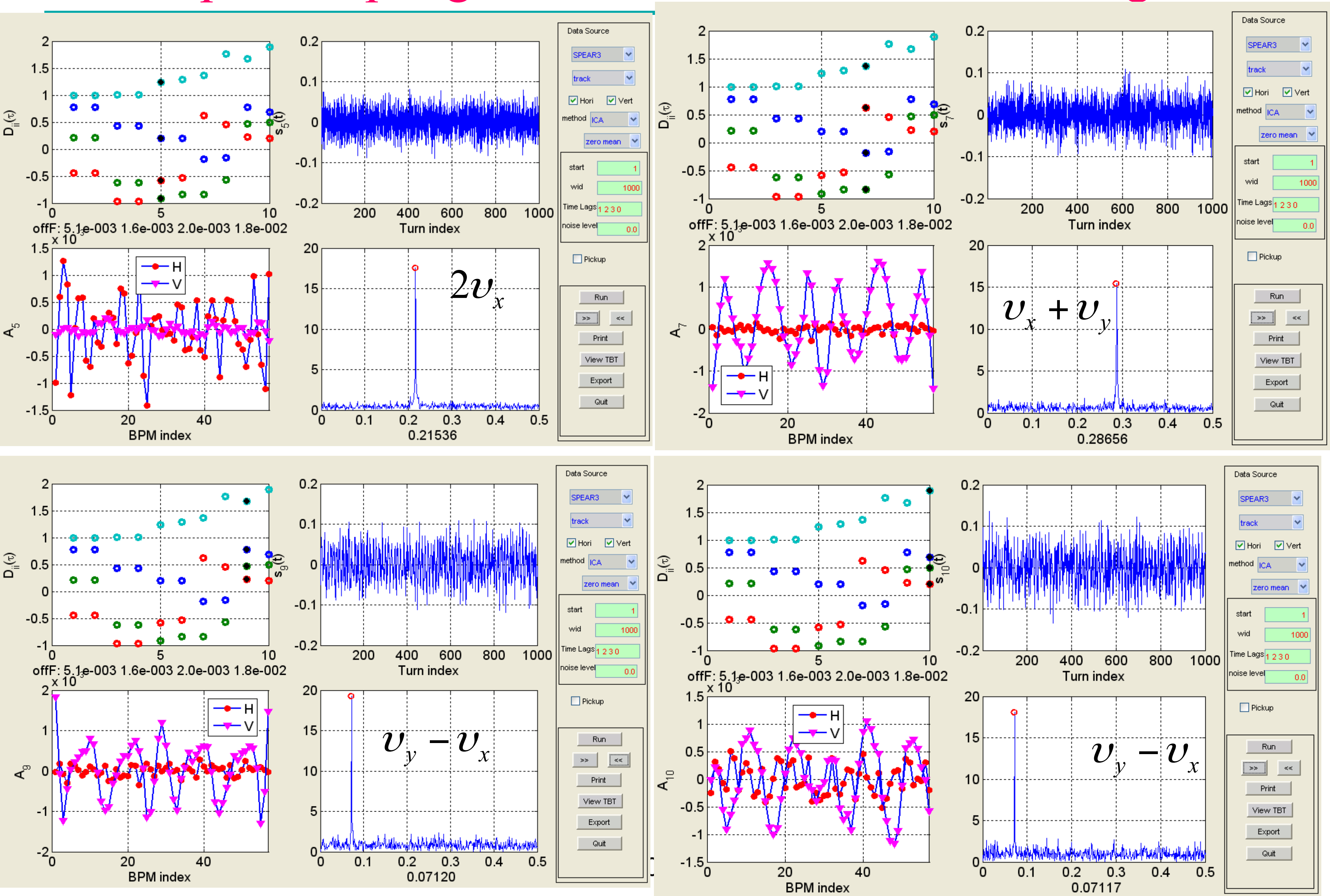
Example: Nonlinear modes in tracking data



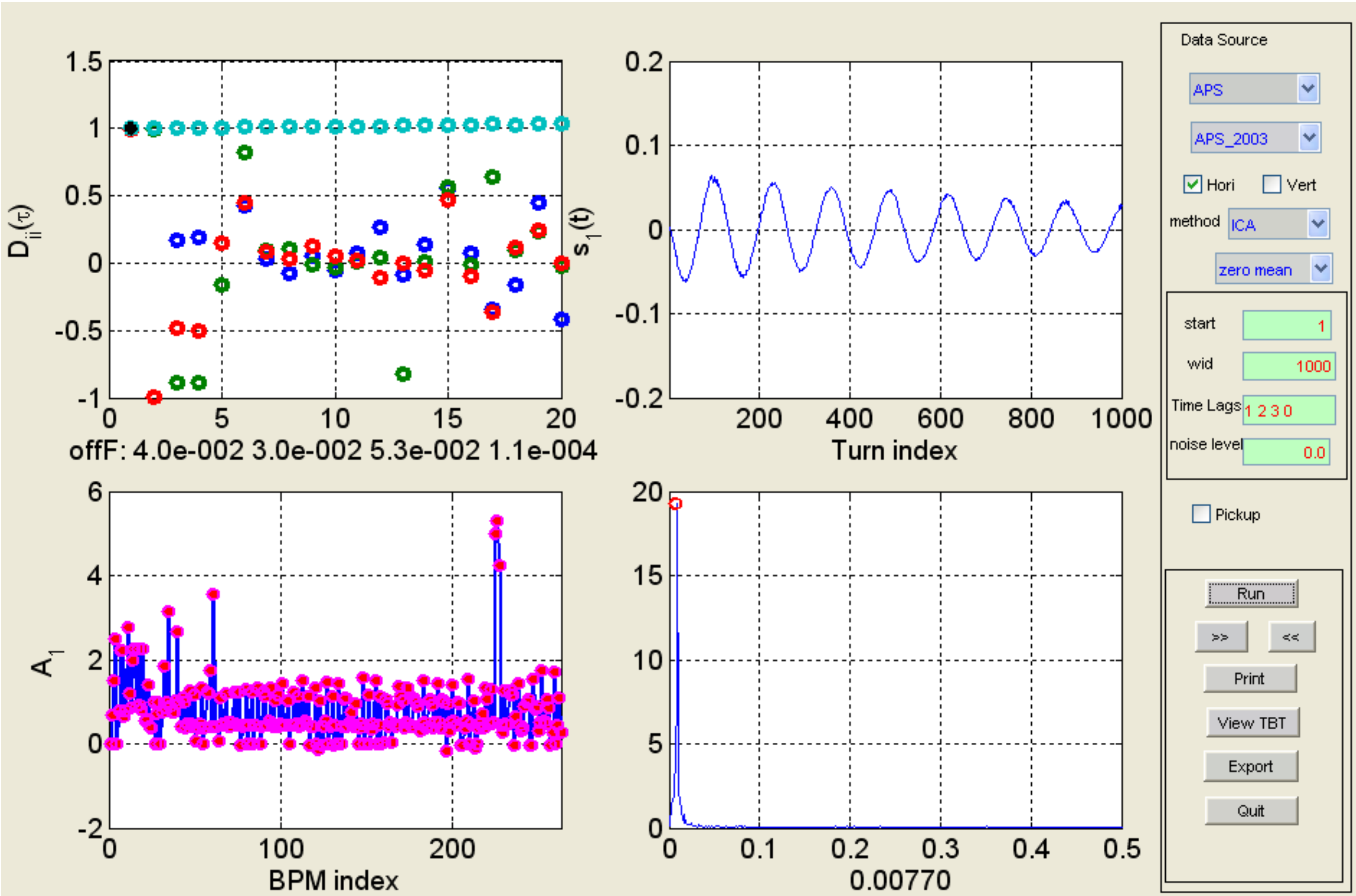


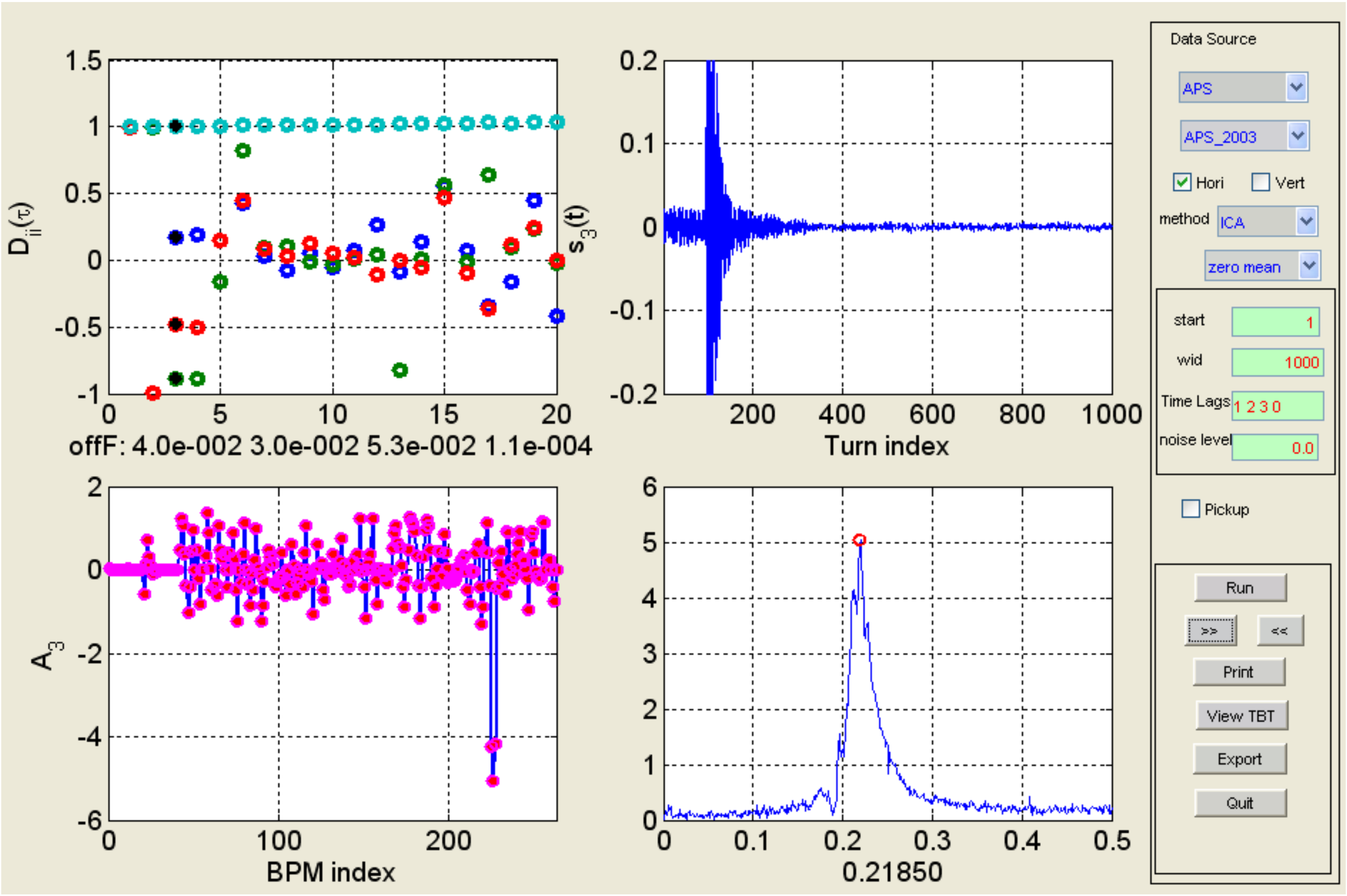
The betatron modes.
Data from tracking SPEAR3 model. There are 57 BPMs.

Example: coupling and nonlinear modes in tracking data



Application: APS data





$D_{ii}(\tau)$
offF: 4.0e-002 3.0e-002 5.3e-002 1.1e-004
$s_3(t)$
Turn index
A_3
BPM index
0.21850
Data Source
APS
APS_2003
Hori
Vert
method
ICA
zero mean
start
1
wid
1000
Time Lags
1 2 3 0
noise level
0.0
Pickup
Run
>>
<<
Print
View TBT
Export
Quit

From Today's computer lab